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Husserl and Bourbaki on Mathematics: Similarities, Possible Influence and some Dissimilarities

Guillermo E. Rosado Haddock
University of Puerto Rico at Río Piedras

Abstract

Husserl's mature philosophy of mathematics has, on the logic side the influence of Leibniz, Bolzano, Lotze and Hume, and on the mathematics side the influence of Leibniz and Riemann. What is not clear are the influences Husserl's views exerted on those of later researchers. There is, however a remarkable similarity between Husserl's conception of mathematics as a theory of structures and the views of the school of Bourbaki. Was there some direct or indirect influence of Husserl on the Bourbakians?

Keywords: Bourbaki, Husserl, logic, mathematics, theory of structures

1. Introduction

The evolution of Husserl's views on logic and mathematics from his youth work, *Philosophie der Arithmetik* (Hua XII), to his mature views of the first volume of his *opus magnum Logische Untersuchungen* (Hua XVIII and XIX), expounded also much later in the first part of *Formale und transzendentale Logik* (Hua XVII), as well as in his posthumous *Einleitung in die Logik und Erkenntnistheorie* (Hua XXIV) and *Logik und allgemeine Wissenschaftstheorie* (Hua XXX) has been the matter of some discussion, including distortions and superficial renderings by scholars (and Fregean fans)[1] working in the so-called analytic tradition, and some of which never studied Husserl seriously.[2]

The fact of the matter, as has been pointed out many times by the present author and by other Husserlian scholars, is that *Philosophie der Arithmetik* is basically a work stemming from Husserl's professorship's thesis of 1887, *Über den Begriff der Zahl*[3], and corresponds to Husserl's views at most up to 1890, being that the main reason why the second planned volume of that work was never published and it seems that never written. In fact, if one examines the writings included in Husserl's posthumous book *Studien zur Arithmetik und Geometrie* (Hua XXI), one can very well trace the evolution of some of Husserl's views on those mathematical disciplines from 1886 to 1894. On the other hand, if one reads Husserl's posthumous paper 'Zur Logik der Zeichen (Semiotik)'[4], written in 1890 and his critical review of the first volume of Ernst Schröder's *Vorlesungen über die Algebra der Logik*[5], which was almost surely written in 1890, since it was already in press in January of 1891 when Frege's 'Funktion und Begriff'[6] was published, there is absolutely no doubt that Husserl discovered the distinction between, in Frege's terminology, 'sense and reference (better: referent)' with complete independence of Frege and probably at the same time of his a decade older rival. In fact, that distinction was clearly anticipated by Bolzano, as pointed out by the present author in a recent paper.[7]

In fact, as Husserl pointed out in the first volume of *Logische Untersuchungen* (Hua XVIII, Ch. X, §§ 60-61, and Appendix; see also Hua XIX, 35-38) and elsewhere, Leibniz, Bolzano, Lotze and Hume were the philosophers who played a decisive role in making Husserl abandon the mild Brentanian psychologism of his *Philosophie der Arithmetik*, not the 1894 late review of that book by Frege[8]. By the way, Husserl's mature conception of logic, mathematics and their relationship dates precisely from 1894 and is clearly different from Frege's. Husserl was never a logicist, and not even a reductionist.

2. The Influence on Husserl of both Leibniz and Riemann

Husserl's conception of the relation between logic and mathematics is certainly different from Frege's, though not unrelated, In fact, both are heirs, as is also David Hilbert, of the seminal contributions to philosophy of the great German mathematician and philosopher Gottfried Leibniz. From the time of its early systematization by Aristotle –and even earlier- in Ancient Greece, logic was conceived as a philosophical discipline with little relationship to mathematics. Moreover, whereas mathematics grew gradually from its early origins in Greece, India and the Middle East, until the revolutions made by Descartes (analytic geometry) and Newton and the same Leibniz (differential and integral calculus) immensely accelerated that process, logic remained basically the same from Aristotle's systematization to the nineteenth century. However, precisely Leibniz had already somehow anticipated the modern view of logic and mathematics by bringing them together as fundamentally intertwined in his conception of a *mathesis universalis* (see Leibniz 1982).[9] That conception of the essential connection between logic and mathematics was taken by the great philosopher and mathematician Bernard Bolzano[10], and later developed in more concrete and diverse fashions by the three illustrious intellectual grandsons of the great Leibniz –and, thus, intellectual cousins – Gottlob Frege, Edmund Husserl and David Hilbert. The three intellectual cousins were originally mathematicians, who turned to philosophy in different degrees. Hilbert was certainly the only one who remained essentially a mathematician and, by the way, probably the greatest mathematician of the first half of the twentieth century. Frege remained a mathematics professor all his life, but his research was essentially in logic and philosophy, being certainly one of the greatest logicians ever, as well as one of the best and most influential contemporary philosophers. Husserl, on the other hand, made the turn from mathematician to philosopher more completely than the other two, being a philosophy professor all his life and, by the way, being one of the greatest philosophers ever.

