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Husserl on Sets and the Causes of the Set-theoretical Paradoxes

Claire Ortiz Hill
Free researcher

Abstract

Few realize that Edmund Husserl theorized about sets and the causes of the set-theoretical paradoxes. Interpreted here are his statements that: 1) the paradoxes show that his contemporaries did not yet have the real and genuine concept of set needed; 2) that if one is clear and distinct with respect to meaning, one readily sees the contradiction involved in the set-theoretical paradoxes; 3) that the solution to them would lie in demonstrating the shift in meaning that makes it that one is not immediately aware of the contradiction and that, once it is perceived, one cannot indicate wherein it lies. I study these convictions in connection with Frege's and Russell's ideas about sets and the conclusions that they came to regarding the causes of the paradoxes derivable within Frege's system.

Keywords: Husserl, Frege, Russell, Set Theory, Set-theoretical Paradoxes, Meaning

1. Introduction

The full story of set theory's role in shaping modern logic and in redrawing the boundaries between mathematics and philosophy in both the analytical and the phenomenological traditions is yet to be told and its full implications drawn. In particular, the fact that Edmund Husserl thought and taught about it well before the logic shaped by *Principia Mathematica* (Russell & Whitehead 1927) came to play a key role in laying the foundations for analytic philosophy has barely been investigated.

Husserl searched for clarity about the meaning of sets all throughout his career. For example, in his late work *Formal and Transcendental Logic* (Husserl 1978), he described his first

book, *Philosophy of Arithmetic* (Husserl 2003a), as an initial attempt on his part "to obtain clarity regarding the original genuine meaning of the fundamental concepts of the theory of sets and cardinal numbers" (Husserl 1978, §27a). He was well-versed in the set theories being created by his contemporaries and lucid about their logical, epistemological and ontological implications. As a colleague and best friend of Georg Cantor, the creator of set theory, during the last 14 years of the 19th century, he had a front row seat at the creation of set theory and the discovery of the paradoxes of the transfinite. So he was sort of a victim *avant la lettre* of the crisis in foundations that broke out once Bertrand Russell publicized the famous contradiction about the set of all sets that are not members of themselves that he discovered while studying Cantor's theories (Hill & Rosado Haddock). Then, appointed to the University of Göttingen in 1901, Husserl engaged in exchanges with mathematicians who knew the set-theoretical paradoxes before Russell did. In November 1903, David Hilbert wrote to Gottlob Frege that Russell's antinomy was already known in Göttingen, that Ernst Zermelo had found it three or four years earlier after having learned of other, even more convincing, contradictions from Hilbert himself as many as four or five years before (Frege 1980a, 51; Peckhaus & Kahle 2000/2001). Concrete evidence corroborating Zermelo's finding of the paradox is found in a note he sent to Husserl on in April 1902. (Husserl 1994, 442; Rang and Thomas 1981)

Guillermo Rosado Haddock drew attention to Husserl's notes on set theory (Husserl, Ms A 1 35) in his 1973 doctoral thesis (Rosado Haddock 1973), but this did not arouse any excitement. Here I seek to provide the conceptual framework for interpreting Husserl's statements in those notes that: 1) the set-theoretical paradoxes show that his contemporaries did not yet have the real and genuine concept of set needed; 2) that if one is clear and distinct with respect to meaning, one readily sees the contradiction involved in the set-theoretical paradoxes; and 3) that the solution to them would lie in demonstrating the shift of meaning that makes it that one is not immediately aware of the contradiction and that once one perceives it one cannot indicate wherein it lies. To do this I present those of

Husserl's ideas that I think are necessary for interpreting those statements. To this end I juxtapose issues involved in Frege's use of the extensions that lead to the contradiction about the set of all sets that are not members of themselves within his system and the conclusions that he and Russell came to regarding the causes of that contradiction. Finally I interpret Husserl's statements in light of what I have said. My remarks are not about mathematics per se, but about set theory and the foundations of analytic philosophy and phenomenology.

