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Husserl's Logic of Probability: An Attempt to Introduce in Philosophy the Concept of "Intensive" Possibility

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Abstract

Husserl's insisting reflections on the question of probability and the project of a logic of probability, although persisting throughout his work, published and unpublished, from the *Prolegomena* to later works (the *Krisis*), has not received any serious attention. While exposing the main lines of his project, this article aims at listing some of the reasons explaining this paradoxical situation. 1) The logic of probability is not conceived by Husserl as an extension of formal logic and especially of an already made logic, but as a reform of logic (from the recensions of Schröder to *Formal and Transcendental Logic*, and beyond). 2) This entails a revised notion of *proposition*, enlarged to every forms of "positions" or "thesis" and, extended, following the correlation of intentionality, to the noematic side. 3) The very notion of the "possible" at the basis of any logical, algebraic, arithmetic and geometric treatments of probability is enlarged and modified accordingly. 4) As a consequence, his position is rather singular and very hard to locate in the battle field among mathematicians, logicians and philosophers around the question of "foundation of probability" and the interpretation of probability calculus (a priori probability *vs* a posteriori probability, subjective *vs* objective probability, logical *vs* psychological probability, etc.).

Keywords: possibility, modality, range, logic, von Kries

In *Formal and Transcendental Logic*, Husserl declares explicitly that, in the *Prolegomena*, he committed a double fault in his presentation of "pure logic", the second failing being a restriction of the scope of "formal logic", by the exclusion of "modal modifications of judgement". Instead of restricting "pure

[*] In the following, I reformulate and complete a thesis that I have exposed for the first time in a Husserl Circle Meeting in Paris, in 2009.

logic" to the sole "logic of certainty and truth", Husserl argues that "in connection with the concept of truth" "the modalities of truth are not mentioned, and *probability* is not cited as one of them"[1] – "as universal formal possibilities, modal variants of judging and of judgments enter into certainty or truth logic." The reason for such exclusion was that ontological and axiological modalities, with their correlative act-modifications, were wrongly "regarded as *extra-formal*". According to Husserl's view on logic and history of logic, this limitation, inherited from the tradition, is explicitly considered as unacceptable. Modalities do belong to the content or "*matter*" (Materie) of judgement and contribute to the constitution of the object referred to, to the "*something*" with its specific determinations. Since Husserl's understanding of "formal semantics" must be seen in the frame of the "correlation" between apophantic and ontology, this means that without those modal determinations, the whole sphere of logic would be drastically amputated, and semantics would remain empty. But what does semantic mean here?

What is at stake here is *not* the proposal of a *modal extension* of formal logic such as it has been promoted at the same time by his former student Oskar Becker (1930 and especially 1952, §2)[2] or others like Lewis (1928 and 1932) nor a prefiguration of a semantical approach *à la* Kripke, but something that has to do with a deeper understanding of formal logic, formalization and formal semantics. Since "probabilities" are themselves modalities, in order to understand what is the profile of the new and enlarged formal logic resulting from the inclusion of modalities strictly understood into the "formal content" of logic, and what motivates this enlargement, I would like to examine more closely Husserl's evolution on the delineation of formal logic, his previous project of a "logic of content", in connection to his project of reform of formal logic (Lobo 2017a; 2017b).

Considering, on the one hand, that, roughly speaking, probabilities are a kind of possibility, which, in contrast with empty formal possibilities, should be called "loaded" or "intensive possibilities" and, under some conditions, countable or measurable possibilities; considering on the other hand, that *transcendental logic* deals with another kind of possibilities:

"real possibilities", or "possibilities of objectivity" or else "possibilities of a full meaning" of concepts and judgements, ruled by certain *a priori* conditions; we are entitled to infer that the inclusion of probability as a modality within the ground framework of formal logic is essential to understand why and how this change of the logical status of probabilities led Husserl to rehabilitate *transcendental logic* and redefine in new terms its essential tasks, as well as those of a critique of logical reason[3]. Furthermore, this helps us to understand why the first task of this transcendental logic is to promote and justify a deep reform of formal logic, by surveying the larger domain of formal disciplines such as axiology, practice, choice, etc. [4]. This should explain also why the project of transcendental logic takes place in the frame of a fully new discipline: transcendental phenomenology. Last but not least, if we take into consideration that the turning point occurred around 1909 (while the outburst of probabilities in contemporary mathematics and physics was not already achieved nor fully acknowledged), this illustrates – if it does not demonstrate it – the epistemological relevance of phenomenology.

In the following, I shall propose a brief overview of the evolution of Husserl's conception of probability; a presentation of the problem of the logical foundations of probability calculus following Husserl (following Von Kries, and others like Boole); a sketchy presentation of Husserl's logic of probability and the foundations it provides to probability calculus, and some prospective views on the larger theory of manifolds obtained by considering "probability spaces" as manifolds of (equally or unequally) loaded possibilities, as fields of possibilities varying in intensity; before concluding with some "snapshots in the twilight".

1. Brief overview of the evolution of Husserl's conception of probability

As long as probabilities were at the periphery of pure logic and pure mathematics, all epistemological issues touching the application of mathematical concepts or of formal forms of reasoning to empirical contexts were relegated or ascribed to the much looked for, but informal "logic of induction" or "inductive logic."[5] The most difficult and epistemologically promising aspect of Husserl's logical approach is the promotion of probabilities thus understood as the *core* and *fundament* of formal logic, and *not* as a secondary extension. As soon as probability becomes part of the ground structure of formal logic and pure mathematics, this hierarchy as well as the distinction between applied and pure mathematics loses their pertinence. Such was the position of Husserl in 1901 until the shift afore mentioned occurred at the turn of 1909.

This shift has not been sufficiently noticed, if not fully ignored or vigorously repressed for various reasons.

The first important reason is that Husserl's position on probability is quite difficult to locate in the battle field of interpretations around probability calculus: *a priori* vs *a posteriori* probability, subjective *vs* objective, logical *vs* psychological, etc. Going beyond some discordances, Husserl seems to belong to a tradition known as "range theory of probability", which is a sub-division of the logical approach to probability, whose major figure are Boole, Stumpf, Peirce, Ramsey and Keynes[6]. Probabilities are attributed in the first instance not to events, processes or things in general, but to propositions (Keynes 1921, 10-19). More precisely looking at the logical concepts and principles promoted by Husserl and Keynes, we could pinpoint the influence of the neo-Kantian Johannes von Kries (Fioretti 1998, 2001; Heidelberger 2001; Rosententhal 2010). But, as we shall see, because of the inclusion of modalities and probabilities in the "formal matter" of terms and propositions, the widened notion of "formal truth" and the theory of judgment and proposition are accordingly deeply reshaped.

A second reason, strictly connected to the former, is that, as long as we identify Husserl's conception of formal logic with one of the former or contemporary conceptions of so-called classical logic, and its theory of multiplicities, with that which prevails as the foundation of modern mathematics, i.e. set-theory, we remain inevitably blind to its modal core and, consequently, Husserl position either remains invisible, or appears as inconsistent within the set theoretical frame promoted by Kolmogorov's axiomatization of probabilities in 1931 (see Kolmogorov 1956).

For these reasons, Husserl's theory of probability has remained almost completely unnoticed up to now, among mathematicians as well as logicians and philosophers, even those interested in the question of the "foundation of probability."[7]

Husserl's conception of probability is clearly exposed for the first time in the *Lessons on Logic and Theory of Science* from 1909 (Hua 30). It belongs to a tradition which goes back to Leibniz (Couturat 1901, 239-240), according to which the foundation and the correct interpretation of probability calculus requires a *logic of probability*, i.e. the consideration of the form and the content of a special kind of propositions. This position, which is that of Keynes too, was strenuously rejected by the dominant figures of the modern theory of probability, such as Borel[8]. In the *Prolegomena*, the name of Von Kries is not mentioned in reference to probabilities, especially when Husserl refers to the status of probabilities as *derived logical forms* compared to plain logical propositions, but in connection to the distinction between deductive sciences and descriptive sciences, i.e. *nomological (deductive) sciences* and *ontological (descriptive) sciences*, which is clearly taken from von Kries and his book on probability (Hua 18, § 64)[9]. But when considering the "ideal conditions of possibility of science in the most general manner", Husserl admits the existence of "ideals elements and laws even in the field of empirical thinking, in the sphere of probabilities", as an "*a priori* basis", as pure conditions of possibility of empirical science in general. Yet Husserl, on one stroke, rejects probabilities outside the sphere of pure logic and, consequently, the possibility of a transcendental logic is brushed aside (Hua 18, § 72). They represent the second fundament of the technology of science at work in empirical sciences to approximate, through successive revisions, the pure form of a scientific theory. Consequently, the manifolds on which probability measures are implemented remain outside the field of pure (definite) manifolds, so to speak, mathematically outlaw.

Probabilities are primarily subjective modifications of judgment which can become objects for new judgements of second order, expressing thus mere asymptotic approximations of truth judgement in the strict sense of the term (Hua 18, § 6). The content of probability judgement does not enter the formal realm of logic, and the wider form of knowledge does not imply any enlargement of pure logic. We can talk of a "logic of empirical sciences", only in a derived sense, as a "technic of evaluating probabilities and founding probability" (Hua 18, § 64). In the lessons from 1906/07, despite recent development in the mathematical treatment of probabilities[10], the constitution of a formal theory of probability seems still dubious. The quantification and numerical determination of "degrees of justified conjectures", built up under the form of a deductive discipline is not enough. What is measured remains something subjective and ambiguous. As in 1901, Husserl's refers to Laplace fundamental notion of "equipossible cases" as "state-of-affairs" of which we have no knowledge, or rather of which we are in a state of "no-knowledge" (*Unkenntnis*). The underlying principle is known as the indifference principle. According to Laplace here rephrased by Husserl, the meaning of probability equations is nothing more than a subjective mixed state of ignorance (*Unwissenheit*) and knowledge (*Wissen*)[11].