Hilbert tried to develop logic and arithmetic at the same time, as parts of a common discipline, without clearly articulating their relationship (see Hilbert 1964 and 2013). Frege articulated the relation between logic and mathematics in a much clearer fashion. Non-geometrical mathematics can be obtained analytically, by definitions and derivations, from logic. The latter is the mother discipline, while non-geometrical mathematics is the daughter discipline.[11] That conception has been baptized "logicism", and since Frege – and also since Richard Dedekind – has played an important role in the discussions on the philosophy of mathematics. And since Frege was not only a logicist, but also a Platonist in the philosophy of mathematics, he was forced to introduce so-called "logical objects", and to conceive the truth-values – the true and the false – as logical objects *par excellence*.

As Leibniz, Bolzano, Frege and Hilbert, Husserl also conceived logic and mathematics as strongly related. But his conception was more articulated than those of his predecessors and his two contemporaries, and that was partially due to the fact that Husserl had another strong intellectual influence from another source, namely, from one of the greatest mathematicians of the nineteenth century, and interestingly not from his teacher Karl Weierstraß, but from Bernhard Riemann.

Already in a letter to his teacher Brentano of the 29th of December of 1892 Husserl informed him that he had accepted Riemann's twofold conception of the nature of geometry, namely, (i) that from a mathematical point of view all geometrical structures, be it of three, four or n dimensions, be it of zero, negative or positive curvature stand at the same level, and geometry in the mathematical sense is the study of all those different sorts of geometrical manifolds (or structures); and (ii) that with respect to physical space one cannot decide *a priori*, but only empirically whether it has zero, negative or positive curvature, as well as three, four or whatever dimensions. In later letters of the 29th of March of 1897 and the 7th of September of 1901 to Paul Natorp[12] – thus, clearly before the advent in 1905 of Einstein's special relativity, Minkowski's 1908 refinement and Einstein's and Hilbert's 1915 general relativity – Husserl reasserted such convictions. On this point Husserl and his since 1901 near friend Hilbert were far ahead of their stubborn older intellectual cousin Frege, who in a paper written between 1902 and 1906, but published only posthumously, compared non-Euclidean geometries to alchemy and astrology.[13]

3. Husserl's Conception of Logic: a Brief Survey

Although our interest here is mainly on Husserl's conception of mathematics and that of the Bourbaki group, a few words have to be said about Husserl's conception of logic, in order to explain Husserl's understanding of the *mathesis universalis* and contrast it to that of Frege.[14]

First of all, Husserl was neither a logicist and, thus, did not need to try to derive mathematics from logic, nor was he a logical Platonist, thus, had no necessity to postulate the existence of any so-called logical objects. To say it briefly, for Husserl there were no logical objects. Logic was for Husserl a syntactic-semantic discipline, based on what he called "meaning categories", which are on the basis of the formation of all sorts of sentences. Besides the formation of elementary (or atomic, in contemporary parlance) sentences, the most important aspect of the formation of the first and fundamental stage in the edification of the logical syntactical-semantic building is what Husserl calls somewhat negatively "the laws that protect against nonsense". Those laws allow us to form complex sentences from more elementary sentences with the help of what are now called logical connectives. Thus, beginning with elementary sentences, by means of the reiterated application of the logical connectives, one could form complex sentences of any finite level of complexity. It should be perfectly clear for anyone with a minimum of knowledge of logic and of contemporary analytic philosophy that this first level of the logical building is that of what Carnap, without citing Husserl or even including *Logische Untersuchungen* in the bibliography, called "formation rules" in his *Logische Syntax der Sprache*.[15]

The second level of the logical building was for Husserl that – once more negatively expressed- of the laws that protect against formal countersense, that is, against contradiction, and more positively expressed, guide derivations. These are what Carnap in *Logische Syntax* called, once more without any reference to Husserl, "transformation rules", and which, as the formation rules, are now part of the standard rigorous presentations of logic in textbooks. Husserl called this part of logic "apophantic logic", that is, the theory of the proposition (or of the sentence), and in more modern parlance could have been called syntax or theory of deduction.