2. Logical Laws and Laws of Meaning

By the time he wrote the *Logical Investigations* in the late 1890s, Husserl considered it very important to distinguish between logical laws and laws of meaning. According to him, logical laws serve to guard against formal or analytical contradiction, what he called *Widersinn*. What violates logical laws, what is contradictory (*widersinnig*), genuinely has a coherent meaning and can be determined to be true or false. However, though meaning is there, no existing object can correspond to the meaning. As examples of contradictions (*Widersinnigkeiten)*, he gave expressions like 'wooden iron'. 'round square', 'all squares have five corners' that have meaning, but no object. No thing or fact such as is described by such expressions exists or can exist (Husserl 1900/01, IV). In his notes on set theory, Husserl studied the sentences, 'The present emperor of France is blond' and 'The present emperor of France is not blond'. 'The present emperor is blond' implies that France presently has a blond emperor, while she has no emperor at all. He contended that the sentence is not valid, because it is objectless either in actual fact or owing to a contradiction.

Along these same lines, he maintained that to the objection that there is no set that contains itself as an element, one need merely respond that that is *widersinnig*. (Husserl, Ms A 1 35)

In contrast to logical laws, laws of meaning serve to distinguish meaningfulness from meaninglessness, sense from nonsense, by providing pure logic with possible coherent, meaningful meaning forms whose formal truth or falsehood and reference to objects, logical laws determine. Meanings, Husserl

repeated over and over, are governed by *a priori* laws that regulate how they may be combined, fit together and constitute meaningful, coherent meanings instead of chaotic nonsense (*Unsinn*). The impossibility of combining meanings in certain ways is not subjective, but objective, ideal and grounded in the pure essence of meaning.

Husserl believed that the primitive, essential distinction between dependent and independent meanings formed the necessary basis for discovering the essential categories of meaning in which were grounded a number of essential laws of meaning. He, like Frege before him and Russell after him, stressed that fundamental differences between dependent meanings and independent meanings lying concealed behind inconspicuous grammatical distinctions are inviolable because they are "founded deep in the nature of things".

Husserl studied how one may be led astray by the fact that meanings of each category may figure in the subject position otherwise reserved for substantival meanings. The words are definitely in the subject position, but their meanings are not the same as they normally are. Not just any meaning can be substituted for *S* or for *p*. Once meaning categories are violated, the coherency of the meaning is lost. The underlining on Husserl's copies of Frege's "Concept and Object" and "Function and Concept" shows Husserl's fundamental agreement with Frege on this matter. (Hill & da Silva 2013)

3. A Natural Order in Formal Logic

In addition, Husserl found a natural order in formal logic and broadened its domain to include two levels above traditional Aristotelian logic, which he saw as being but a small area of pure logic that needed to be distinguished and segregated from the extended sphere of pure logic that includes the mathematical disciplines and is immense in range and wealth of content in comparison. He considered his understanding of the structure of the world of pure logic to be a radical clarification of the relationship between formal logic and formal mathematics and that it led to a definitive clarification of the sense of pure formal mathematics as a pure analytics of non-contradiction. (Husserl 2008, §§18-19; Husserl 1978, 11; Hill 2013; Hill 2015)

On the first level of Husserl's hierarchy, the traditional Aristotelian logic of subject and predicate propositions and states of affairs deals with what is stated about objects in general from a possible perspective. In the disciplines of the two higher levels, it is no longer a question of objects as such about which one might predicate something, but of investigating what is valid for higher-order objective constructions that are determined in purely formal terms and deal with objects in indeterminate, general ways. (Husserl 2008, 18c)

On the second level, Husserl located the basic concepts of mathematics, the theory of cardinal numbers, the theory of ordinals, set theory, mathematical physics, formal pure logic, pure geometry, geometry as *a priori* theory of space, the axioms of geometry as a theory of the essences of shapes, of spatial objects, but also the pure theory of meaning and being, *a priori* real ontology of any kind, ontology of nature, ontology of minds, natural scientific ontology, the sciences of value, pure ethics, the logic of morality, the ontology of ethical personalities, axiology or the pure logic of values, pure esthetics, ontology of values, the logic of the ideal state or the ideal world government as a system of cooperating ideal nation states, or the science of the ideal state, the ideal of a valuable existence, objective axioms (relating to *a priori* propositions as truth for objects, as something belonging in the objective science of these objects, or of objects in general in formal universality, essence-propositions about objects insofar as they are objective truths and as truths have their place in a truth-system in general. (Husserl 2008, §§18-19, 434-35; Husserl 1996, Chapter 11)

The third level is that of his theory of manifolds (Husserl 1900/01, *Prolegomena*, §§69-70; Husserl 1962, §§71-72; Husserl 1978, §33), which we shall not be concerned with here. The key thing to realize at this point is that, according to Husserl's theory, sets and numbers function in an entirely different way on the first level than in set theory and arithmetic, which Husserl put on the second level.