In order to eliminate the thread of "probabilism" (Hua 18, § 22, [65]), – a variant of Psychologism and Skepticism – the first reaction of Husserl is to expel probabilities from the sphere of pure logic. The mathematics of probability are not formal mathematics ("*aber formale Mathematik ist das nicht*"). Probability is nothing more than a modalized certainty[12] and the correlative "state-of-affaire", each time at stake, appears as such, *just as it would be posited in an apodictic evidence as certain, if we could reach the level of evidence for such a judgement*, i.e. *ideally*, — but haloed or fringed by subjective modifications of the "holding-for-true", namely that of presumption. In other words, the "logical content" or "matter" is exactly the same; i.e. nothing of the modification of belief enters the content of judgment and the probability apparatus remains outside of the logical sphere of judgement. The change of modality or modalization is presupposed to be parallel to that of fulfilment, of making-evident, but fully independent of the process of identification and determination[13]. Probability calculus remains just a sophisticated device, among many others epistemological technologies and a substitute (or surrogate) for an evident and certain judgment. At any rate, "probability cannot rival truth, nor can any presumption rival intellectual evidence (*Einsicht*)" (Hua 18, 75); the more that probability can do is to strive asymptotically toward such a full and complete evidence, to approximate it (*ibid*. 29).

The knowledge, broadened through inclusion of a wider range of belief "distinguishing reasonable from unreasonable, better from worse-founded assumptions, opinions and surmises", is not knowledge in pregnant sense. Correlatively, the ontological domain of probabilities relies ultimately on empirical facts, on individual existences. Since *every probability*, even the highest and most valuable, refers back to an "existential content" (Hua 18, 84), even a deductive theory of probability must be expelled from the sphere of *Mathesis pura*, which is, by definition, freed from any existential supposition, or, which amounts to the same, the demonstration or the postulation of possibility is tantamount to that of existence, while mathematical impossibility is identical to inexistence[14].

But as we said above, the relation of formal mathematics to "material mathematics" is regulated by ideal laws, which are epistemologically relevant for the understanding of the applicability of mathematics and the progressivity of knowledge. The modal (probable) characters of the facts on which empirical theories rely, impregnate, so to speak, the theories and the knowledge themselves. On the other hand, *these epistemological situation and relations* must be logically exposed and explained. And indeed, a "pure theory of probability" should study the different ways in which an empirical theory is modified, and occasionally enlarged, as well as the ways in which a formal domain (such as that of arithmetic or any definite axiomatic system) is modified and enlarged (Hua 12, 452 *et seq*.). Nonetheless, Husserl maintains that this pure theory of probability is not part of pure logic.

The *Prolegomena* conclude with a programmatic "pure theory of probability", whose status and tasks remain ambiguous and rather undetermined, because of the amphibology of the very notion of probability, which reaches here its climax. Following the classical definition (at least until its absorption into measure theory), probability designates sometimes the rational number resulting from the comparison of favorable cases over the totality of equipossible cases ; sometimes the substitute of evidence for empirical knowledge as such, or for the application of mathematical models, or for the degree of approximation to the postulated ideal theory obtained through this modelling; sometimes the ideal hypothesis or subtractions underpinning this application. In this later sense, the pure theory of probability should be the logic posing the ideal laws governing the production and the validation of such idealizing fictions, and explaining why a theory is enlarged in order to cope with experimental facts contradicting it, or why another theory is rejected although no experimental fact invalidates it. But rather equivocally, Husserl concludes that the "ideal elements and ideal laws" founding the "possibility of empirical sciences in general" and the "idea of the unity of empirical explanation" belong to "pure logic" *only in a correspondingly extended sense* (Hua 18, 258]) — an external extension which constitutes the second theoretical fundament of logic normatively converted: i.e. technology properly speaking (Hua 18, § 72).

In the *Introduction to Logic and theory of knowledge* from 1906-1907, the idea of a pure logic of probability is not yet clearly defined. These lessons provide an exposition of a noetic as a "pure theory of law (*Rechtslehre*) of knowledge", i.e. as a theory of validity and possible validity of knowledge in general and an exploration of the eidetic frame of logic, i.e. the eidetic underpinnings of its pretension to set the norms of valid knowledge. The mathematical treatment of probabilities is thus promoted to the rank of *fundamental* technology of knowledge[15]. And this is a real promotion indeed, since the vast domain of effective mathematics is equipped with powerful symbolic technics and tools. And beyond the mathematical discipline which deals quantitatively and numerically with degrees of presumption and belief, Husserl acknowledges the right of a new deductive discipline *closely linked* to "formal logic as *mathesis*" (Hua 24, 132). Overall, the reduction of probability to a numerical calculus nourishes a reasonable hope of bringing the theory of probability inferences, which play an essential role in all empirical sciences, closer to the theory of science. Yet this deductive theory of probability remains outside formal logic and is not part of the noetic; since it is itself in need of justification and formalization.

In the lessons on *Logic and theory of science*, which started in 1909, the delimitation of logic proposed in the *Prolegomena* becomes controversial as well as the division of labor between mathematician, logician and philosopher (Hua 18, 255). As in 1901, Husserl still denies any polemical intention against logicians and mathematicians, and any revisionist intention[16]. Mathematicians remain the "only competent engineers for effective constructions", while the philosopher's task "resides in a totally different direction": that of "proposing a complementary reflection in the essence and the meaning of fundamental concepts and prevailing fundamental laws, and not, at least, in the considerations of the internal relations of those disciplines to all other disciplines." (Hua 24, 163) The aim of such a philosophical consideration entails immediately a redefinition of the relations of logics to mathematics. We are invited even to "skip traditional syllogistic, which is only a piece of pure mathematics, i.e. pure mathematics of possible propositions and possible predicates in general, and the whole field of theoretic analysis".

But since the main goal is precisely defined as a "deepening and enlargement of the idea of the theory of science", we cannot escape the conclusion that, at least, the delimitation of pure logic and of pure mathematics will not keep untouched.

Since the culminating point appears under the heading of "logic of probability" it is of interest to grasp what Husserl understands under this expression.

2. The problem of the logical foundations of probability calculus following Husserl (following Von Kries)

The turning point leading to a logic of probability as a fundamental part of pure logic is placed under the influence of von Kries. Before Husserl, the opposition between *a priori* and *a posteriori* probability was already criticized, displaced and complicated by Johannes von Kries's investigations on the *Principles of probability calculus* (Von Kries 1886).

This reference, as it is better known by recent publications, although discrete, is important for the subsequent historical development of the logical theory of probability. As we already saw, Husserl borrows from him the distinction between nomological sciences and ontological sciences, but above all the concept of "range" ("range of play" or *Spielraum*) (Heidelberger 2001; Fioretti 1998; Keynes 1921, chap. VII, 97 *passim*)[17]. This notion will become a central concept in the description of the subjective constitutive manifolds, of the horizon structure of consciousness, of the processes of determination, and the dynamical relation between intention and fulfillment. But before that, this notion provides the fundamental concept of Husserl's logic of probability and the fundamental principle for a logical interpretation of probability which be illustrated by Keynes.

Von Kries's investigations on the logical principles of probability calculus help us to understand Husserl's subsequent evolution. Von Kries is aware that a purely subjectivist interpretation of probability theory reaches an impasse and that a logical foundation of the calculus could not depend on ambiguous or arbitrary principles such as the *indifference principle* or the *principle of insufficient reason* (Laplace 1921, 2). He criticizes also the interpretation of "probability propositions" in terms of practical expectation. Because probability calculus does not measure a mental state of belief, but expresses a logical relation between two quantities[18], the transition from the logical concept of probability to measure needs itself to be logically justified and clarified. This way of setting probability calculus, if not circular, as it is frequently argued (Poincaré 1896; von Mises 1957, 67)[19], is at least arbitrary and apply only to countable sets of cases[20]. Even if we put aside the important problem of continuous (or geometrical) probabilities, this amounts to an assumption which holds only for a limited number of cases and limit-cases.

If probabilities are grounded on a set of equipossible *items* (cases, events, experiments, etc.), then "all the secret of probability calculus" lies in the *construction* or the *setting* on a manifold of equiprobable cases, or which amounts to the same of an homogeneous distribution. The principle of indifference or insufficient reason states: two or several cases must be considered as equally possible if, whatever our state of knowledge, we cannot find any reason to hold one for more probable than the other[21]. This presupposition requires a cautious investigation, since otherwise and in case its consistency is fully demonstrated, probability calculus will amount just to a useless symbolic game. This certainly justifies that we *do not* attribute *more weight* to one of the possibilities than to the others, but that surely does not justify the *positive attribution* to each one of a strictly equal weight. The calculation is thus groundless.

That does not mean that we should dismiss any logical approach in favor of an *a posteriori* or frequentist approach. Since we do not want either to dismiss the requirement of a logical clarification and foundation, in favor of an *a posteriori* or frequentist approach or to depend on psychological investigations, we must look for a sound and non-arbitrary logical principle, fundamental enough to account for discrete and continuous, but also homogenous as well as non-homogenous manifolds of possibilities, of possibilities equally and stably loaded and possibilities unequally and unstably loaded (Von Kries 1886, 15). What is at stake here is the enlargement of mathematics to inhomogeneous fields, i.e. fully random spaces, where homogenous distributions are just an important but limit-case, or to put it in modal terms, where possibilities are diverse in intensity or variably loaded. The parallel between the geometrical starting point of the theory of manifolds (with Riemann) and the one dealing with random or stochastic manifolds emerges here[22].

Von Kries investigation is motivated not only by a purely philosophical interest on the so-called foundation of probability, but rather because of persisting difficulties and paradoxes which have accompanied the birth and the development of probability calculus since the time of Fermat and Pascal. As it has been observed from the beginning of its development, this calculus sets innumerable problems of interpretation in its "pure" form, as well as in its so-called "applications", with a constant shift and confusion between statistics and probabilities. It has given way to famous paradoxes (such as D'Alembert's or Bertrand's paradoxes, without mentioning those of quantum physics). The conflict between the combinatorial starting point (enumeration of possibilities) and the continuity character of many probabilities has been partially tamed by Kolmogorov's axiomatization (1931). But such a taming presupposes, as Pierre Cartier notices, rather strong assumptions, those of measure theory and Lebesgue's integral, generalized to abstract spaces (Cartier 1985, 15).