In *Logische Untersuchungen* Husserl had still not neatly distinguished between syntax and semantics. This distinction was clearly made in *Formale und transzendentale Logik* when he added above the level of apophantic logic a level of the logic of truth. One obtained this level from the previous one by introducing the notion of truth and related notions of a semantic flavour. On this point Husserl also anticipated a little what occurred a few years later at the hands of the great Tarski.

But though logic was a syntactical-semantic discipline and mathematics was not derivable from logic, it does not mean that logic and mathematics were separated from each other. For this intellectual grandson of Leibniz, logic and mathematics were very related, but not as mother and daughter as in Frege, but as sister disciplines. Mathematics, geometry included, was also a formal discipline, though not a syntactical-semantic discipline, but an ontological one. Mathematics was a sort of ontological counterpart of logic, the ontologically fat sister discipline of logic, which Husserl used to call “formal ontology”.

4. Husserl’s Views on Mathematics as a Theory of Structures

Husserl considered mathematics a formal ontology. From an etymological standpoint that means a domain of purely formal objects, in contrast to the regional (material) ontologies that are the objects of study –or prospective objects of study- of the material sciences. But what was meant by ‘formal ontology’ was a plurality of formal structures. Husserl had generalized Riemann’s conception of geometry as the study of geometrical manifolds or structures to the whole of mathematics. For Husserl there was a plurality of fundamental formal-ontological categories, which served as the building blocks of the most basic and fundamental mathematical disciplines. This point should be stressed, since Husserl never envisaged the reduction of all mathematical concepts to a single one: he was certainly not a reductionist. The lists of formal-ontological categories – as he called them- fluctuated a little from exposition to exposition, but it usually included the notions of set, relation, whole and part, and of number (presumably cardinal number) and of ordinal number.

On this point, mathematicians, logicians and philosophers schooled in the set-theorist tradition would certainly point out that the notions of cardinal and ordinal number, as well as the notion of relation can be defined in terms of that of set. Leaving aside whether those definitions are natural or somewhat forced, it should, firstly, be pointed out that the notions of set and of relation, and with them the fundamental mathematical notion of function are really interderivable. In set-theoretical mathematics a function of n arguments can be defined as a relation of n+1 arguments univocally determined in its last argument. On the other hand, Frege defined the notion of relation in terms of the notion of function: a relation of n arguments being a function of the same number n of arguments, whose value is a truth-value.[16] But the notion of set can also be defined, as pointed out by Saunders Mac Lane[17], in terms of that of relation. In fact, the notion of set can also be defined directly in terms of that of function.[18] And as is very well known, the notion of set can be defined in terms of the notion of category, as shown in any textbook on category theory.

The most interesting and less considered of the formal-ontological categories is that of whole, or if you prefer, of whole and part. Probably most mathematicians do not consider the notion of whole a mathematical notion, probably because it is too loosely characterized. Nonetheless, firstly it should be pointed out that the great Polish logician Stanislaw Lesniewski developed a theory of parts and wholes, a mereology, as a fundamental part of his alternative logical building and presumably a nominalist replacement of set theory. On the other hand, in *Logische Untersuchungen* Husserl considered a somewhat particular case of the notion of whole, namely, the notion of extensive whole, which certainly admits a formal mathematical treatment, and even Whitehead considered a rigorous treatment of a theory of parts and wholes in his theory of the extensive continuum (see Whitehead (1979, 294-301). One could ask whether the notion of (extensive) whole is definable in terms of that of set or (and) the other way around.[19]

Continuing with Husserl, the formal-ontological categories give rise to the fundamental mathematical disciplines, for example, to set theory, to a mereology, to a theory of relations, to cardinal number theory. The remaining mathematical disciplines are obtained, according to Husserl by one of two procedures or by a combination of the two procedures. Firstly, we obtain new areas of mathematics by specialization. Secondly, we obtain new areas of mathematics by bringing together two or more of the fundamental mathematical disciplines. And thirdly, we obtain new areas of mathematics by combining the two procedures of specialization and of bringing together structures to form more complicated structures.[20]