In the case of numbers, in expressions of the first level, for example, '2 men', '3 houses', numbers occur as form, but not as independent objects about which something is predicated. In that case, the sentence "Jupiter has four moons", to use Frege's

example in the *Foundations of Arithmetic* (Frege 1884, §57), is a statement about Jupiter's moons in which the number characteristic four occurs as form and is thereby dependent. If one says *w* and *x* and *y* and *z* are *φ*, Husserl explained, then one has combined the objects *w*…*z* by 'and'. The 'and' is form and grounds the coherent form of the plural predication. Corresponding to this is a cardinal number, which is a new thought configuration. It is one thing, he stressed, to make statements about objects in which number properties occur as form, and are thereby dependent, and another thing to make statements about numbers as such in such a way that the numbers are the objects. We can make such forms independent, but then new higher-order objects, hypostatizations of forms, emerge that are not objects in their own right. This is why numbers function entirely differently in the propositional logic of the first level than they do in the arithmetic of the second level, where statements about numbers in which numbers are the objects are found, for example,

1. "Any number can be added to any number".

2. "If *a* is a number and *b* a number, then *a* + *b* is as well".

3. "Any number can be decreased or increased by one".

4. "The numbers form a series continuing from 0 *in infinitum*". (Husserl 2008 18c)

4. Analytics

Instead of pure logic, Husserl taught, one might speak of analytics, or the science of what is analytically knowable in general, the science that establishes and systematically grounds analytic laws (Husserl 2003b, 244). He conceived of the second level of pure logic as an expanded, completely developed analytics in which one proceeds in a purely formal manner since every single concept used is analytic. One calculates, reasons deductively, with concepts and propositions. Signs and rules of calculation suffice because each procedure is purely logical. One manipulates signs that acquire their meaning in the game through the rules of the game. One may proceed mechanically in this way and the result will prove accurate and justified. (Husserl 2008, §§18-19, 434-35; Husserl 1996, Chapter 11)

In his logic courses, Husserl taught that the mathematical disciplines of the purely logical sphere proceed from given, purely logical concepts and axioms that are grounded in the essence of purely logical categories. It is a matter of a rigorously scientific, a priori theory that builds from the bottom up and derives the manifold of possible inferences from the axiomatic foundations a priori in a rigorously deductive way. (Husserl 2001b, 32-35, 39; Husserl 2008, §§13c, 19d, 25b)

From the late 1890s on, Husserl held that the "*world of the mathematical and purely logical is a world of ideal objects*, a world of 'concepts'…. *There all truth is nothing other than analysis of essences or concepts",* and pure logical, mathematical laws are laws of essence. (Husserl 2008, §13c)

In affirming this, he wanted to make it clear that he was not hypostatizing ideal entities or talking about the unwelcome, obscure "special and irreducible intermediary entities called meanings" that Quine called "illusory". (Quine 1961, 11-12, 22)

Husserl said that it was his failure "to obtain clarity regarding the original genuine meaning of the fundamental concepts of the theory of sets and cardinal numbers" in *Philosophy of Arithmetic* that had "compelled" him to recognize the purely logical ideal (Husserl 1978 §27a; §24 and note; Husserl 1975, 34-35). It is worthwhile pointing out in this regard that in Russell's article on the philosophical implications of mathematical logic that is translated in Husserl's notes on set theory, Russell affirmed that "all knowledge which is obtained by reasoning, needs logical principles which are *a priori* and universal" and that mathematics and logic force us "to admit a kind of realism in the scholastic sense… to admit that there is a world of universals and of truths which do not bear directly on such and such a particular existence". (Russell 1973, 292-93)

Husserl said that his concepts of ideal meanings and contents and the idea of transferring all of the mathematical and a major part of the traditionally logical to the realm of the ideal derived from Hermann Lotze, who had been Frege's teacher. Husserl repeatedly defended the view, which he attributed to Lotze, that pure arithmetic is a branch of logic that had undergone independent development. Husserl taught

that the unending profusion of theories that arithmetic develops is already fixed, enfolded in the arithmetical axioms, and deduction effects the unfolding of them following systematic, simple procedures. Each genuine axiom is a proposition that unfolds the idea of cardinal number from some side or unfolds some of the ideas inseparably connected with the idea of cardinal number. (Husserl 2001a, 241-42, 271-72; Husserl 2001b, 19, 32-35, 39; Husserl 2008, §15, Hill & da Silva)

