META: RESEARCH IN HERMENEUTICS, PHENOMENOLOGY, AND PRACTICAL PHILOSOPHY
VOL. XI, NO. 2 / DECEMBER 2019: 569-588, ISSN 2067-3655, www.metajournal.org

Oskar Becker or the Reconciliation of Mathematics and Existential Philosophy

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Abstract

Oskar Becker's work in the philosophy of mathematics makes an important contribution to the philosophical understanding of the constructivist program. Becker (1889-1964), a student of Edmund Husserl and an associate of Martin Heidegger, initially sought to ground a constructivist view of mathematics in Husserl's transcendental phenomenology; subsequently he adopted a Heideggerian and existential view of mathematics that, he argued, would allow one to rescue large parts of classical mathematics from an intuitionist and constructive perspective. In his later writings he finally turned to a radically historicist interpretation of the constructivist program.

Keywords: Edmund Husserl; Hermann Weyl; L.E.J. Brouwer; Martin Heidegger; Mathematical intuitionism; Constructivism; Historicism

I begin with a piece of autobiography. When I first heard of symbolic logic I was a schoolboy, about fifteen years old. By chance, I had discovered a little book on the topic in some local bookstore and had found its treatment of the propositional and the predicate calculus an endlessly intriguing subject matter. It was a time when I dreamt at night of sex and truth-tables. I don't know any more in what order. Then, as a freshman at the University of Bonn, I discovered a course in symbolic logic taught by one Oskar Becker. He turned out to be a charming, elderly, slightly dotty professor who still taught his regular classes even though he was retired and who was scheduled that semester not only for his logic course but also a seminar with the title "The Principle of Reason." In my innocence I conjectured that this would be more logic and so enrolled in

both classes. The seminar, however, turned out to be on the later Heidegger, of whom I had never heard till then, and the text for the course was a small book of lectures entitled "The Principle of Reason," that quickly intrigued me with its darkly poetic prose. Heidegger was asking himself in the work why calculative reason has become so dominant in our culture and was warning of the limits of all such reasoning. While the principle of sufficient reason tells us that everything has its rational ground, its claim to universal validity was itself mysterious; the human condition remained, in fact, unexplained by it. In short, as Heidegger said at the end, "Being: the Abyss."

For Becker, as I saw him in that semester, the conjunction of logic and later Heidegger seemed to come naturally. I learned afterwards that he had been a mathematician at first, then a student of Edmund Husserl's, and finally a personal friend and philosophical associate of Martin Heidegger, that he had written extensively on the history and philosophy of mathematics and had made contributions to modal logic as well as to aesthetics and other parts of philosophy. How interconnected these interests were in Becker's own mind became clear to me only many years later when I studied his book on "the logic and ontology of mathematical phenomena," one of his major pieces of writing, published in 1927 under the title *Mathematische Existenz*. The first thing that struck me about that work was that it had appeared in volume 8 of Husserl's *Yearbook for Philosophy and Phenomenological Research* side by side with Heidegger's *Being and Time*. The two works made up the entire content of that particular yearbook. At first sight, they seemed to have nothing in common. Becker's treatise belonged, after all, to the philosophy of mathematics: it examined the conflict between L.E.J. Brouwer's mathematical intuitionism and David Hilbert's formalism and sought to put mathematics on a new philosophical footing. Heidegger's work, on the other hand, concerned itself with the conditions of human Dasein and with what its author called the temporality and historicality of Being. But it seemed that Husserl, the editor of the *Yearbook,* had considered the works as related and complementary. Part of the explanation I

found in Husserl's conviction that Becker and Heidegger represented the two sides of the phenomenological movement in philosophy - the scientific and the humanistic one, for short - and that he hoped his two disciples would bring his own work to further fruition in these two directions.

This was not to be, as we now know. Heidegger's *Being and Time* was, in fact, a radical turn in the road of phenomenology and not a continuation of Husserl's direction of thought. And by 1927, Becker was walking more in the footsteps of his friend Heidegger than those of his teacher Husserl. *Mathematische Existenz* constituted, in fact, an attempt to re-think mathematics with the help of Heidegger's insights. Becker made that explicit in his preface in which he announced that his treatment of the philosophy of mathematics was relying primarily on Heidegger's hermeneutic-phenomenological method of investigation – in addition, he hastened to add, to the methods of Husserl's formal transcendental-constitutive phenomenology. (442) With a further bow to Heidegger, Becker also declared it to be his intention "to put 'mathematical existence' in the context of human Dasein which must be regarded everywhere as the fundamental context of interpretation." (442) In other words, he proposed to treat the problem of the existence of mathematical objects with the tools of existential philosophy; hence, presumably, the title of his work