5. Bourbaki's Views on Mathematics as a Theory of Structures

The collective French mathematician Nicholas Bourbaki is certainly one of the most distinctive components of twentieth century mathematics. In the early 1930s a group of young but already distinguished French mathematicians, among them Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Dieudonné, André Weil, Charles Ehresmann and René de Possel, organized the Bourbaki group, whose project was to rewrite the whole of mathematics under strict foundations.[21] The group was constantly renewed, both because new members were added and others opted to abandon the group, as well as because the organizing members established an age limit of 50 for all (original and future) members of the group. Due to the development of set theory as a founding discipline during the first decades of the twentieth century at the hands of Ernst Zermelo, Abraham Fraenkel, John von Neumann and others, the notion of set was taken as the most basic mathematical concept. Nonetheless, there was no attempt at reducing mathematics to set theory, but instead the notion simply served as the language used for the introduction of the basic mathematical structures, what the Bourbakians called the "mother structures".[22]

According to Bourbaki, the mother structures were threesome, namely, algebraic, topological and order structures. These were the ground structures. All other mathematical structures were obtained from them by three processes with which the reader should already be familiar, namely, the specialization of a structure, the connecting of two (or more) structures and the combination of those two procedures. Thus, Hausdorff spaces are specializations of topological spaces, being uniform spaces and metric spaces further specializations in that order, whereas topological groups are structures obtained by connecting topological and algebraic structures. Banach spaces and the structure of the real numbers bring together specializations of different sorts of mathematical structures.

It is easy to see that the conception of mathematics as a theory of structures of the Bourbaki group is very similar to that of Husserl even in fundamental details.[23] In both cases there are what the Bourbaki group called "mother structures" –three in the case of the Bourbaki group, a not definitely determined, but probably a little larger number in the case of Husserl. The rest is basically identical, namely, one can obtain other structures either (i) by specialization of the mother structures –that is: incorporating additional structure-, (ii) by connecting two or more mother structures to form a complex one, or (iii) by combining the two procedures of specialization and connection of structures at any level, for example, bringing together specializations of mother structures to form complex less abstract structures, or obtaining specializations of complex structures resulting from the connection of mother structures, etc.

6. Small but Important Divergences

From a purely theoretical –not necessarily historical- standpoint, the Bourbaki group's conception of mathematics as a theory of structures can be seen as an elaboration or refinement of Husserl's views. It seems as if the Bourbaki mathematicians had made Husserl's views more precise. Whether a historical link could also be traced is another matter, one that we will touch briefly below.

There are however, some small differences of detail, on which we want to dwell now. The most obvious difference between the two theories of structures is that the Bourbaki group, probably influenced by the development of set theory, takes the notion of set as the basic notion that even the mother structures should take for granted. The three mother structures

–topological, algebraic and order structures- are, thus, not based on a fundamental mathematical notion, but on a notion that can be expressed in set-theoretical language. On the other hand, in Husserl's theory the notion of set is only one of the fundamental notions of mathematics and each fundamental notion originates a mother structure. Set theory is just one of maybe five or six mother structures, though such a number could be reduced in view of the definability of some of the candidates for formal ontological categories –for example cardinal number and ordinal number- in terms of the notion of set and the interdefinability of others – namely, the notions of relation and function- with the notion of set.

A second seemingly small but very important difference is the inclusion by Husserl of the notions of whole and part in the list of formal ontological categories, and the urge to develop a mathematical theory of wholes and parts. Tough as stated above, in some mathematical contexts the notions of set and of whole seem not to be clearly distinguished –is the spatial continuum the set of which a point is a member or the whole of which the point is a part?-, the notions are not only different but not definable in terms of the other in a non-artificial way. Of course, in Bourbaki's views, as in those of all of current mathematics, the notions of whole and part are not mathematical. However, the fact that Lesniewski could develop a formal theory of wholes and parts should be a reminder for mathematicians that such a mathematical theory is feasible and not an unfounded speculation of Husserl.