This is not necessary to my argument here, but because of the literature making Husserl into a sort of Brouwerian intuitionist (for example, Tieszen 1989; Van Atten 2007), it needs to be made clear that Husserl repeatedly, explicitly and emphatically stressed that, because they belong in the world of the purely logical, arithmetic and set theory are not phenomenology. He maintained that as long as we remain in pure theory of meaning and being, we need not concern ourselves at all with cognitive formations, with consciousness. He believed that everything 'purely' logical was an 'in itself', an 'ideal' that included in its proper essential content (*Wesengehalt*) nothing mental, nothing of acts, subjects, or empirically factual persons of actual reality. He believed that in the case of pure logic, of an 'analytics' in the broadest, radical sense of the word only certain of the most general cognitive-formations enter the picture for purposes of phenomenological elucidation. (Husserl 1975, 20, 31; Hill 2013)

5. Husserl on Sets and the Set-theoretical Paradoxes

So how do Husserl's ideas about sets and the set-theoretical paradoxes fit into the conceptual framework I have just described?

First, it is imperative to keep in mind that sets have an entirely different meaning in the subject-predicate propositions of the first level of Husserl's hierarchy than they do in the set theory of the second level. In the theory of proposition forms or forms of states of affairs of the first level, individual objects are the terms of the predication. Sets, however, do not occur as objects in the subject-predicate propositions, but function in them as dependent forms.

In contrast, in the set theory of the second level, truth is the analysis of essences or concepts, where "we make judgments universally about sets that in a certain way are higher order objects. We do not make judgments directly about elements, but about whole totalities of elements and arbitrary elements, and the whole totalities, the sets to be precise, are the objects-about-which.

He gave these examples of statements about sets on the second level,

1. "2 sets can each be joined into a new set".

2. "2 sets *a b* are each related to one another in such a way that either *a* is part of *b* or *b* is part of *a*, or that they intersect (a set having a part in common), or that it turns out that they are identical, coincide".

3. "The set formed of the elements *A B C* is part of the set formed of the elements *A B C D* containing 'more elements'". (Husserl 2008, 18-19)

On the second level, set theory is derived analytically from the concept of set, which if it is to be mathematical must have a "set essence" in view. This set essence is expressed in the relation between a set itself and its elements. An essence relation makes it impossible for the members of the relation to be identical. So it belongs essentially to the concept of set that no set can contain itself as an element without contradiction.

For Husserl, it is part of the idea of set to be a unit, a whole, comprising certain members as parts in such a way that it is something new that is first formed by them. It belongs essentially to the concept of whole that no whole can contain itself as a part. So, as a kind of whole, a set is subject to the formal rules governing wholes and parts that stipulate that a whole cannot, without contradiction, be its own part. So no set can contain itself as a member. Sets are *a priori* different from their members. (Husserl, Ms A 1 35)

Husserl's 1902 exchange with Zermelo turned upon remarks Husserl had made in 1891, in his review of Ernst Schröder's *Vorlesungen über die Algebra der Logik*, in which Schröder had tried to show that bringing all possible objects of thought into a class gives rise to contradictions. In his review, Husserl wrote that in "the sense of the calculus of sets as such,

any set ceases to have the status of a set as soon as it is considered as an element of another set; and this latter in turn has the status of a set only in relation to its primary and authentic elements, but not in relation to whatever elements *of* those elements there may be". He warned that if "one does not keep this in mind, then actual errors in inference can arise". (Husserl 1994, 84-85, 442; Rang & Thomas 1981)

Third, Husserl repeatedly relegated the set theoretical paradoxes to the category of *Widersinnigkeiten*. For him, a set that contains itself as an element was *widersinnig*. By saying that the set of all sets that were not members of themselves is a *Widersinnigkeit*, Husserl was putting it into the same category as the round square, the golden mountain, and the present emperor of France. The formal logical construction "set of all sets which do not contain themselves as parts", he argued, may not be presupposed to be about something that already exists. Just as it is contradictory for a whole to be its own part at the same time, so it is contradictory for a set to be its own member. It proceeds from the paradox that a set that contains itself as an element or a set that does not must be a *Widersinn*. The classification is *widersinnig* as well.