1. Husserlian Beginnings

Mathematische Existenz was, in fact, Becker's second attempt at a foundational theory for mathematics. In 1923 he had developed an account of the foundations of geometry that had drawn not yet on Heidegger but instead on the philosophical work of his teacher Husserl. Relying on both published and unpublished writings he summarized and commented extensively on Husserl's views in that essay. "In writing this treatise I owe gratitude in the first instance to Edmund Husserl," he declared, "whose research is the foundation on which it arises." At the same time he had spoken of his reliance on the work of the mathematician Hermann Weyl "whose account of the mathematical and physical

problems," he added, "provided particularly suitable material for phenomenological analysis because he himself is close to phenomenology." (388) Weyl was a mathematician sympathetic to "the need of a phenomenological perspective on all questions of the clarification of basic concepts." (van Dalen, 3) Husserl had even invited him to submit an article to the phenomenological *Yearbook* and had attached "very high value" to Weyl's proposed contribution, an essay on "The New Foundational Crisis in Mathematics." To Husserl's regret, Weyl, though, finally decided to publish the piece instead in a mathematical journal.

In that essay Weyl had contrasted the classical, atomistic view of the continuum as an ordered set of points (a view he himself had espoused in earlier writings) with Brouwer's conception of "the continuum as medium of free becoming." (Weyl 1998a, 93) Identifying now with Brouwer's views, Weyl had written: "It would have been wonderful had the old dispute led to the conclusion that the atomistic conception as well as the continuous one can be carried through. Instead the latter has triumphed for good over the former. It is Brouwer to whom we owe the solution of the continuum problem." (Weyl 1998a, 99) In a later paper Weyl spoke of the conflict between Hilbert and Brouwer as deeply grounded in fundamental questions of epistemology. "The old opposites of realism and idealism, of the *Being* of Parmenides and the *Becoming* of Heraclitus, are here again dealt with in a most pointed and intensified form." (Weyl 1998b, 141) With Brouwer, he wrote, "mathematics gains the highest intuitive clarity; his doctrine is idealism in mathematics thought to the end." (Weyl 1998b 136) But "full of pain" Weyl also regretted that in Brouwer's reconstruction "the mathematician sees the greatest part of his towering theories dissolve in fog." (Weyl 1998b, 136) This gave him renewed sympathy for Hilbert's attempt to salvage the entire edifice of classical mathematics. He noted that Hilbert was, in reality, not as sharply separated from Brouwer as the polemical tone of their debate made it appear. For Hilbert, too, was "completely convinced that the power of interpreted thought does not reach further than is claimed by Brouwer, that it is incapable of supporting the

'transfinite' modes of inferences of mathematics, and that there is no justification for all the transfinite statements of mathematics qua *interpreted, understandable truths*." (Weyl 1998b, 136) Hilbert had sought to rescue transfinite mathematics, however, by treating its formulas as un-interpreted, contentless combinations of signs whose formal consistency could be established by means of an interpreted finitary metamathematics. For Weyl this proved in the end unsatisfactory. "If Hilbert is not just playing a game of formulae, then he aspires to a theoretical mathematics in contrast to Brouwer's intuitive one. But where is that transcendental world carried by belief, at which its symbols are directed?" (Weyl 1998b, 140) It was these ideas of Weyl's that provided Becker in 1923 with the material for his own thinking about mathematics. Husserl himself summarized Becker's work at the time in a letter to Weyl as coming to the conclusion "that the Brouwer-Weyl theories are the only ones that stand up to the strict, indispensable demands of a constitutive-phenomenological research into foundations." (van Dalen, 7)

Husserl's enthusiasm for Weyl's intuitionism and for Becker's appropriation of Weyl's view calls for explanation. For Husserl is not generally considered to have been inclined towards a constructive view of mathematics. He is, perhaps, best known for his defense of the objectivity of logic and the rejection of all forms of psychologism as spelled out in the first volume of the *Logical Investigations* of 1900. Husserl appears there committed to an uncompromising Platonic realism similar to the one generally ascribed to Frege. In a review of Husserl's total work Becker found it therefore necessary to address this apparent discrepancy between Husserl's objectivism and his apparent inclination towards constructivism. He pointed out that neither Husserl's Philosophy of Arithmetic of 1891 nor the later volumes of the *Logical Investigations* are committed to any form of realism. Becker rejected, moreover, the claims of those interpreters who believe that "Husserl had developed from an extreme representative of psychologism (in *The Philosophy of Arithmetic*) to the most radical anti-psychologist (in volume one of the *Logical Investigations*) and had afterwards (beginning already in volume two of the *Logical*

Investigations and then in further writings) relapsed more or less back into psychologism." (B/H, 120) Instead, he spoke of a continuous development in Husserl's thought which had led phenomenological research step by step to an explicit recognition of the transcendental idealism that was the hallmark of Husserl's later philosophizing. Already in *The Philosophy of Arithmetic*, Becker argued, one could find an endorsement of "the principle of transcendental idealism" which asserts "the universal accessibility in principle to all objects of which philosophy can speak with any sense at all." (B/H, 123) Volume one of the Logical Investigations had to be seen in this context.