Things get more interesting if we bring to the fore the most important structuralist rival of Bourbaki's views, namely, category theory, developed a decade after the surge of Bourbakian mathematics by some collaborators of the Bourbakians, especially Saunders Mac Lane and Samuel Eilenberg. In category theory the notion of set does not play any decisive role. It is one of many mathematical notions that can be dealt with without difficulty in the context of category theory. A category consists of two components, namely, objects and morphisms (or arrows) between the objects. Somewhat more precisely, a category **K** consists of a collection of objects Obj(**K**), together with, for each pair of objects A and B in the collection of objects **K**, a possibly empty collection of morphisms $f{:}A\rightarrow B$ such that:

(i) For any three, not necessarily distinct objects A, B, C in Obj(**K**) and any $f{:}A\rightarrow B$ and $g{:}B\rightarrow C$, there is an operation "$*$", called the composition of f and g such that $g*f{:}A\rightarrow C$, which is associative, that is, $h*(g*f)=(h*g)*f$.

(ii) For every object A of **K**, the collection **K**(A,A) contains an identity morphism id_A, that is, one such that if f is a morphism in **K**(A,B), respectively, in **K**(B,A), we have $id_A*f=f$, respectively, $f*id_A=f$.[24]

Category theorists, in their textbook expositions sometimes use the word "set" instead of the more neutral word "collection", but we have preferred to avoid it, in order not to create the suspicion that when one later in such textbooks introduces the notion of set in terms of categories we are operating in a circle. Nonetheless, the fact that categories can also be introduced with help of the notion of set debilitates the claim that category theory can serve as the foundation of the whole of mathematics. In the best of cases, it would seem to be on equal stand with set theory

A very different objection could come if we take Husserl seriously and consider the theory of (at least extensive) wholes and parts as a mathematical theory. Probably the attempt to define wholes in terms of categories would be at least as difficult or artificial as to define them in terms of sets. Hence, in reality there are not two but three rival foundations of structural mathematics, namely: (i) set-theoretic foundations, (ii) categorical foundations and (iii) none of the above, but a foundation on a plurality of fundamental (formal ontological categories), none of which seem more fundamental than the others and some that could only be, in the best of cases, artificially defined in terms of some other.

7. Did Husserl influence the Bourbaki group?

This question admits presently no definitive answer, and we will only pave the way for future investigations –most probably by younger authors. Nonetheless, there are some interesting factors that point to a possible somewhat indirect influence of Husserl on the Bourbaki group. First of all, there are two general circumstances present in Germany and France in the first decades of the twentieth century that should not be ignored. The first one is very simple and general, namely, that contrary to what one could think today, especially if you see the history of contemporary philosophy through the muddy lenses of North American empiricism[25] and of the multiply-broken ones of its magician cousin nominalism, in Germany, France, Poland and other European countries the spectre of empiricism did not blind the spirits of mathematicians and philosophers, forbidding them to read non-empiricist philosophers.

The second more specific one is that in some of the most important centres of mathematical research both in Germany and in France in the first decades of the twentieth century mathematicians and philosophers were in near intellectual contact. In Gottingen, for example, which had been one of the most important centres of mathematical research since Gauß, before and after 1900 mathematicians and philosophers were not only administratively linked, but in some cases also intellectually and personally strongly related. That was the case from 1901 to 1916 between Felix Klein and, especially David Hilbert with precisely the mathematician turned philosopher Edmund Husserl. In fact, Husserl initiated his tenure as philosophy professor in Göttingen with a double conference at meetings of their mathematical society, and that just a year after the publication of the first volume of *Logische Untersuchungen*, in whose last chapter he had presented for the first time his mature conception of logic and mathematics. In particular, Husserl and Hilbert developed a friendship that lasted for their whole lives. Moreover, many of Hilbert's collaborators, like Ernst Zermelo and Constantin Carathéodory were also in friendly terms with Husserl, and many students of Hilbert, as Hermann Weyl and Max Born, were also students of Husserl, and these two also developed a lifelong friendship with the great philosopher.