Of what he referred to as "Zermelo's paradox", Husserl wrote that Zermelo argued that a set M that contains each of its partial sets as elements is an inconsistent set. 1) We consider those partial sets that do not contain themselves as elements. 2) In their entirety these form a set M′ that is contained in M. 3) M′ is thus an element of M. 4) M′ is not an element of M′. Proof: were M′ an element of M′, then it would contain a partial set of M (namely M′) that contains itself as element. However, M′ is to contain *ex definitione* partial sets of M that do not contain themselves as elements. 5) Thus M′, since it is not an element of M′, is a partial set of M, which does not contain itself as an element. But all such sets are *ex definitione* contained in the concept of M′, thus in opposition to 4. But M′ is an element of M′. We come to a direct contradiction. If it essentially belongs to the concept of set that (without contradiction) no set can contain itself as an element, then M′ and M are identically the same set, and we show that the whole reasoning was untenable. (Husserl, Ms A 1 35)

6. Frege's Recourse to Extensions

Husserl, Frege and Russell came to many of the same conclusions about the causes of the set-theoretical paradoxes, so we now need to look at the reasoning that led Frege to introduce sets and at Russell's struggles to avoid the contradiction derivable in Frege's system.

Frege thought that wherever we are concerned about truth, we must attach a reference to proper names and concept-words and that we are making a mistake that can easily vitiate our thinking if we do not to do this. So he considered the prime problem of arithmetic to be that of how one apprehends logical objects, in particular numbers. (Hill & da Silva 2013)

Operating only on the first level of Husserl's hierarchy, Frege argued that numbers were independent objects that must always be conceived substantivally and not as dependent attributes. He believed that the presence of the definite article 'the' in an expression like 'the number 4' served to class it as an object and that in arithmetic this independence comes out at every turn, as for example in an identity like 4 + 4 = 8. He thought that we should not be "deterred by the fact that, in the language of everyday life, number appears also in attributive constructions" for that "can always be got around". He proposed that:

> "Jupiter has four moons" can be converted into "the number of Jupiter's moons is four"… we can say: "the number of Jupiter's moons is the number four, or 4". Here "is" has the sense of "is identical with" or "is the same as". So that what we have is an identity, stating that the expression "the number of Jupiter's moons" signifies the same object as the word "four". (Frege 1884, §57)

He added that the independence that he was "claiming for number was not to be taken to mean that a number word signifies something when removed from the context of a proposition, but only to preclude the use of such words as predicates or attributes, which appreciably alters their meaning".

Seeing that many of the inferences that could be made by appealing to his formula for treating what is dependent as independent led to evidently false or nonsensical conclusions, or

was sterile and unproductive, Frege settled for the definition: "The Number which belongs to the concept F is the extension of the concept 'concept equal to the concept F'" and for his axiom of extensionality, which he considered "an unprovable law" authorizing a transformation to "take place, in which concepts correspond to extensions of concepts…" (Frege 1979, 182)

Upon learning of the contradiction about the set of all sets that are not members of themselves that Russell derived in the system of *The Basic Laws of Arithmetic*, Frege tested the validity of the chain of inferences leading up to the contradiction and concluded that his law about extensions was false. He confessed that he had been reluctant to use classes, but had found no other answer to the question as to how to apprehend logical objects. (Frege 1980b)

He later described the shift of meaning that had made him not immediately aware of the contradiction. The paradoxes of set theory arise, he said, because a concept is connected with something that is called the set which appears to be determined by the concept and determined as an object. Such a transformation of a concept into an object is inadmissible, because the set formed only seems to be an object, while in truth there is no such object at all. He summed up the "essence of the procedure which leads to the thicket of contradictions":

> The objects that fall under *F* are regarded as a whole, as an object and designated by the name 'set of *F*s'. This is inadmissible because of the essential difference between concept and object, which is indeed quite covered up in our word languages…. Confusion is bound to arise if a concept word, as a result of its transformation into a proper name comes to be in a place for which it is unsuited. (Frege 1980a, 55)

In *Foundations of Arithmetic*, he had warned that it was a mere illusion to suppose that a concept can be made into an object without altering it. (Frege 1884, X)