One of those paradoxes, quoted by von Kries, stems directly from the equivalence between "the principle of ignorance" or "insufficient reason" and the equipossibility or equiprobability — to hold a proposition *A* for true or false. Von Kries alludes to other paradoxes such as Bertrand's or to d'Alembert's objections[23]. I follow here Zabell:

> "Before we possessed any means of estimating the magnitudes of the fixed stars, the statement that Sirius was greater than the sun had a probability of exactly 1/2; it was as likely that it would be greater as that it would be smaller; and so of any other star" (212). Using the very same example of Sirius (making it clear that Jevons is von Kries's target), von Kries showed (10–11) how this type of reasoning could be used to arrive at contradictory results. Thus, arguing one way, the probability that there is gold on Sirius is 1/2, that there is iron is similarly 1/2, and therefore that there is neither is 1/4. (Of course there is an— unargued — assumption of *independence* being made here.) Taking the 68 elements known at the time and arguing in similar fashion gives a very small probability that none are present, or equivalently a very large probability that at least one is present. On the other hand, starting immediately from the proposition "Sirius has an earthly element", one immediately arrives at a probability of 1/2." (Zabell 2016, 135-136)

The experiment is the following. In throwing a coin twice, calculate the probability of showing Heads. The classical enumeration of cases (HH, HT, TH, TT) answers 3/4. D'Alembert (1784, 471) argues that if I got H with the first throw, the game is over and a second throw is pointless. The *order* is thus essential and the present order of outcomes produces, so to speak, *a reduction or a collapse of the manifold of possibilities*[24]. This objections seems itself pointless and has been indeed unanimously rejected, to the exception of Bayes in his *Essay* from 1764 (See Zabell 2016, 136). Nonetheless nobody could demonstrate where the argument went wrong.

Von Kries gives other examples which indicate clearly that any probability depends on the assumption that the fundamental manifold is complete – one would say, in Husserl's sense, *definite*. Targeting Stanley Jevons reasoning, Von Kries develops a new paradox, taken from Jevons: "If A and C are wholly unknown things, we have no reason to believe that A is C rather than that it is not C; the antecedent probability is then 1/2." (Jevons 1874; cf. Keynes 1921, 46)

In order to avoid similar paradoxes, Von Kries (1886, 36-37) introduces the principle of ranges (of play) (*das Prinzip der Spielräume*), which states "that assumptions are in a numerically probability relation, if and only if they include *mutually indifferent (original and comparable in size) ranges of play* (*Spielräume*)", and "that certain probability values arise, *where the totality of all possibilities can be exhausted by a number of such assumptions*." (ibid. 36)

This principle is fundamental and logically sufficient. It will be rephrased by Husserl under the title of *fundamental field principle*. If the "fundamental field of equal possibilities is not defined univocally" or if "it is mixed up with a different field", we fall then inevitably into wrong inferences and paradoxes. Husserl considers these paradoxes as "deceiving inferences" (or paralogisms) stemming from the lack of clarity and soundness of the logical foundations of probability calculus. In order to resolve them all on one stroke, he adopts a principle equivalent to that of Von Kries, although disguised under new terms, the "fundamental field" ("*Grundfeld*") principle[25]. "All the deceiving inferences in probabilities and in the theory of probability itself, so much dreaded but still non cleared, are based on the fact that either the fundamental field of equal opportunities has not been defined exactly or univocally, or, in spite of rigorous definitions, that, in the course of reasoning, the initial field has been confused with another one. In this interweaving of probabilistic inferences there are, as a rule, different fields, but always one field is the fundamental field so much so that all the other fields are extracted from it exactly or loosely." (Hua 30, 253-254)

If the expression of "*Spielraum*" is eclipsed as a mathematical or logical term, designating a more subtler form of manifolds, it will reappear as a fundamental phenomenological term, as a fundamental character of any subjective constitutive manifolds, i.e. the fact that it is *open* and never fully *saturated*, i.e. as a constitutive moment of the horizon structure. All the subsequent difficulty lies thus in the conceptualization and formalization of such non-definite (i.e. incomplete) manifolds.

3. From the project of a logic of probability to the project of a reform of formal logic

We must now explain how this principle leads Husserl, contrary to von Kries, to a reform of logic and an enlargement of the formal theory of manifolds. Here phenomenology as such steps into the game.

The major change is the *phenomenologically* enlarged notion of *proposition*, which covers all forms of "positions" or "thesis". These modalities that Husserl names "doxic" along with "axiological and practical" modalities must be understood as noetic as well as noematic determinations. Their explicit thematization opens the larger fields of formal disciplines (including formal axiology, formal theories of action, decision, choice, collective choices, games, etc.).

This enlargement is necessary to fill a gap which persists throughout the historical development of formal logic, as Husserl repeatedly says, in 1923 (in *Erste Philosophie* (Hua 7, 21-22) and in *Formal and Transcendental Logic*, in 1929, as we saw above). The default of formal logic (old and new) resides in the fact that every material and intuitive content is eliminated, because it is wrongly assumed that any intuition and any content are necessarily empirical. Blindness to categorial intuition goes obviously on a par with deafness to modalizations. Nevertheless, it is a disastrous mistake for the very understanding of formalization to eliminate the very possibility of a formal content, a "formal matter" that is conveyed by qualities (i.e. modalities) of acts, in general, and of judgement in particular. Hence Husserl argues that traditional formal logic does not "include amongst its theoretical elements neither the concept of truth nor its derivatives and modalities", i.e. concepts such as "possibility, necessity, probability etc. and their negations". And this represents an "inadmissible restriction" (*unzulässig Beschränkung*) (Hua 7, 26), which has hindered the development of an efficient logic of truth, describing formally "how judgements can reach material adequacy" and "how their truth and falsehood are decidable" (Hua 7, 25). From this "very important lack" (*sehr bedeutsamer Mangel*), ensued serious imperfections of logic, especially in its "methodological procedures" and in the constitution and understanding of formal mathematical fields, and more precisely regarding the definition of probabilities.

The controversies around probabilities, and especially between subjectivist and objectivist, *a priori* and *a posteriori* stem from the fact that modalities (on each side of the battle field) are considered as exclusively subjective, psychological and empirical modifications. From this psychologist prejudices stem also false analogies, such as that between degrees of sensation and degrees of belief (with Wundt, Fechner and Meinong). The discovery of intentionality should prevent from such misleading analogies. If any act has its sensuous and emotional substrate, objectifying acts as well as axiological acts (acts of feeling and willing) have among their inner intentional constituents modalities in the broad and the narrow sense of the term.

This enlargement and deepening, following the intentional correlation, goes on a par with an enlargement of the noematic thesis or "propositions". Moreover, considering that each position and proposition is produced by a kind of modification or "function" that Husserl calls either "qualitative" modification (or "modalization"), the notion of predication and predicative function should be enlarged accordingly. Last but not least, acts of reflection, whatever their kind, for instance those underpinning an act of nominalization (Husserl 2006, 75, 97-105; Husserl 1969, 113-118), are themselves such modifications, and eventually combinations of modal modifications or neutralizations. But to restrict ourselves to probability modifications, phenomenologically speaking, probabilities are *relational modalizations*, comparisons and evaluations of the respective "weight" of manifold possibilities (and within samplings of such manifolds) emerging from spontaneous "thematizations" of specific intentional modifications, "modalizations" of the moment of "belief"[26].

Without dwelling on the details of the theory of science thus promoted, we must insist on the deeper and larger concept of science, resulting from the inclusion of modalities. Instead of being restricted to the "sole knowledge of apodictic truth" i.e. to demonstrative knowledge, the methodology and theory of science must "explore the immense variety of the concrete life enfolding in man's mind, during his intellectual work" in "which he lives without noticing it" (Hua 7, 39-40)[27]. These investigations are not purely informal, and without inputs in the determination of the tasks of formal logic. One of the most noticeable consequences is precisely the proposal of a logic of probability as a fundamental part of formal logic. The inclusion of probabilities in the sphere of formal logic entails thus a reframing of formal logic and mathematics.

The turning point to my view, despite visible hesitations, can be dated from the lessons on *Old and New Logic* from 1909, in which Husserl explicitly mentions the possibility of an enlargement of formal logic through inclusion of probabilities[28]. I can here but give some spot checks on the lessons on *Logic and Theory of Science* given from 1909 onward.

3.1. An enlarged and deepened concept of proposition

First of all, what is the wider formal concept of proposition which gives way to "intensive" or "loaded" possibilities understood as specific propositional functions?

Let us start with judgments and their predicative propositions. The traditional view point considers propositions such like "it is certain that *p*" or "the certainty that *p* is justified" as well as "it is doubtful that *p*", or "there is a doubt whether *p* is valid", etc. as belonging to logic in an enlarged sense. But they are excluded from the description of the basic forms of propositions, i.e. from morphology of meaning or logical grammar defining what are well formed formulas. It is even possible to examine the conditions under which a proposition expressing a doubt, a question, is valid, i.e. is rational. But "they have no place in the frame of the theory of judgement, understood as meaning of acts of judging". "The same thing holds for judgements of possibility and necessity". "As long as their meaning includes, contains, a subjective and empirical content about he who judges, on his opinions, his knowledges, conjectures etc.", "these distinctions have no room in formal logic."[29]

But as soon as we get rid of psychologist assumptions, we must admit that every judgement contains a certain "quality", i.e. a certain modality (inclusively plain "assertions") and that an "unqualified" (or "non-modalized") judgement is nothing but an *abstract constituent* obtained through a *sui generis* modification (precisely that of "bracketing" or neutralization), which instantly displaces outside the brackets the original mode of assertion (of certainty) — Frege's famous assertion stroke. Yet a universal bracketing remains ideally possible.

The fact that currently probability judgements appear as *secondary forms* of judgments, as judgments *about previous* judgements, does not entail that elementary (or "first order") judgements be deprived of any quality; more precisely, that the content of original judgments or even of primary representations should be deprived of any modal component[30]. The primitive and fundamental form of judgement is always a compound of modalities (characters of positionality) and a mere "as such" (*als Was*), a mere something which, without those modal characters, remains formally an empty "something whatever" (*eine leere beliebige Etwas*) (Hua 30, 106, 140). Consequently, we must admit as original propositional forms: the "proposition of truth" (in the narrow sense of the term), the proposition of probability, the proposition of question of knowledge (*Wissensfrage*), the proposition of doubt but also propositions of will, of wish, and their corresponding sub-modalities etc. (Hua 28, 119 *et seq.*)[31]

Moreover, and generally speaking, against the common prejudice at the basis of the so-called "linguistic turn" in philosophy, the proposition in the narrow sense of the term, i.e. as *expression* of a predicative act presuppose the later one as such, i.e. as an *expressed* act, which involves or at least presupposes pre-linguistic and pre-grammatical acts. By limiting our consideration to the series of propositions characterized by their doxical character, as "holding-for-true-something" and correlatively "holding-for-being-something" in the various modes of "holding-for-certain", " for-possible", "probable" etc. these acts are presupposed by their expression, and exist, at least ideally, be they expressed or not[32].