The situation was very similar to that in Göttingen two to three decades later in Paris at the École Normale, which usually assembled the best French mathematicians, both as students and as faculty. Mathematicians, like Henri Cartan, André Weil and others of the founders of the Bourbaki group had a strong intellectual and personal relation with philosophers also schooled in mathematics, as Jean Cavaillès and Albert Lautman. Cavaillès was an excellent Husserlian scholar intensively working both on Husserl's views on mathematics and logic and on the philosophy of mathematics, in general.[26] Henri Cartan had not only been one of Cavaillès' teachers, but he was also in the doctoral committee of Jean Cavaillès' dissertation and, moreover, wrote the Preface to the second edition of one of Cavaillès' two doctoral theses, *Méthode Axiomatique et Formalisme*. But Cavaillès seemed to have been near also to many other members of the Bourbaki group, among them André Weil, René de Possel, Paul Dubreil and, especially, Claude Chevalley,[27] though his strongest relation with any member of the Bourbaki group was that with another of its founders, Charles Ehresmann, who was one of Cavaillès best friends, and after Cavaillès death in 1944 was one of the editors of Cavaillès' posthumous *opus magnum*, *Sur la Logique et la Théorie de la Science*, and co-author of its Preface. Thus, Cavaillès belonged to the periphery of near friends of the Nicholas Bourbaki group, and received regularly their manuscripts before their printing. Most surely, besides Ehresmann, Chevalley and Cartan[28], also other members of the group took seriously any constructive criticism of his and respected his well-founded philosophical views on mathematics. Cavaillès' views on mathematics, however, had essentially two fundamental sources, namely, Husserl's conception of mathematics as a formal ontology, that is, as a theory of structures as we described above, and the development of set theory from Cantor to its axiomatization beginning with Zermelo and up to at least 1930. We can suppose that the "working mathematicians" of the Bourbaki group were at least partially acquainted with the development of set theory, to which French mathematicians of the prior generation, like Émile Borel, had contributed. It is certainly not excluded that one or two of them had read some Husserl, since Husserl was highly esteemed in those days in French philosophical circles, and precisely in 1928 he had lectured in Paris at La Sorbonne on the foundations of our knowledge, lectures that were the basis for his book of the same year under the title

Cartesianische Meditationen (Hua I). But even in the case that none of the founders of the Bourbaki group had ever acquainted himself directly with Husserl's views on mathematics, they were most surely informed of them by their highly respected Cavaillès. It is by no means preposterous to sustain that the Bourbakians were somewhat at least indirectly influenced by Husserl's views on mathematics as a theory of structures -or at least Husserl's views strengthened theirs-, with its mother structures and all the ways in which one can obtain new structures from already existing structures, and the fundamental similarities between both conceptions were not a purely lucky coincidence.[29] Nonetheless, as we stressed above, there were also two major discrepancies, namely, (i) the presupposition by the Bourbakians of the set-theoretic language and of set theory as a sort of basis of even the mother structures, and (ii) the inclusion by Husserl of a mereology – in Lesniewski's parlance – as a fundamental mathematical structure, that is, as a mother structure. Concerning the first discrepancy, due to the fact that sets can be defined in terms of other mathematical concepts, it should be clear that Husserl was right and the Bourbakians wrong: the notion of set is not the fundamental mathematical notion, not even in a linguistic-pragmatist sense. There does not seem to exist a unique most fundamental mathematical notion. Concerning the second discrepancy, both the Bourbakians and the category theorists would have to show that everything that can be obtained in a mereology can be obtained in their respective conceptions, making the introduction of the notions of whole and part in mathematics superfluous. If that were not the case, then Husserl would have been justified in introducing the notions of part and whole as fundamental mathematical notions. Only the future development of mathematics can decide this question.[30]

NOTES

[1] Dagfinn Føllesdal (1958, 1969) has been very influential in the propagation of this incorrect view, as was also Evert W. Beth (1959) and much earlier than both Alonzo Church in his review of Marvin Farber's book *The Foundation of Phenomenology*. For the three authors, see the references.