7. Russell's Attempts to Evade the Paradoxes

As for Russell, he said that his struggle with the contradiction he derived in Frege's logic had taught him that if a word or a phrase that is devoid of meaning when separated from its context is wrongly assumed to have an independent

meaning, false abstractions, pseudo-objects, and paradoxes and contradictions are apt to result (Russell 1973, 165). He had originally believed:

> When we say that a number of objects all have a certain property, we naturally suppose that the property is a definite object, which can be considered apart from any or all of the objects, which have, or may be supposed to have, the property in question. We also naturally suppose that the objects which have the property form a *class*, and that the class is in some sense a new single entity, distinct, in general, from each member of the class. (Russell 1973, 163-64)

However, the contradiction about the classes that are not members of themselves showed him that classes must be something radically different from individuals (Russell 1956, 81). He came to believe that if one assumes that the class is an entity, one cannot escape the contradiction (Russell 1973, 171). As he explained,

> if you think for a moment that classes are things in the same sense in which things are things, you will then have to say that the class consisting of all the things in the world is itself a thing in the world, and that therefore this class is a member of itself. (Russell 1956, 261)

Russell decided that he needed a way to make classes disappear from the reasoning in which they were present without really completely letting go of them (Russell 1919, 184), because he believed that "without a single object to represent an extension Mathematics crumbles". (Russell 1903, §489)

While wrestling with the problem of fake objects, he saw parallels existing between the problems arising when classes are treated as objects and those arising when descriptions, 'like the present king of France is bald', are treated as names. So, satisfied that classes and descriptions both fell into the same logical category of non-entities (Hill 1997), he reasoned that since:

> we cannot accept "class" as a primitive idea. We must seek a definition on the same lines as the definition of descriptions, i.e. a definition which will assign a meaning to propositions in whose verbal or symbolic expression words or symbols apparently representing classes occur, but which will assign a meaning that altogether eliminates all mention of classes from a right analysis of such propositions. We shall then be able to say that the symbols for classes are mere conveniences, not representing objects called

> "classes", and that classes are in fact, like descriptions, logical fictions…. (Russell 1919, 181-82)

Russell believed that his means of drawing objects out of descriptions provided a practical model of how to make non-entities function as entities without incurring contradictory results.

Early in his search for ways to evade (his choice of verb) the problem of the contradiction about the class of all classes that are not members of themselves, Russell thought that "the key to the whole mystery" was to be found by inventing (his choice of verb) a hierarchy of types (Russell 1903, §104). It had become clear to him that the contradiction about the classes that are not members of themselves could only be avoided by realizing that no class either is or is not a member of itself, that the entire question as to whether a class is or is not a member of itself is nonsense (Russell 1956, 261-62). So, he invented a hierarchy of classes according to which the first type of classes would be composed of classes made up entirely of particulars, the second type composed of classes whose members are classes of the first type, the third type composed of classes whose members are classes of the second type, and so on. The types obtained would be mutually exclusive, making the notion of a class being a member of itself meaningless (Russell 1973, 201; Russell 1903, §§104-105; Russell 1956, 264). His hierarchy of types was to perform "the single, though essential, service of justifying us in refraining from entering on trains of reasoning which lead to contradictory conclusions. The justification is that what seem to be propositions are really nonsense". (Russell 1927, 24)

Russell believed that no solution to the contradictions was technically possible without his theory of types, but he realized that it was not "the key to the whole mystery". After all, it was but an ad hoc effort to restore the hierarchical structure established by the fundamental differences between dependent and independent that ordinarily protects against invalid inference that was broken by Frege's Axiom of extensionality. He saw that deeper problems caused the old contradiction to break out afresh and he realized that "further subtleties would be needed to solve them". (Russell 1919, 135; Russell, 1956, 333; Hill & da Silva)

8. Interpretation of the Statements

In light of what I have said, how do I interpret the statements I said I was going to interpret?

The *first* statement concerned the set-theoretical paradoxes showing that Husserl's contemporaries did not yet have the real and genuine concept of set needed.

Those paradoxes were derived using a concept of set that allows one to form the expression "a set may be a member of itself", which Husserl judged to be *widersinnig*. In contrast, as we have seen, he would derive set theory analytically from the real and genuine a priori concept, or essence, of set, for which no set can be a member of itself and for which reasoning appealing to the notion of sets that do not contain themselves as members is entirely untenable. A set is a kind of whole and is subject to the formal rules governing wholes and parts that stipulate that a whole cannot be its own part.