3.2. Consecutive enlargement of formal logic and constitution of logic of probability

The domain thus delineated is "nomological" in the proper and deeper sense of the term, since logic is fundamentally an examination of all the modes and essential laws governing the "position-of-truth" (*Wahr-Setzung*) as a quality of act. Husserl not only admits a "formal logic of qualitative modalities (*eine formale Logik der qualitative Modalitäten*), of possibilities and probabilities as well as a formal logic of problematicities (*eine formale Logik der Fraglichkeiten*)", as a discipline belonging to the same ideal sphere (that of pure logic), but he asserts that "pure logic" is two-fold, and that we must admit as a first and fundamental group of logical laws, "the laws of probabilities, of presumed possibilities", i.e. the rules that are at the basis of the rational norms of validity of probabilistic inferences, of questioning, problematizing, doubting, etc. (Hua 30, 79). This norms (and the *Normierung* modification as such) are not grounded on psychological empirical findings, but on an eidetic analysis of intentional essences, and the delineation of the central constituent of every intellectual activity, the "sphere of doxic positionality", of "acts of belief", of "holding-for-true" in the larger sense of the term.

The perspective of the theory of justification of acts of knowledge is not limited to the connection of assertive judgments, nor to proof theory, but extends to the all sphere of acts partaking in the process of justification (*Rechtsausweisung*) of judgements (such as perception, memories, etc.). Husserl describes this as an enlarged theory of epistemological norms encompassing a wide range of "pure disciplines" still to be constituted, which, if they were constituted, would enable us to reduce to ideal principles and decide according to those principles, any epistemological situation, any actual case of judging in all its form (presumptive judgement, probable judgement, etc. as well as any actual case of founding, justifying, inferring deductively, explaining, or inferring inductively, etc.).[33]

We understand better why the critic of logical reason started here entails a reform of logic and an interventionist conception of the epistemological role of phenomenology, for this enlargement underpins a new settings of the norms of validity, starting from those of the holding-for-true. These norms don't apply exclusively to the "lived experience" of assertive judgements, but to every act of judgement "in the widest sense of the term", i.e. "lived experiences of holding-for-possible and holding for probable, of questioning and doubting". And since those norms are intimately connected to mathematical forms and norms, "a new perspective of interpretation of many norms of the pure mathematics is thus opened up."[34]

3.3. Logic of probability and manifolds of intensive or loaded possibilities

This Idea of a logic of probability leads to that of a formal logic in a deeper sense, to "an enlargement of the idea of pure logic into a pure logic of probabilities and possibilities", and, correlatively, intertwined with it "to an enlarged pure arithmetic and pure theory of manifolds."[35]

Probability fields as modal manifolds. The pure formal manifolds must be classified following deeper principles that the usual one (discrete *vs* continuous, measurable, countable, signature and degree of curvature, with or without torsion, connected or not, etc.), i.e. the *forms of possibility*. Beyond the distinction between physical and logical and mathematical possibilities in the usual sense of the term, we must distinguish between analytical possibilities, synthetic *a priori* possibilities. But the latter ones must be divided in turn into "extensive" possibilities and intensive possibilities.

Probabilities are "intensive" i.e. founded possibilities, in as much as they are "loaded", so to speak, because something "talks in favor" of them. In contrast, classical mathematical possibilities are "empty possibilities", "mere imaginations", in as much as nothing talks in favor of them, except the general fact that they can be deduced or constructed. *They are modally flat, of nil curvature*, to speak analogically. A probability as an empirical conjecture or a presumption (*Vermutlichkeit*) is a founded possibility. A likelihood or a plausibility is the same thing than a "probability": a *founded possibility*, a possibility implying and presupposing fundaments of plausibility — a possibility loaded with diverse fundaments, variable in number and weight. Contrary to a misleading analogy, these possibilities are not necessarily discrete and independent, nor "continuous" and extensive, but they must be connected. They represent a primitive form of connected manifold, maybe more fundamental than the connected manifolds developed in the wake of Riemann's prophetic conference by Weyl and E. Cartan (Cartan 1923, 326).

Measure. What is usually expressed in terms of *degrees of intensity*, or in subjective terms, of degrees of belief[36], are an improper expression of the *number of foundations of probability*. This point is very important for a justification and a setting of the fundamental algebraic operations of addition and multiplication, before secondary distinctions such as that between discrete and continuous probabilities.

The subjective expression talks of: "*more or less strong or weak presumption*". In contrast Husserl says: "the *presumption is reinforced by the number of foundations* of *probabilities*: the more things speak against a probability and the more the probability decreases." (Hua 30, 252) As a founded mode of possibility, an intensive probability implies a countable sub-manifold of fundaments of possibility.

Husserl goes on analyzing this modal field. He who has learned, through the analysis of intentionality, to separate what is on the side of consciousness (or *noesis*) and what is on the side of meaning (or *noema*), will recognize, without difficulty, in the present instance, an "*objective expression*", i.e. that the founded possibilities as well as their fundaments are the correlates, contents or "significations" of new acts.

These founded or loaded possibilities are originally ruled by a *relation of preference and intensification*, and consequently by *laws of increase*. The manifolds of possibilities are ordered following these relations of intensification, with a simple possibility (without negative or positive fundaments), positive and negative intensities of possibility, comparable to positive and negative quality in the sphere of assertions. A difference remains, for in the sphere of assertions, "there are no preferences and intensifications", whereas in the former, "in the domain of probability", "they play such an important role." (Hua 30, 253)

Negation and quantification. A negative presumption, something speaks against "A is B" is equivalent to "something speaks in favor of the fact that 'A is not B'". For this reason, we have a rich variety of negatives possibilities. Before any partition, we have here an original domain of additive positive and negative magnitudes. Two possibilities with equal weights are indifferent in a totally different sense than the empty indifference ("nothing speaks in favor of *p*"). The first indifference is an equality of weight between negative and positive fundaments, an "absolute problematicity". In case there is no indifference between two possibilities, one is necessarily heavier than the other one. It is then strictly more probable that *p* than *non p*. Probability in the strict sense is thus the *relative overweight* of the positive motives – compared to the negative motives, if any. In some circumstances, vague expressions "strong", "weak", "very restricted" probability can be converted into exact ones (numerical or measures), but often it is not possible (Hua 30, 253).

4. Snapshots in the twilight[37]

On the footsteps of Husserl, the mathematician and phenomenologist Gian-Carlo Rota suggested in different papers, that Husserl's phenomenology was aiming at providing logic with new fundamental concepts, new constants. He insisted simultaneously on the limits of the logical syntactical approach to probability and even expressed strong doubts that probability theory, despite its axiomatization (after Kolmogorov), possessed any true syntax. Rota in his incentives investigations on the foundations of probability theory and statistics aimed at bridging the gap between probability theory and other mathematical theories, including some parts of first order logic and algebra. Rota mentions, as a possible application and as "the most promising outcome", the translation of "the notion of quantifier on a Boolean algebra" into that of "linear averaging operator": "in this way, problems in first order logic can be translated into problems about commuting sets of averaging operators on commutative rings" (Rota, 1973).

The semi-formalized analyses of phenomenology anticipate thus quite strikingly Rota's suggestion that the general form of conditional probability is similar to Reynolds operator (Lobo 2017b, 156-170). The fact that this operator is used in what is considered as belonging to "applied mathematics" is not a reasonable objection. Historically, most of the formal mathematical theories (Euclidian geometry, vector calculus, graph theory, etc.) have emerged from semi-formal fields, and have been only secondarily "purified", that is detached from their empirical or "material" (*sachhaltig*) clothing.

My guess is that by symbolizing the relations and the laws exposed by Husserl in this text and later writings, especially *Ideas I*, we get very close to a form of operator. Mathematically: the logical expression and formalization of this system of modifications gives way to a *linear functional* which constitute the hidden hypothesis of the so-called axioms of probability calculus (in Kolmogorov). By formalizing it, we obtain an operator which is similar to Reynolds operator, which is known in algebra as *Reynolds operator* and in other fields as *averaging operator,* and writes: $Au = u$, $Az = z$. Would not it be possible, by introducing "belief" functions, under the form of modal functions, to obtain, via an adequate formalization, an operator of the type $A(fAg) = AfAg$?

4.1. Phenomenology, algebraic logic and logic of probability

But this requires more generally to understand better Husserl's position toward algebraic logic. Beyond the current characterization of formalization as algebraic transformation, that is an emptying of any material reference or content, Husserl's reception of algebraic logic (as promoted by Boole, Peirce, and subsequently by Halmos and others) has been concealed by the focus of the dominant debate (formalism *vs* logicism *vs* intuitionism; Hilbert, Frege and Heyting). Yet, Husserl constantly considered Boole as "an outstanding technician in logic", although "a very mediocre philosopher of logic" (Husserl 1994, 59; Hua 22, 9)[38]. One must consequently not use the philosophical occasional nonsenses in which he fell as an excuse to reject his "splendid" logical construction. Husserl's enthusiasm of the early years is still perceivable in later texts from 1913 onward (Hua 24,162; Hua 17, 83; Hua 30, 271-272). Against the critiques from the side of logicians, as well as the attacks from philosophers (such as Lotze or Windelband) (Hua 24, 162, trans. 160; cf. Hua 30, 248-249), he praises Boole for having achieved "at one stroke" almost miraculously (Husserl 1994, 88; Hua 22, 40), a logical calculus. The "reduction", i.e. "ingenious transference of the arithmetical algorithm over the domain of class, through which the class calculus stood forth at one stroke, is almost a miracle" (Husserl 1994, 88; Hua 22, 40; Husserl 1994, 441), showing convincingly that class calculus and arithmetic were but two provinces of the same country (Hua 17, 203; Husserl 1969, 78). The interpretations of the 0 and 1 (as meaning respectively the logical universe or the total class and the null class) may lead to absurdities, as demonstrated convincingly by Schröder (Husserl 1994, 84; Hua 22, 35-36)[39]. But Schröder's argument is itself considered by Husserl as "sophistical": it rests on a confusion between "subordinate class" and "element", and correlatively between two separate relations (inclusion and membership). This demonstrates only that Boole's method must not be applied blindly, but this does not concern the "technical" as well as the "mathematical" presentation, which Husserl considers "exemplary" (Husserl 1994, 88; Hua 22, 40) and of indubitable "superiority" over the old methods of inferring (Husserl 1994, 90; Hua 22, 42). Similarly, Schröder and Venn's substitution of the concept of "identity relation" to the Boolean concept of "exclusive addition" demonstrates rather the superiority of the later concept, since this concept alone enabled "the ingenious transference of the arithmetical algorithm to the domain of classes" and the miracle of a "logical calculus".