[2] Some scholars cannot even distinguish well between Husserl and Kant, and have argued that Kant was the main influence on Carnap's *Logische Aufbau der Welt*, whereas in reality it was Husserl who exerted a not acknowledged fundamental influence on that work, to the point that we should seriously speak about plagiarism. See on this subject endnote 14 below as well as writings of Verena Mayer and the present author in the references.
[3] Published for the first time as Appendix A to the *Husserliana* edition of *Philosophie der Arithmetik* (Hua XII; Husserl 1970b).
[4] 'Zur Logik der Zeichen (Semiotik)', was written in 1890 but published for the first time only as Appendix B.(I) to the Husserliana edition of *Philosophie der Arithmetik* (Hua XII; Husserl 1970c).
[5] 'Besprechung von E. Schröders *Vorlesungen über die Algebra der Logik I*' 1891, reprinted in (Hua XXII; Husserl 1979b).
[6] Frege's 'Funktion und Begriff' was most probably also written in 1890. It was reprinted in his *Kleine Schriften* (Frege 1967/1990).
[7] See Rosado Haddock (2018, 199-219).
[8] 'Rezension von E. G. Husserl, *Philosophie der Arithmetik I*' 1894, reprinted in *Kleine Scriften* (Frege 1967/1990, 179-192).
[9] For an excellent treatment of Leibniz and the *mathesis universalis*, see the recent paper by Centrone and Da Silva (2017, 1-23). For Husserl's assessment of Leibniz' influence, see also Hua XVIII, §§ 60-61.
[10] See Hua XVIII (Appendix to Chapter 10), for Husserl's assessment of Bolzano's work and influence and, very especially, the section 26d of his *Formale und transzendentale Logik* (Hua XVII). See also Centrone and Da Silva's (2017), as well as Casari (2017, 75-91), and the references therein. For a thorough treatment of both Leibniz and Bolzano's views on mathematics, see Danek (1975).
[11] On Frege's logicism and Platonism, see his philosophical masterpiece *Die Grundlagen der Arithmetik* (Frege 1986), as well as the Introduction to his *Grundgesetze der ArithmetikI (*Frege 1962), a certainly failed but nonetheless impressive attempt to derive arithmetic and mathematical analysis from logic.
[12] Since we have quoted extensively from those three letters in two older papers, namely, in the already mentioned 'Husserl and Riemann' and in 'Husserl's Conception of Physical Theories and Physical Geometry in the Time of the Prolegomena: a Comparison with Duhem and Poincaré', we refer the reader to those papers (Rosado Haddock 2012 and 2017).
[13] See Frege's posthumous 'Über Euklidische Geometrie', in Frege (1983, 182-184).
[14] We will follow basically *Logische Untersuchungen*, though *Formale und transzendentale Logik*, *Einleitung in die Logik und Erkenntnistheorie* and *Logik und allgemeine Wissenschaftstheorie* could very well had been used. There is only one point, emphazised in *Formale und transzendentale Logik*, in which we will refer especially to this last work.
[15] This is a clear case of dishonesty by Carnap without any possible excuse. In former books of his, namely, in his dissertation, *Der Raum*, and in *Der logische Aufbau der Welt*, Carnap included *Logische Untersuchungen* in the bibliography, but not in his 1934 book nor in 'Die Überwindung der Metaphysik durch logische Analyse der Sprache', in both of which he

appropriated material from Husserl's *Logische Untersuchungen*. Of course, as Verena Mayer and the present author have shown in various writings (see Mayer 2016), in *Der logische Aufbau der Welt* he appropriated many more ideas from Husserl, this time precisely from the latter's "Ideas", namely, from *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie I* (Hua III) and from other then unpublished manuscripts of Husserl, most probably from the then still unpublished second volume of that work.

[16] Frege really mentioned explicitly only the case in which n=2, but the generalization to any finite n is trivial.

[17] See, for example, Mac Lane (1986, 359, 407).

[18] Already in 1925 John von Neumann defined the notion of set in terms of that of function, as pointed out to me many years ago by Philippe de Rouilhan.

[19] By the way, it is interesting that when studying general topology and considering notions like that of neighbourhood I have always wondered whether in that fundamental area of mathematics –and maybe in parts of mathematical analysis- one could very well replace the notion of set with the notion of extensive whole, thus, obtaining another sort of "non-standard" analysis.

[20] See on this whole last point, e.g. Hua XVIII, §§ 69-70.

[21] For a general exposition of the views of the Bourbaki group see Corry (2004, chapter 7, especially, pp. 292-293).

[22] For more detailed expositions of the views of the Bourbaki group, see Bourbaki (1949 and 1950).