The *second* statement says that if one is clear and distinct with respect to meaning, one readily sees the contradiction involved in the set-theoretical paradoxes.

It follows from the above that, if we are clear and distinct about the meaning of the real and genuine concepts of "set", "member", and more universally about the meaning of the real and genuine concepts of "wholes" and "parts", we readily see that all talk of sets being members of themselves is *widersinnig*.

As we have seen, for Husserl, being clear and distinct about meaning involved recognizing the primitive, essential, a priori, inviolable differences between the dependent and independent meanings that form the necessary basis for discovering the essential categories of meaning in which are grounded laws of meaning that provide logic with possible coherent, meaningful meaning forms whose formal truth or falsehood, reference to objects, *Widersinnigkeit* or lack thereof, is determined by logical laws.

For him, being clear and distinct about meaning also involved recognizing that sets have an entirely different meaning in the subject-predicate propositions of the first level of pure logic where they function as dependent forms, than in

set theory of the second level where they function as higher order ideal objects and where truth is the analysis of essences or concepts.

In comparison, Frege reasoned on the first level, which obliged him to treat sets and numbers as objects. For example, he mixed the first level subject-predicate proposition "Jupiter has four moons" with what Husserl considered to be the second level arithmetical statement that 2+2=4. He considered numbers to be independent objects that must always be conceived substantivally and not as a dependent attributes (Frege 1884, §106 and note). He confused statements about objects in which number properties occur as form, and are thereby dependent, and statements about numbers in which numbers are the objects. This led him to introduce a law which he thought would permit him to treat what he recognized as dependent meanings as independent meanings. By making such forms independent, he generated new higher order objects, hypostatizations of forms that are not objects in their own right.

I interpret the *third* statement about the solution to the set-theoretical paradoxes lying in demonstrating the shift of meaning that makes it that one is not immediately aware of the contradiction and that once one perceives it one cannot indicate wherein it lies as having to do with Husserl's insistence upon the importance of the fundamental distinction between independent and dependent meanings that lies concealed behind inconspicuous grammatical distinctions.

Husserl and Frege were in fundamental agreement about what Frege called the "fatal tendency" of our "word languages" to cover up essential differences between concepts and objects and allow a concept word to be transformed into a proper name and so to come to be in a place for which it is unsuited. By unavoidable "awkwardness of language", by "a kind of necessity of language", one mentions an object, when one intends a concept.

Frege had thought that the presence of the definite article 'the' in an expression like 'the number 1' sufficed to class it as an object and that we should not be "deterred by the fact that in the language of everyday life number appears also in attributive constructions" for that "can always be got around".

He ultimately concluded that this propensity language to undermine the reliability of thinking by forming apparent proper names to which no objects correspond had allowed concept-words to be transformed into proper names and come to be in places unsuited to them and so had "dealt the death blow" to his set theory.

On his copy of Frege's "On Concept and Object", Husserl marked the sentence that reads, "Language has means of presenting now one, now another, part of the thought as the subject". And he tellingly underlined the word 'language'. According to his theory about the differences between logical laws and laws of meaning, something that violates logical laws can genuinely have a coherent meaning and can be determined to be true or false, but since it is *widersinnig*, no object can correspond to the existing meaning. So the formal logical construction "set of all sets which do not contain themselves as parts", may not be presupposed to be about something that exists any more that the expression "the present emperor of France" denotes something that exists. (Hill & da Silva 2013)

Such shifts of meaning allow the pseudo-objects and type ambiguities to creep into reasoning unnoticed that Russell struggled to eliminate in his attempts to evade the paradoxes. As he once warned, when two words have two different types of meanings, the relations of those words to what they stand for are also of different types and the failure to realize this is "a very potent source of error and confusion in philosophy". (Russell 1956, 133)

In addition, if, as Frege stressed, concept words and proper names must occupy essentially different places, and it is obvious that a proper name will not fit into the place intended for a concept word (Frege 1980a, 54-55), if, as he wrote, there is a radical difference between dependent and independent meanings concepts, which is such that an object can never stand for a concept or concept for an object (Frege 1980a, 92), then basic rules of inference like the principle of substitutivity of identicals and existential generalization will fail when one puts one in the place intended for the other.