Above all: Husserl adopts the same attitude towards Venn's critiques of Boole's choosing the "exclusive addition" instead "identity addition" and so-called improvements (Husserl 1994, 88; Hua 22, 40). Against Venn, Husserl argues that the importance of the calculus does not lie in its practical applications and value (for instance, spare of time in reasoning). Boole's algorithms are worth logically and mathematically, not practically, although Husserl does not exclude that future epistemological situation requiring such sophistication and other "fruitful applications". It is true that, from a theoretical point of view, the current scientific forms of reasoning (in mathematics and physics) do not enter into inferential complexity such as to require the sophisticated apparatus provided by Boole[40]. But once again this is no objection against Boole main theoretical goal, among which the application of its logical algebra to the calculation of probabilities, and its analysis and elucidation of probability. For, this "application" presupposes the distinction between "conceptual extensions" and "sets", and of the formally defined identity relation over sets[41]. Boole has thus shown that the mathematical treatment of probability "*is itself a part of logic*" (Hua 17, 203; Husserl 1969, 78). This "application" is absolutely central in the project of Boole[42] and has survived through the contemporary branch of algebraic logic (see Rota 1973; Ellerman & Rota 1978). The brilliant insight of Boole of the deep formal analogy between arithmetic and syllogistic (Hua 30, 272) has given the first sample of successful formalization in the field of logic, so frequently misunderstood by philosophers in their polemics against "mathematizing logic", as well as by the mathematicians, unaware of this essential distinction within the field of arithmetic (Hua 30, 271) and of the relation from purely formal theories (which are merely hypothetical theories) to mathematical theories (which without being applied nor material are nonetheless true, i.e. categorical) (Hua 30, 273)[43]. Such is the meaning of Boole's application to probability calculus. Its purpose is to logically clarify the logical underpinnings of probability inferences as a special form of deductive inference.

Nonetheless, Boole's logical analysis of probability calculus appears incomplete to Husserl from a logical point of view viz. from a noetic point of view. As we already saw, one of the turning points dates 1909[44]. A systematic exposition of the forms of judgements stemming from the modal modification of belief (of the "holding-for-true") and their noematic correlates is missing. By restricting the focus on one form of judgment and proposition (the categorical form), the noematic notion of proposition has been mixed up with the "apophantical" one, and, subsequently, the full extension of both has been narrowed (Hua 3, §§ 133-134; see Lobo 2011).

In order to formulate the most general formal laws of thought, it is crucial to start from the larger sphere of "judgment forms" and grasp the relation of categorial form to the other apophantical forms. There is also a need of a closer definition of the "qualities" (i.e. modalities) of judgement ("possibility, probability, problematicity"). Husserl's logic of probability appears as an extension and mutation of its "formal logic of content". But the "formal content" is a "modal content" gained in "a modalized intuition" or "*a modal intuition*" (sic) (Hua 23, 418) which is, as Husserl suggests, a "*categorial intuition*", and the larger theory of proposition shall cover all forms of propositions those expressing modes of the "holding-for-true" (such as possibility, probability, etc.) which enter the sphere of formal logic (Hua 30, 250-251; Hua 17, *n*. § 35, *n*. § 50; Lobo 2018), but also, axiological and practical propositions, entailing the diversity of modes of the "holding-for-worth" and, correlative, manifolds of forms of values. This enlargement is required in order to distinguish between different forms of possibility (analytical possibility, synthetic *a priori* possibilities, conditioned possibilities, rationally motivated possibilities, ordered or not, diversely loaded, etc.), which are usually mixed up in mathematics and in probability theory, and ignored or misinterpreted by the logicians.

> "The up-to-now unresolved philosophical and factual difficulties connected with the founding of probability theory are all based on the fact that, on the one hand, the distinction between the psychological and the theoretical side of meaning has not be carried out, and, on the other hand, one does not understand the concept of possibility, which is fundamental for probability theory, which must be distinguished from other logical concepts of possibility. This must, from the outset, relate to and recall us of our previous remarks on modalities." (Hua 30, 250)

This analysis refers back to a phenomenological analysis and its noetic-noematic distinctions, and within it, to the analysis of the central sphere of modalities (i.e. of positionality)[45].

In the battle field of probability, Husserl takes a stance, on the footsteps of Boole (but also Von Kries)[46], on the side of the "logic of probability". This position goes on a par with his theory of modalities, rooted in the phenomenological investigation of the "correlational a priori" (i.e. intentionality), and forbids any assignment of his projected formal logic neither on the side of subjectivist interpretation of probability, nor on the naively objectivist side[47]. Meanwhile, there is room for a non-naive objective theory of probabilities and a transcendentally rooted subjective interpretation of probabilities as modal modifications of the holding-for-true, of intentional *meaning*.

For Husserl, the study of belief and its modifications belong in the theory of judgment and its forms (Lobo 2018a; 2018b; 2017). In the light of the "correlational a priori" (i.e. noetic-noematic) analysis, to each mode corresponds a mode of sense of being, i.e. a new mode of being. The task of formal logic is to express both, and the investigation of modalities shall appear in this light as the fundamental and larger basis of formal apophantic and formal ontology. In other words, modes of belief are the correlates of a dependent moment of any objectifying intentionality, not exclusively of judgment. A judgement, which is a "holding-for-true" (in any of its modes) and would not *opine* (without a *Meinen*) is just as absurd as the denial of the intentional character of perception. The *Psychological Studies in the Elements of Logic* [or *Elementary Logic*] from 1894, represent already a breaking point with those who deny any modal (qualitative) content to representations such as perception (*Wahrnehmung*), whereas any perception entails intentionally a holding-for-true (*Für-wahr-halten*)[48].

In the review of W. Jerusalem's "Glaube und Urteil" (1894) which aims at clarifying "the rather confused relationship between belief and judgment" (Hua 22, 135; Husserl 1994, 181), Husserl's pithy objection to the main thesis ("Belief is nothing other than a *feeling which accompanies the* *judgment's holding of something to be so*") is unambiguous: "One can hardly expect advancement of the theory of judgment from such fictions as these". In the recension of J. Bergmann's *Die Grundprobleme der Logik*, (Hua 22, 180-245) Husserl assumes that the proposition which supports the negation is not the affirmative judgment, but "only to the *signification content* of the judgment, *abstracted from the belief character*, that talk of containing or being contained could have any reference – and even then it must not be taken literally." (Hua 22, 185; Husserl 1994, 230) But further on, against Brentano and Bergman, he considers that the meaning of the proposition, deprived of this moment of belief, must at the same time retain something of the belief. Their position and the correlative classification of acts (of psychical phenomena[49] (in three classes: representations, judgments, feelings and volitions) is criticized and, in the case of judgment, the main argument presented above rests on a confusion between "two essentially different relationships: 1. the relationship between the mere representation underlying the judgment and the *belief-Moment consummating the judgment,* and 2. the relationship between a plain and simple judgment and the judgment on it. Belief is not something "added" to "representation" in order to convert it into a judgment. Husserl claims: "in my opinion, we have to include under 'representation' the total signification content of the judgment, *the whole of the meaning of the assertion*". Consequently, "we cannot, *as in Brentano and Bergmann*, restrict 'representation' to the (nominal) representation of the object taken as subject, even though it were to include in one content the representations of the determining properties predicated of the object." (Hua 22,186; Husserl 1994, 231). It seems that we should divide accordingly the notion of "belief": (1) "the 'belief," as "characteristic of certainty or conviction" and (2) "the belief belonging to the *content or matter* of the judgment as such." (Hua 22, 186; Husserl 1994, 231) Moving away from Brentano's lesson (distinction between matter and quality of the act of judgement, i.e. meaning and belief), Husserl holds that "*the matter [i.e.* "*what* is believed"] *is not the representation*, possibly as it existed prior to predicative articulation, and it is no representation expressible by a name";

the quality of judgment is to Husserl's view "no acknowledgement or rejection directed upon such a representation", but belongs, as a constituting part, to the content, to the meaning. The copula "the 'is' […] is nothing less than an expression of 'belief,' and much less then is the "*is not*" an expression of a co-ordinate 'unbelief.' *Rather, the positing and 'certainty' characteristic belongs to the matter as a whole*, regardless of however it may further articulate itself into parts. The usual expressions for this characteristic – '*holding to be true,' 'believing,' 'consciousness of validity*,' and the like - all *suggest the erroneous view that we have here a predication of truth, validity or correctness upon the matter, and moreover, that we must here distinguish two co-ordinate* qualities: a *holding-to-be-true and a holding-to-be-false***.** Even this latter point does not seem to be absolutely beyond all doubt. Every (normal) assertion expresses a judgment, but every judgment also finds its expression in a possible assertion". And the final conclusion: "incorrect". "In each case the expression of the rejection, of the non-belief or the untruth, *pertains to the matter of the assertion*; and what makes it an assertion is not the non-belief predicated, but rather the *character of conviction or of 'believing' which as it were animates the matter*. *Every asserting is a believing*." (Hua 22, 185-186, emphasis mine)

To reject the foundation of validity upon belief considered as a feeling or a habit as well as its defence (as we find in von Kries) amounts to rest on symmetric confusions. One does not take into consideration that the modes of belief "belong primarily to the content: more precisely, to its logical forms" (Hua 22, 226; Husserl 1994, 280) This is true for each categorial form and their validation. Each one has a "type of intuitive realization which is just its own." But these differences effect the "validity feeling," the belief, only insofar as it is belief with a content of this or that form. They belong primarily to the content: more precisely, to its logical forms.

4.2. Formal operators stemming from pure phenomenological reflection

In order to understand more precisely how logical forms related to probability emerge from the phenomenological analysis, let us follow here some indications given by Husserl, which are concentrated on the forms of belief, their correlates and their modifications. As every phenomenologist knows, any lived experience is intentionally structured and, roughly speaking, constituted of two components: a real (reell) and a unreal or ideal one, called also respectively, *hyletic* and *intentional* components.

Let us denote this lived experience: *e* and express its composition by $e = \{r; i\}$ or $e = \{e_r; e_i\}$. And we can, by mere abstraction, explore either sides, and for a start, in a purely static manner. The analytic of real components and their "combinations" amounts to the analysis of the connections between productive (*erzeugende*) modifications, in other words, of their syntax or synthesis, which, by *modifying modifications of real components, modify the whole lived experience*. This analysis represents the new path of the phenomenological transcendental aesthetics or "hyletic" phenomenology, which covers diverse groups, systems or structures of modifications (temporal, spatial, kinesthetic, not to mention impulses, passive affectivity and tendencies). Historically Husserl started with the constitution of time and space, *as systems of modifications*. The combinations of these groups of modifications are called "*continuous syntheses*" by Husserl.