> The correctly understood principle of transcendental ("constitutive") idealism is an integral component of phenomenology as such. Accordingly it can be pointed out in different forms in every phase of Husserl's philosophizing. (B/H, 123)

Volume one of the *Logical Investigations* was therefore not to be read as an argument in favor of a Platonic realism, but as an attack on the "empiricism, anthropologism, relativism, and psychologism of the time." (B/H, 124) Husserl's later views on his principle of transcendental idealism should, however, treated as an expression of a "constitutive," and that is constructive view of reality. Husserl had thus been, in essence, a constructivist throughout his career.

2. The Heideggerian Becker

In trying to think of logic and mathematics from the perspective of existential philosophy Becker drew on two propositions fundamental to that tradition, propositions characteristic of Heidegger's thought up to and including *Being and Time* which entirely bypass his later hostility towards logical reasoning. The first of these concerns what Becker called "an 'existentialist' identification of 'reality' with the reality of factual life." (Gr., 61) We can express the idea succinctly in the assertion that (1) the real is the temporal.

Let us call this the anti-Platonic principle of existential thought. It derives, of course, from Nietzsche and is not specific to Heidegger. The principle refuses an interpretation of the temporal world in terms of a supposedly a-temporal realm, be it

that of the Platonic ideas, of concepts, numbers, and values conceived as "real" entities, or of transcendent supernatural powers. The proposition is also directed against Kant and the post-Kantians who insist that empirical reality can only be understood by appeal to "transcendental", that is, by appeal to necessary and hence timeless principles of human reason.

This anti-Platonism the existential tradition shares with a number of different movements of nineteenth and twentieth century thought, in particular with the kind of philosophical naturalism and positivism characteristic of some early phases of so-called analytic philosophy. The second proposition on which Becker builds his philosophy of logic and mathematics is, by contrast, more specifically tied to existential thought and even more specifically to the Heideggerian version of it. It maintains that (2) Human existence is through and through historical. We can call this, for lack of a better name, the historicist principle. It implies, in particular, that our understanding of the world, our determination of meanings, that is the whole process of interpreting the world and our symbols (including the linguistic and mathematical ones), is historical in nature. In clarification of the concept of the historical here appealed to Becker agrees with Heidegger that historical time experience has a specific structure which is not captured by the scientific notion of an infinite, linear, neutral time-series. Human time experience is characterized rather by its finitude (hence the central significance of death for interpreting ourselves); it is secondly characterized by our directedness towards the future, "our projective running forwards towards the future," as Heidegger puts it; and it is characterized thirdly by the experience of the uniqueness of the historical event.

I will now try to describe the effect these two propositions have on Becker's thinking about logic and mathematics. Proposition (1), the claim that the real is the temporal, leads him to conclude immediately that logic and mathematics, too, need to be interpreted in temporal terms. He writes:

> Time is not only the form of inner sense, but the fundamental structure of human life altogether... Our existence can be characterized as temporality. Time is not a mere form that surrounds

> us, but permeates our total being and essence. That shows itself also – even though it is often overlooked - in mathematics... We can and must count and calculate only because we are temporal beings. An eternal, infinite being does not need to count. (Gr., 158)

Mathematics, too, must then be interpreted in terms of the notion of time. Becker argues that such an interpretation has already been undertaken by Brouwer. Dutch intuitionism is thus the position in the philosophy of mathematics that corresponds to the existential point of view. Intuitionism can and must be supported by means of considerations drawn from existential philosophy.

Existential philosophy recognizes, first of all, no a-temporal notion of truth. Elaborating on Becker we can point out that Nietzsche already urges us to abandon our Platonistic notion of "objective" truth and to replace it with that of "my truth", that is with the recognition that which propositions we can assert will depend on the occasion and the moment. At every occasion, I will be in a position to assert some propositions and to deny others, but there will also be many propositions about which I am not in a position to make a judgment. Having replaced the notion of absolute truth by that of assertibility, we must conclude that the principle of excluded middle no longer holds. We are not in a position to say of every proposition that either it or its negation are to be asserted. Existential philosophy leads thereby directly into intuitionistic logic. With respect to mathematical objects the existential philosopher and the intuitionist once again agree; they both regard them as temporal constructs.