[23] It should be pointed out, however, that Bourbaki's conception, though fundamentally similar to Husserl's, is far more elaborated than Husserl's sketches. For example, in order to combine two structures, some law of compatibility is usually necessary. Thus, in the case of topological groups, which combine topological and algebraic structures, a law of compatibility requires that homomorphisms between groups be continuous. However, though Husserl did not make explicit such a requirement, there is little doubt that he, as a well trained mathematician, would have accepted it. There are other components in the more sophisticated presentation by the Bourbaki group, though they are not essential for the conception itself, but for the particular presentation. See on this issue Bourbaki (1966, chapters 1 and 4).

[24] We have followed here, with small modifications, the definition of category in Michael A. Arbib's and Ernest G. Manes book *Arrows, Structures, and Functions: The Categorical Imperative*, though we could very well had used Saunders Mac Lane's classic book *Categories for the Working Mathematician* or any other textbook on category theory.

[25] A friend of mine who made her undergraduate studies in philosophy in one of the most renowned North American universities told me that one of her philosophy professors – around 1970 – told her that after 1905 all good philosophy was written in English and, thus, it was not necessary to learn other languages, especially German. *Lang lebe die Unwissenheit und Ihre Schwester die Doofheit!*

[26] For the life and work of this philosopher and hero of the French resistance against the Nazis during the second-world war, we refer to the very valuable biography written by his sister Gabrielle Ferrières (1982).

[27] A possible example of this nearness is the following: In a footnote on p. 78 of his 1937 *Méthode Axiomatique et Formalisme* Cavaillès mentions that the most general definition of integration had been given just recently simultaneously and independently of each other by Hans Hahn and René de Possel. In the references there is a paper of Hahn (1933) – to which Cavaillès most surely refers – but no writing of de Possel is included. That points either to having obtained that information directly from de Possel or from another member of the Bourbaki group, in any case to the information being obtained from the inner circle of the group.
[28] See Cartan's assessment of Cavaillès in the already mentioned preface to *Méthode Axiomatique et Formalisme* (Cavaillès 1981).
[29] Besides Cavaillès two books already mentioned, his biography written by his sister, Gabrielle Ferrières, is also very informative with respect to his relation to members of the Bourbaki group. Thus, for example, on pp. 106-107 Ferrières quotes a letter from her brother, in which Cavaillès not only shows his great esteem for Chevalley, but also mentions that the latter is working on one of the monographs for the Bourbaki group and that he – Cavaillès – will be taking part in the discussion of the monograph. Moreover, and also as an example, Ferrières (1982, 124) refers to Cavaillès' great friendship with Ehresmann; Cavaillès (125) mentions that the Bourbaki group continues to send him parts of their projected treatise on analysis; and Ferrières (211) mentions that in a book on algebra published after Cavaillès' death Paul Dubreil recommends the reader to read Cavaillès, Thus, it is very difficult to argue that the Bourbakians were not informed about Husserl's views on mathematics.
[30] For an exposition of both the Bourbakian conception of mathematics and of that of the category theorists, as well as a comparison between them that favours category theorists, see Corry (2004, Chapters 7-9). Of course, nothing is said about Husserl or the possibility of conceiving a theory of wholes and parts as a mathematical discipline.

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Guillermo E. Rosado Haddock is a retired (December 2010) full professor of philosophy at the University of Puerto Rico at Río Piedras – where he had obtained his BA (1966) and MA (1968). He obtained his doctorate in 1973 at the University of Bonn, under the supervision first of Gottfried Martin, who died in October 1972, and then of Gisbert Hasenjaeger. Rosado Haddock is the co-author with Claire Ortiz Hill of the book of essays *Husserl or Frege? Meaning, Objectivity and Mathematics* (Open Court 2000) and the author *of A Critical Introduction to the Philosophy of Gottlob Frege* (Ashgate 2006), *The Young Carnap's Unknown Master* (Ashgate 2008), and the books of essays *Against the Current* (Ontos Verlag 2012) and *Unorthodox Analytic Philosophy* (College Publications 2018), He is the editor and co-author of *Husserl and Analytic* Philosophy (de Gruyter 2016), and is also the author of multiple essays and critical studies appearing in some twenty journals and ten publishers in a dozen different countries, mostly dealing with the philosophy of logic and the philosophy of mathematics, be it in Husserl, in Frege, or not restricted to any particular philosopher.

Address:
Guillermo E. Rosado Haddock
Estación Central de Correos
Apartado 367893
San Juan, Puerto Rico 00936-7893
Email: gerosadohaddock@gmail.com