In *Ideas I*, §§ 84 and following, the analysis of the group of modifications qualified as real (*reell* in contrast with *real*) are characterized as "productive" (*Erzeugende*), since they give way to new phenomenological unities. Through such synthesis, we obtain, for instance, a temporalization of lived experiences, and eventually their insertion into one unique flow. Such is the case of retention, which apply to a former retention and transitively to the whole lived experience just retained. This is, for example, the case for "consciousness of delight". It is given "in a continuum of consciousness, which forms remains firm". It presupposes an "impressional phase", which is just a "limit-phase with regard to the continuum of retentions". The following analysis indicates precisely the way this continuum takes on the form of *a flow*. Since the impressional-phase and the retentional-phase do not belong to the same level, the retentional flow is "conjugated" to the continuous import of new impressional phases as limits and starting points of ever new retentional-phases; "they are related to each other continuously and intentionally under the form of a continuous embedding of retentions of retentions." The former retentional phase "combines" (*fügt sich*) with the new impressional phase, and the new retentional phase applies not only to the new impressional phase, but to the whole conjunction; "continuously the impression converts into a retention, and the latter continuously in modified retention, and so forth". All this is, of course, described through eidetic variation and under transcendental reduction.

This amounts to build up a kind of linear operator of the type $Au = u$, $Az = z$ and more precisely of the type $A(fAg) = Af\ Ag$. In order to justify this symbolic transcription, let us repeat and abbreviate the former analysis of retention. Any e (lived experience or *Erlebniss*) is submitted to a retentional modification, which is a continuous synthesis of retentions of retentions. For any e, holds the following proposition:

$$r(e) = e$$

And as each new retention-phase is conjugated, so to speak, with a new impressional-phase and its new retentional phase, we get:

$$r(e)\ r\ (e') = r\ (e'\ r\ (e))$$

This is an analog of a "averaging operator", or else, Reynolds operator used in fluid dynamics, and in functional analysis or in invariant theory. A Reynolds operator is, algebraically, a linear operator acting on algebraic functions. If any e defines an intentional function, as Husserl suggests, the retentional modification as a real modification applies as a similar operator. And as we write Reynolds operator $R(\varphi)$, $P(\varphi)$, or $\rho(\varphi)$, we may write the retentional operator under the form R and the every time new e using prime numbers as indices or the prime symbol $'$. And as, for every two functions φ, ψ, Reynolds operators satisfy the following condition:

$$R(R(\varphi)\psi) = R(\varphi)R(\psi) \text{ for every } \varphi, \psi.$$

The linear retentional modification should be written:

$$r(e)\ r\ (e') = r\ (e'\ r\ (e))$$

And, as for the Reynold operator, here too the following condition holds:

$$R(\varphi\psi) = R(\varphi)R(\psi) + R((\varphi - R(\varphi))(\psi - R(\psi)))$$

for all φ, ψ.

We should consequently find for the retentional operator, for every two lived experiences *e, e',* the following condition:

$$Rét(e.e') = Rét(e)Rét(e') + Rét\ (e - Rét(e))\ (e'\text{-}Rét(e')))$$

Which is to be interpreted: *the retention of two lived experiences* (for instance the hearing of sound *a* and the sound *b* or *a'*) *is the product of the retention of* a *and the retention of* b *to which is added* (fügt sich) *the retention of the product of the difference of* e *minus the retention of* e *and the difference of* e' *minus the retention of e'.*

Other condition, which holds also:

$$R(R(\varphi)\psi) = R(\varphi)R(\psi) \text{ for every } \varphi, \psi.$$

Phenomenologically:

$$R(R(e)e') = R\ (e)R(e') \text{ for every } \varphi, \psi.$$

In words: *Every retention of the product of a retention of an* e *and an* e' *is the product of the retention of* e *and the retention of* e'.

Last, the condition :

$$R(R(\varphi)) = R(\varphi),$$

which states that for every φ *R* is an averaging operator if and only if it is a Reynolds operator. Similarly, we have for retentions: *R(R(e) = R(e).* A retention of a retention of a lived experience is itself a retention of a lived experience. We are her at the starting point of a *chronometry* on which is rooted the constitution of the consciousness of the *etc.*

In order to understand why and how, we must shift to the other great group of modifications and the correlative components, those which we named "unreal" or "ideal" components.

Among those components we find all the constituting elements of the noetic and noematic structures, analyzed as meaning (noetic and noematic). And in order to go straight to the nucleus which is directly concerned by the present issue (that of foundation of a logic of probability), let us focus on the group of *modal modifications*, i.e. of modalizations, and more specifically the subgroup of *doxic modifications*.

As we learn from Husserl, the sphere of modal modifications does not restrict itself to the sphere of judgement in the strict sense of the term, nor to that of predication. All lived experiences (nomination, perception, memory, and even imagination) have a kinship with judgements understood as predicative certainty. A first enlargement concerns the sphere of derived modal forms of certainty: *suppositions, conjectures, doubts, refusal* as well the correlates corresponding to them. The noema nucleus is just the invariant through a series of different characterizations. "The same *S is P* which represents the noematic nucleus can be part a certainty, the supposition of a possibility or of a conjecture, etc." (Hua 3, 196-197).

Let us denote the set of modal modifications by Greek capitals with indices M_1, M_2, ... M_n in order to remind that those *modifications* (of the acts of taking for true, or for real, for being), are operators rather than mere functions.

But with that reservation in mind, we may use small letters $\mu_1, \mu_2, \ldots, \mu_n$. If we specify, these modification as *thetic* modifications, we should write: $\theta_1, \theta_2, \ldots, \theta_n$.

Last, in order to distinguish within the sphere of thetic modifications, the two major subgroups of axiological and doxic modifications, let us use respectively write: $a_1, a_2, \ldots, a_{n1}$ and $\delta_1, \delta_2, \ldots, \delta_n$.

This subgroup is a sub-sphere of the "sphere of positionality", which obeys certain laws, from which stem the so-called laws of logic – at least if we admit the goals assigned to transcendental logic by Husserl, which is to describe the emergence and growth of logical forms from the soil of the most primitive forms of synthesis (passive synthesis, continuous synthesis, kinesthetic synthesis, etc.)

What is particularly new in the way phenomenology conducts this investigation into the original soil of logic is that they lead to the structural fundamental laws which are at the same time laws of the "fundaments" of new logics. Those fundaments involve precisely the laws of "combinations" exposed by the newer and enlarged logical grammar promoted by Husserl, which is undoubtedly larger than that which was examined by the traditional or current logical grammars.

Once again this enlargement stems from the implementation of the *epokhè*, i.e. the bracketing of the natural thesis. The phenomenological laws define well-formed acts, whether those acts be performed psychologically, or by human consciousness, or not. For instance, we know that it is *a priori* possible to reflect on a reflection, *in infinitum*, even though no human mind has ever actually done it, for obvious reasons.

Among the original constants or modifications introduced by phenomenology which opens this larger field of possible syntaxes, we must count the modification of neutrality, or neutralization. *Every lived experience can be "expressed" under the form of a combination of modal or thetic modifications and neutralization.* And more especially: every objectifying intentionality as a doxic form can be expressed under the form of a combination of neutrality character and doxic characters. Following the correlational *a priori*, this holds for the noetic as well as for the noematic side. And since the modalization apply to both sides, as in real modification, noetic modalities have their noematic counterparts; but each noetic modification produces a new modified correlate in which the former noetic modal character is infused. A doubt about a perception of A transforms the "*perceived A*" into a "*maybe perceived A*", or a "*possible illusion or semblance of A*", or a "*misidentified non-A*", etc. Or to stand by the sphere of judgment, let us think of a nominalization of *S est P*, though which the assertive force is not excluded but "thematized", i.e. incorporated to the "matter" of the new consciousness, that of the nominal form *"the S which is P"* or "*the Sp"*, which in turn can be transformed (as we have just done it by quoting this nominal form into inverted comas, as an example).

The sphere of modalizations forms a group, in as much as they form a "monoid" of the type $\{M, *, \mu\}$ where every *e* (*Erlebniss*) is, among other things, but essentially, a combination of the type: $\mu_1 * \mu_2$. Thanks to the neutrality modification, the internal law of composition, and associativity,

1. $\mu_1 * \mu_2 = \mu_3$
2. $(\mu_1 * \mu_2) * \mu_3 = \mu_1 * (\mu_2 * \mu_3)$
3. $\mu * \nu = \mu$

the modal group of modification is a monoid. But in order to get a full group structure, a sub-sphere of symmetrical elements should be added, corresponding to *opposite* elements, denoted $\mu^{-1}$and such that $\mu * \mu^{-1} = \nu$. But this seems rather artificial. Moreover we must observe a peculiarity of the neutrality, which causes some perplexity about this attempt of notation.

The first complication touches the ambiguity of the counterparts. The neutrality operation, which could be that of an explicit *épochè*, can also take the form of the *quasi-modification* (i.e. a neutrality modification of the "pure fantasy" type) which produces an imaginary counterpart. Of course, we can admit as many actual replicas of any lived experience as possible, all the more if we remember that any actual *e* is just an arbitrary instantiation, an example taken out of an eidetic extension of the type to which *e* belongs. The same holds for the modal components of *e*. Through the neutrality modification, the first manifold faces a manifold of imaginary counterparts.

The second complication comes from the use of the parenthesis, as neutrality operators. If we admit that the parenthesis themselves are a form of neutralization, this renders even more problematic our attempt, since the associative propriety expressed above would amount to a mere tautology:

$$\mu * \nu = \nu * \mu = (\mu)$$

In order to avoid such collapse, we should write:

1. $\mu_1 * \mu_2 = \mu_3$ with $\mu_1, \mu_2, \mu_3 \in \mathrm{M}$
2. $\nu * \mu = (\mu)$ for all $\mu \in \mathrm{M}$
3. $(\mu_1 * \mu_2) * \mu_3 = \mu_1 * (\mu_2 * \mu_3)$

But maybe, this collapse is *significant* and *useful*. It would unable to understand (and solve) many paradoxes, such as d'Alembert's or a heavier ones, such as the renormalization in quantum mechanics known as wave packet collapse.

Associativity appears in this case a special form of synthesis, corresponding to that which Husserl calls « polytheses », combination of theses of the same level, without any foundation (*Fundierung*) relation or other kinds of modification producing differences of levels.