Conclusion

In conclusion, I wish to emphasize that Husserl did not say that set theory itself was false. He considered it to be a legitimate mathematical discipline of the second level of the purely logical sphere. For him, set theory was a matter of a rigorously scientific, a priori theory that proceeds from the purely logical concepts and axioms that are grounded in purely logical categories such as those discovered by the essential distinction between dependent and independent meanings. He concluded that it was faulty reasoning about a faulty concept of set that had led to the set-theoretical paradoxes.

In particular, he found himself at odds with the concept of set underlying popular axioms of extensionality. While Russell's tactic was to invent ways to avoid the contradictions (Hill 1997), Husserl advocated making a fresh start and deriving set theory from a non-contradictory concept of set and element, or more universally of whole and part, without resorting to an axiom of extensionality. He was most disparaging when it came to the popular extensional definitions of sets *Principia Mathematica* and related systems and he was lucid enough to see that Mathematics would not crumble if it did not have "a single object to represent an extension". All the rigmarole that Russell went through to evade the contradictions derivable from Frege's system with its axiom of extensionality serves to illustrate what Husserl meant in *Formal and Transcendental Logic* when he said that extensions generate contradictions requiring every kind of artful device to make them safe for use in mathematical reasoning. (Husserl 1978, 74, 76, 83)

In comparison, Husserl's friend and colleague, David Hilbert, determined not to be thrown out of the set-theoretical paradise that Cantor had created (Hilbert 1967), seemed to think that the laws of inference were faulty. As he wrote,

> In their joy over the new and rich results, mathematicians apparently had not examined critically enough whether the modes of inference employed were admissible; for purely through the ways in which notions were formed and modes of inference used–ways that in time had become customary–contradictions appeared.... In particular a contradiction discovered by Zermelo and Russell had, when it became known, a downright catastrophic effect in the world of mathematics....

> The reaction was so violent that the commonest and most fruitful notions and the very simplest and most important modes of inference in mathematics were threatened and their use was to be prohibited.… Just think: in mathematics, this paragon of reliability and truth, the very notions and inferences, as everyone learns, teaches and uses them, lead to absurdities. (Hilbert 1967, 375)

In contrast to Hilbert's assessment of the problem, viewed from the angle of Husserl's theories about the inviolability of the laws governing the use dependent and independent meanings, Russell's contradiction is just faithfully telling us that: the set X of x's is not a member of what it is a set of (Hill 1997); what is predicated of an object is of a different logical type from the object itself; a concept is not an object; what is dependent is not independent.… In short, logic is doing what logic is supposed to do. Blurring distinctions between talk of sets on different levels by allowing the sets as dependent forms of the first level be transformed into proper names and come to figure on the wrong tier in the hierarchy of meaning breaks the logical structure. Flattening logical structure smooths the way for things to come into places not intended for them. Once logical structure is broken and meaning categories are violated trouble is ahead in the form of failures of inference. (Hill & da Silva, 2013)

Why should *Widersinnigkeiten*-producing theories about sets and the foundations of arithmetic have any lasting "downright catastrophic effect in the world of mathematics?" If those theories are producing contradictions, if they lead to the failure of the simplest and most important modes of inference, it is not logical to see that as posing any particular threat to the modes of inferences themselves and does not indicate that their use should be prohibited. It is more reasonable to conclude with Husserl that those logical laws are determining the truth or falsehood of conclusions just as they are supposed to do.

In my opinion, there is nothing particularly paradoxical or mysterious about the contradictions derivable in Frege's logical system. They are just cheap contradictions generated by an unclear theory of meaning. There is no reason at all why the paragon of reliability and truth that is mathematics should "crumble" as a consequence, as Russell once said it might or

that basic rules of inference should be abandoned as Hilbert suggested.

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Claire Ortiz Hill has specialized in the Austro-German roots of twentieth century philosophy in relation to contemporary issues in Analytic philosophy. She holds a BA and MA from the University of California, Riverside and a Maîtrise and Doctorat from the Université de Paris, Sorbonne. She has never been institutionalized and has been a religious hermit with the archdiocese of Paris since the early 1980s.

Address:
Claire Ortiz Hill
39 rue Gay Lussac, 75005 Paris
France
E-mail: claire.hill@orange.fr