The sub-group of doxic modifications is essential to understand the emergence of probability. The doxic theses satisfy the following conditions:

1. $\delta_1 * \delta_2 = \delta_3$ with $\delta_1, \delta_2, \delta_3 \in \mathrm{M}$
2. $\nu * \delta = (\delta)$
3. $(\delta_1 * \delta_2) * \delta_3 = \delta_1 * (\delta_2 * \delta_3) = \delta_1 * \delta_2 * \delta_3$

But

$$\delta_1 * \delta_2 * \delta_3 \neq (\delta_1 * \delta_2 * \delta_3)$$

i.e. a *combination of doxic thesis is different from the neutralization of this combination*. (2) must be read: *a doxic thesis combined with a neutralization equals the same thesis*. (3) is a more fundamental form of associativity, in comparison with which the usual associativity appears as a particular and derived case. This associativity states: *two combined neutralized theses combined with a third one, which is not neutralized, equals the first one not neutralized combined with the neutralized combination of the other two*. Or else, *a neutralization inserted in a combination of the same level but not fully neutralized does not change the doxic compound form*.

The linearization of those doxic modalities is obtained by the distinction of two levels of combination, with neutrality ν and doxic thesis δ, and the introduction of a modalization of any of the two, or of both. Let us call δ_1 and δ_2 two doxic thesis, M any modalization. The following operator

$$M(\delta_1 M\delta_2) = M\delta_1 M\delta_2$$

means that *the modalization of the product of* δ_1 *and the same modalization of* δ_2 is tantamount *the product of the modalization of* δ_1 *and the same modalization of* δ_2. As an approximation: "the doubt about (the certainty) δ_1 and the doubt of the certainty δ_2 is equal to the product of the doubt on

δ_1 and the doubt of the certainty δ_2". The substitution of ν to the neutralization-parenthesis () gives the following formula:

$$M\nu\delta_1\, M\nu\delta_2 \,=\, M\delta_1\, M\delta_2$$

and means exactly the same thing: "the doubt about the certainty δ_1 and the doubt of the certainty δ_2 is equal to the product of the doubt on δ_1 and the doubt of the certainty δ_2" , as long as the whole doxic compound is not neutralized, any compound is *absorbed* into that modalization.

From the application of the parenthesis or neutralization to the whole doxic compound results a neutral compound, an "imaginary" counterpart. And we understand why and in which very precise sense the *imaginary or fictitious represents the element of phenomenology*, here under the elementary form of doxic syntaxes giving and founding the meaning of probabilities.

NOTES

[1] "A more serious fault of the *Prolegomena* is, by the way, the following: In connection with the concept of truth the modalities of truth are not mentioned, and probability is not cited as one of them. When they are taken into account, an enlargement of formal logic becomes necessary: to the effect that, as universal formal possibilities, modal variants of judging and of judgments enter into certainty- or truth logic - because any such variant can enter into the predicational content of the judgment and, when it does it must not be regarded as extra-formal. In other words, only the content that goes beyond anything-whatever is the « matter » of judgments, in the sense proper to formal logic; all the forms in which one judges _ not only with certainty but also in the mode of possibility, or in other modalities - belong to anything-whatever. A kindred enlargement results from taking into consideration the fact that emotion, and volitions also bring modalities of anything-whatever', which are introduced in the same manner into the dox sphere. (On this last point ct. *Ideen* . pp. 243f1. [English translation, pp. 531f1.]: also § 50 pp, 135 ff., *infra*.)." (Husserl [Cairns] 1969, 101)

[2] Starting with a S5 reduced system of modalities, Becker proposes a "statistical model" or interpretation for the modal calculus of first degree, then for composed modalities (§2). This statistical interpretation is founded on an analogy between modal calculus and classical probability calculus. The quotient 1 for necessary; the quotient 0, for impossible or necessary not. For the interpretation of the range of possibilities, Becker proposes that a proposition holds as true if at least superior to 1/2 otherwise the proposition must hold as not true.

[3] This is the subtitle of *Formal and Transcendental Logic* (Husserl 1969).
[4] Against Kant's conviction as well as that of its modern enemies, that it was *almost* achieved from the beginning; Husserl thinks that it is only at its very beginnings (see Hua 28, 244-245).
[5] Before the influential contribution of Keynes, and the later works of Carnap (1945), Husserl had in view Boole's *Laws of Thought* (1854) and John Venn's *Logic of Chance* (1876).
[6] Keynes (1921, *nn.* 2, 5) notes that the first who took notice of that was Ancillon, in *Doutes sur les bases du calcul des probabilités* (1794), before being « emphasised by Boole », Czuber (in his *Wahrscheinlichkeitsrechnung*,) and Stumpf (1892).
[7] To the exception of Albino Lanciani (2012) and more recently Carlos Vargas (2018).
[8] Émile Borel (1939, §§ 18-19) refers to his recension from 1921 of Keynes "nice book", and to Jean Nicod's critique in "Le problème logique de l'induction" (1924).
[9] This is attested by Husserl's recension of von Kries's "Zur Psychologic der Urteile", *Vierteliahrsschrift fur wissenschaftliche Philosophie*, 23, 1899, S. 1-48, (Hua 22: 224- sq.).
[10] The Russian school (Tchebychev, Liapounov et Markov) is apparently ignored by Husserl. No mention of the French tradition (Poincaré, Borel, Lebesgue) either. But he knows the English logical tradition (Boole, Venn), and could not ignore the development of statistical physics (Gibbs, Boltzmann, and Einstein). He was well acquainted with the logical and philosophical investigations on probability calculus (Wundt, Stumpf, Meinong). The contribution of Per Martin-Löf on random sequences (1966) and on logic are a typical example of a cross-over influence (between Kolmogorov, Frege, Brouwer and Husserl).
[11] "Was bei ihr beirren kann, ist nur der Umstand, daß die Theorie der Wahrscheinlichkeiten als eine mathematische Disziplin konstituiert ist. In gewissen Sphären, die genau zu umschreiben sind, sind die Grade berechtigter Vermutung zahlenmäßig bestimmbar, und die zugehörigen Grundsatze ermöglichen eine rein deduktive Disziplin und eine in quantitativ-mathematischen Formen sich entwickelnde. Aber formale Mathematik ist das nicht, wie schon die Grundsatze und Grundbegriffe lehren. So erklärt Laplace den Grundbegriff der Wahrscheinlichkeit mittels des der *gleichmöglichen Falle*, und diese erklärt er als so1che, über die wir in gleichem Maße in Unkenntnis sind. Von der Wahrscheinlichkeit sagt er, sie beziehe sich zum Teil auf unsere Unwissenheit, zum Teil auf unser Wissen usw." (Hua 24, 132)
[12] "It is in this latter sense, with an eye to degrees of probability. that one speaks of a greater or lesser degree of knowledge. Knowledge in the pregnant sense , – its being quite evident that S is P – then counts as the absolutely ideal limit which the graded probabilities for the being-P of S asymptotically." (Hua 18, 30 ; Husserl 2001, 18)
[13] Husserl will discover in the Fifth Logical Investigation, § 27, that the processes of modalization intersects with and is connected to that of determination, and reciprocally. Not only with expressed judgements, but

with acts of perception (such as the perception of a person and/or a mannequin). Pointed at but not yet clearly analyzed in Lobo (2000, 246-270),

[14] The essentiality (*Wesenshaftigkeit*) of a mathematical constructs amounts to its logical possibility and its mathematical existence. It is diametrically opposed to the "essencelessness" (*Wesenslosigkeit*), which means "impossibility" (*Unmöglichkeit*) or "imaginarity" (*Imaginarität*) (Hua 18, 242).

[15] "The principal part of the art of logic that governs the sciences of matters of fact is the art of judging probability and providing grounds for probability. It plays the greatest role everywhere, even where it is not expressly a question of probability" (Hua 24, [17] 12).

[16] I fully agree with M. van Atten's suggestion that Husserl should have been a strong revisionist in mathematics, and that, after many hesitations, he was. (Van Atten 2007, 59-67).

[17] Felix Hausdorff has developed an incredibly original and profound theory of probability through a reworking of von Kries's notion. See, Carlos Lobo, "Espace, espace de jeu, jeu de hasard. *Position philosophique du problème de l'espace et des probabilités chez Felix Hausdorff*" (Forthcoming)

[18] Measure presupposes a fundamental operation setting the equipossibility of a set of cases (possible events) (figuring as denominator) and a propriety discriminating within that set a portion (of favorable cases) (the numerator).

[19] For an answer to this so-called vicious circle, see Borel (1924, 19): "En réalité, il n'y a pas de cercle vicieux à supposer l'on a la notion vulgaire du sens des mots 'également probable', lorsqu'on veut définir le sens mathématique précis du mot probabilité. Les logiciens qui prétendent construire des systèmes entièrement logiques, sans cercle vicieux, oublient qu'il est impossible de ne pas utiliser le langage usuel, ne serait-ce que pour définir les termes scientifiques que l'on emploie et pour construire les phrases dont on se sert; or, le langage usuel doit être considéré comme une acquisition globale de chaque individu, acquisition qui suppose un grand nombre de cercles vicieux".

[20] "The last *provisio* makes the definition circular, for the concept of probability then is dependent upon the concept of equiprobability. From the purely technical point of view, Laplace's definition reduces calculation of probabilities to counting" (Kac and Ulam 1968, 36).

[21] See Laplace 1921, 10.

[22] This parallel was clearly evidenced by Poincaré (1896). Despite formal imperfections (up to modern mathematical standards and those of the German school of the time), P. Cartier (2006) considers that Poincaré's *conceptual analysis* is the right one ("*C'est encore un ouvrage du 19-ième siècle pour les méthodes analytiques, mais c'est un des premiers ouvrage à faire percevoir les enjeux de la physique statistique, bien au-delà des habituels exercices de combinatoire liés aux jeux de hasard. En ce sens, ce livre appartient déjà au vingtième siècle, frayant la voie à Einstein, Ehrenfest, Wiener, Landau, et tant d'autres…* "). Among the treasures of this book "*à la fin du chapitre 12, une définition très claire de l'ellipse de dispersion, c'est-à-dire le fait que la loi de Gauss à plusieurs dimensions dépend du choix d'une forme quadratique définie positive sur l'espace de configurations*". Normal distribution or Gauss's curve of errors represents a special case amongst a

rich variety of possible "abstract random spaces", i.e. manifolds (with negative curvature).
[23] In *Doutes et questions sur le calcul des probabilités*, 1770, Mélanges de littérature, d'histoire et de philosophie, Tome V. Amsterdam, quoted by Von Kries (1886, 278-279).
[24]Which works as a renormalization comparable to the wave-packet collapse of quantum mechanics.
[25] "Alles kommt dabei darauf an, in objektiv gültiger Weise ein Feld von gleichen Möglichkeiten herzustellen von Möglichkeiten, für die in exakt nachweisbarer Weise genau dasselbe spricht, die gleichen Gewichte, positiv wie negativ - und nun jeden geschlossenen Kreis von Wahrscheinlichkeits-erwägungen auf dieses Grundfeld zurück zubeziehen." (Hua 30, 253).
[26] For a phenomenological exploration of the "central sphere of positionality" (or "modal core") see Lobo (2017).
[27] To the exception of great scientists such as Maxwell who declares: "The actual science of logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore the true logic for this world is the calculus of Probabilities, which takes account of the magnitude of the probability which is, or ought to be, in a reasonable man's mind." (Keynes 1920, 172; quotation from J.C. Maxwell)
[28] "Es ist nun leicht einzusehen, daß Für-möglich- und Für-wahrscheinlich-Halten und auch Fragen und Zweifeln Bewußtseinsarten sind , die mit dem setzenden, behauptenden Urteilen gattungsmäßig verwandt sind, so daß eine Reihe von Akten von propositionalem Inhalt unter einer höheren Gattungs-idee, unter der Idee des prädikativen Urteils oder, noch allgemeiner, eines prädikativen Aktes in einem außerordentlich erweiterten Sinn stehen. (Ob das Sich-denken in diese selbe Reihe gehört oder ob es nicht eine eigene Behandlung erfordert, wollen wir hier nicht entscheiden.)" (Hua 30, 59).
[29] "Ebenso wenig aber auch Sätze, die, statt vom urteilenden Ich auszusagen, vielmehr objektiv hinstellen: „Es ist gewiss" („Die Gewissheit ist berechtigt"), „Es ist zweifelhaft" = „Es besteht ein berechtigter Zweifel", ebenso: „Es ist wahrscheinlich" usf. Es mag innerhalb der Logik im weiteren Sinn eine objektive Wahrscheinlichkeitslehre, objektive Lehre von vernünftigem Zweifel, von vernünftigem Fragen etc. geben: in der Lehre von den Urteilen im Sinn von Urteilsbedeutungen ist nicht ihre Stelle. Dasselbe gilt für die Möglichkeits- und Notwendigkeitsurteile, sofern ihr Sinn mitbeschlösse irgendeinen subjektiven und empirischen Gehalt, der Beziehung hätte auf den Urteilenden, auf seine Meinungen, Kenntnisse, Vermutungen u.dgl. Solche Unterschiede gingen die formale Logik nichts an." (Hua Mat 6, 2003, 230).
[30] It is by a reflection on the content of perception that Husserl discovered that the "perceived" (*wahrgenommen*) as such entailed a qualitative character as part of its content: and makes in case of a "perfect trompe-l'oeil" , the whole difference between a mannequin (*Wachspuppe*) and a lady (*eine Dame*), i.e. between two different noemas (of empathy on the one hand and of picture or sculpture consciousness on the other), with their own essential modal characters. Fifth Logical Investigation, § 27 (Hua 19/1, 176-178)

[31] About the analogon of doxic doubt in the sphere of will (simple and disjunctive doubt-of-will distinct from question-of-will properly speaking). My comment Lobo (2006, 35-68).
[32] The question whether thinking in general and rationality in particular cannot be worked without any symbolic activity is another question, which is not addressed here.
[33] "Alles nun, was wissenschaftlich zu leisten ist unter dem Gesichtspunkt der Rechtsnormierung der Erkenntnisakte, d.h. der Akte, die entweder selbst Urteile im weitesten Sinn sind oder bei der Rechtsausweisung von Urteilen als rechtsverleihende Wahrnehmungen, Erinnerungen usw. eine wesentliche Rolle zu spielen berufen sind, das weisen wir der noetischen Normenlehre zu, und sie hat, nach dem Gesagten, es nicht im eigentlichen Sinn mit Akten als menschlichen oder sonstigen Erlebnissen, sondern mit den entsprechenden Aktideen zu tun. Wäre diese reine Disziplin ausgeführt, so wären wir also in der Lage, jeden Fall aktuellen Urteilens (im engeren Sinn, des aktuellen Vermutens oder Für-wahrscheinlich-Haltens usw.), ebenso jeden Fall aktuellen Begründens, aktuellen deduktiven Schließens und theoretisierenden Erklärens, induktiven Schließens usf. auf ideale Prinzipien zurückzuführen und nach seiner Normalität prinzipiell zu beurteilen. / Von vornherein wollen wir dabei den wissenschaftstheoretischen Charakter unserer Untersuchungen zur Geltung bringen; also wir wollen uns von vornherein in jedem Schritt deutlich machen, daß das Gebiet reiner Erkenntnis, das wir jetzt wissenschaftlich begrenzen, ein Grundstück einer allgemeinen und reinen Wissenschaftslehre sein muß. Wir verstehen darunter eine Wissenschaft, welche in systematischer Weise die zur Idee echter Wissenschaft gehörigen Wahrheiten erforscht, somit alles, "'Das notwendig gelten muß, wenn eine Wissen." (Hua 30, 38)
[34] "So, wie wir Urteilserlebnisse als Erlebnisse des 'So ist es!' unter Idee bringen und in der Einstellung der ldeensetzung nach Geltung und Nicht-Geltung normieren können, so (auch) Urteilserlebnisse in einem erweiterten Sinn, ich meine hier Erlebnisse des Für-möglich und Für-wahrscheinlich-Haltens, des Fragens und Zweifelns. Auch in dieser Hinsicht gibt es Formen und Normen der Gültigkeit, die mit diesen Formen zusammenhängen, und man kann mancherlei Normen der rein mathematischen Wahrscheinlichkeitslehre in dem angedeuteten Sinn interpretieren. Und schließlich kann man auch alle Erkenntniserlebnisse überhaupt, nach allen immanenten Charakteren unter Idee bringen und mit Bezug auf die Rechtsprechungen der Wahrheit, Möglichkeit, Wahrscheinlichkeit, Fraglichkeit durchforschen." (Hua 30, 39)
[35] "Ziehen wir neben dem Urteilen als In-Gewißheit-Behaupten, Behaupten, Aussagen, Für-wahr-Halten auch in Erwägung die mit ihm wesentlich verflochtenen Modalitäten, so das Vermuten, das Für-möglich Halten, so ist das darin Bewußte nicht vermeinte Wahrheit oder Satz, sondern vermeinte Wahrscheinlichkeit oder Möglichkeit. Das gibt Anlaß zur Erweiterung der Idee einer reinen Logik um eine reine Logik der Möglichkeiten und Wahrscheinlichkeiten. Mit der Logik der Behauptungen, der apophantischen Logik, zeigt sich aber aus wesentlichen Gründen auch verflochten, obschon in ganz anderer Richtung, die reine Arithmetik und weiterhin die gesamte

formale Mathematik oder Mannigfaltigkeitslehre. Diese Disziplinen bilden sozusagen ein höheres Stockwerk der Apophantik, und es ist von größter philosophischer Bedeutung, sie in diesem Zusammenhang zu erkennen und zu charakterisieren." (Hua 30, 29)

[36] See Recension of J. Bergmann's *Die Grundprobleme der Logik, zweite, völlig neue Bearbeitung* in Hua 22 (186 *et seq.*); Wundt standpoint exposed in 1894 (Hua 22, 129.); compared in both case to Brentano's theory of judgement.

[37] I shall repeat here the last subtitle of Weyl's magnificent paper "The Ghost of modalities" dedicated to Husserl (Weyl 1940, 278-304).

[38] "And, again, one can be an outstanding mathematician, while being a very mediocre philosopher of mathematics. Boole provides an outstanding example of both.)" (Husserl 1994, 59; Hua 22, 9).

[39] Similar objection by Venn and similar answer from Husserl. "Also, that the Boolean method so frequently utilizes senseless symbols does not yet in itself serve as the basis for a logical objection. We can only object, rather, that that method does not adequately justify the use [399] of such symbols" (Husserl 398-399).

[40] "As a rule they are of such a simple type that to solve them by means of the calculus would be the most laughable of detours." (Husserl 1994, 90; Hua 22, 42)

[41] "This latter, which coincides formally with the former, can indeed be profitably applied in many particular fields of mathematics- for example, in the theory of functions, where manifolds of values of arguments frequently come into consideration. *Likewise in the calculation of probabilities, where sets of chances make the application possible. Beginnings have already been made in these matters, but here too we do not have sufficient results definitively to decide the question about practical value.* But I would in no case wish to cast doubt upon the extraordinary theoretical interest that belongs to the algorithmic treatment of the theory of pure deductions, as well as of pure set theory." (Hua 22, 43; Husserl, 1994, 90)

[42] See Boole (1952, 239) which reformulates retrospectively the purpose of the *Laws of Thought* (Boole 1854).

[43] This represents a strong opposition to Peirce, who considers *all* mathematical reasoning as hypothetical. (See, for instance, Baldwin's Dictionary).

[44] *Lessons on Logic and epistemology*, published under the title *Vorlesungen über Logik und Wissenschaftstheorie*, Husserliana 30, … and *Alte und New Logik*, Husserl, Mat. 6.

[45] For further development of those points, I must refer once to Lobo (2018; 2017a; 2017b).

[46] This discrete and discreet reference in the historical development of probabilities and philosophy of probability is better known now (see Keynes 1921; Rosenthal 2010; Zabell 2016; Lobo 2018).

[47] To be compared with Hausdorff, *Beiträge zur Wahrscheinlichkeitsrechnung*, from 1901, in Gesammelte Werke, Vol. 5, pp. 531-532, and compare to Husserl (Hua 30, § 51-52) and Peirce (Probable Inference, Baldwin's Dictionary, 354).

[48] Rather than a "holding-to-be-so" (Husserl 1994, 149; Hua 22, 102): "we have the inconvenience of denying its name to the perceptual representation [*wahrnehmende Vorstellung*] as it naturally presents itself, for what we in

fact have in such cases is a holding-to-be-so [*Fürwahmehmen*] of what is represented (even if only inauthentically) in the perceptual 'representation'."
[49] Not to be confused, as this frequently occurs, with "physical phenomena" in Brentano's sense, i.e. sensuous and emotional data.

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