What changed for Becker in 1927 were then not his views on intuitionistic constructivism but rather his philosophical justification for those views. Where he had previously supported them by appeal to Husserl's principle of transcendental idealism; he now appealed to Heidegger's hermeneutic and historical conception of human understanding. It is not difficult to see that he may have conceived of the latter as a reworking and extension of Husserl's view, as a historicizing of Husserl's transcendental position. In 1927 he writes accordingly in a critical tone about Husserl's position:

> Seen from the conception of a fully "historical" life experience, transcendental idealism (at least in its usual form) appears as an abstract modification of the original historical standpoint. In this transcendental idealism human life manifests itself only "in the faded form of a 'pure consciousness.'" (Becker 1927, 626)

Becker's move from Husserl to Heidegger should, however, prove as more than a change in philosophical foundations. Becker began to argue now that only from the historical-hermeneutic position could intuitionism be fully understood but that such an understanding would at the same time necessitate modifications in the intuitionistic view. Brouwer had tried to construct the numbers in temporal terms. But he had identified only a single characteristic of the experience of time, the fact that every current moment parts in ever repeated form into a past and future. Brouwer had spoken accordingly of the two-oneness of the moment as the fundamental phenomenon for intuitionism. But this aspect of time, Becker argued now, could only justify the construction of rule-governed series of numbers, such as the sequence of natural numbers, that is the construction of series of numbers according to a repetitive, rule-governed principle. Brouwer's thinking about time was, however, insufficient to make sense of free-choice sequences, that is, non-rule-governed, non-repetitive constructions which according to Brouwer's own view were needed to introduce the real numbers. Such sequences could only be conceived in terms of a notion of historical time, Becker argued now. For only historical Dasein could undergo a process of free becoming.

Human thought could, moreover, so Becker, pass in such a process of free becoming through a series of stages of reflections in which the totality of the previous stages becomes the point of a departure for new reflections. This series is potentially infinite. In thought we can, moreover, once again reflect on this series of possible stages of reflection as a whole and can thus initiate an entirely new and higher level form of reflection. To this process there exists, moreover, no inherent upper bound. Becker writes: "In the repeated iterations the uniformity of the concrete sequential stages of iteration become evident. This leads to the idea of envisaging the whole infinite possibility of iterations, in numerical terms, to speak of

iterations of stage n." (545) And with this we bring "the finite mechanism to light... which so to say governs the transfinite structures in their peculiar movement and which allows our finite human consciousness to grasp them." (548) He hopes that in this way it might be possible, in contrast to Brouwer's expectations, to salvage large parts of Cantor's theory of transfinite numbers. He allows in any case that at least in the theory of pure transfinite ordinals "one can speak of an ontological foundation of the theory of transfinite numbers." (561)

I have reported these considerations not in order to endorse them, but in order to show how murky the philosophical discussion becomes once one raises the question what is to count as an admissible method of construction. In standard expositions of intuitionistic constructivism these difficulties are usually hidden from view because of insufficient attention to the philosophical details. It is a merit of Becker's account to have sought a systematic exposition and to have thereby exposed problems inherent in the constructivist project.

Becker's attempt to extend this method in order to reach a constructivist mathematics is motivated by his concern with the application of mathematics in scientific theorizing. He realizes that mathematics finds its fulfillment only in its use in the mathematical theories of natural science. But here, it turns out, we need and use parts of mathematics that cannot be justified by intuitive and constructive means. The natural world cannot be completely grasped with the tools of a constructive and interpreted mathematics. But given that we are essentially temporal beings and that our understanding proceeds always in temporal and historical terms, we must grant that in science we make use of parts of mathematics which we can no longer intuitively interpret. This kind of mathematics can only be treated as an un-interpreted formal calculus. We are confronted here, as Becker puts it, with the problem of the alienness and incomprehensibility of nature.

Precisely here mathematics helps us further and it does so precisely in its abstract and formalistic shape which depends no longer on intuition. In quantum mechanics it is the concept of Hilbert space of infinite dimensions which brings clarity into the matter just as the four-dimensional 'space- time-union'

Minkowski's played already a fundamental role in relativity theory. (Gr., 167*ff*.) Neither of these can be explained in terms of an intuitively grounded geometry. Becker finds himself forced back here on a distinction made already by Hilbert between an intuitively interpretable and a purely formal mathematics. It is a distinction which in a modified fashion had also already been appealed to by Husserl in his early Philosophy of Arithmetic.

Where Becker differed from both Hilbert and the early Husserl was in his view of what intuitively interpretable mathematics includes. It includes for him all of intuitionistic mathematics in the extended sense he had delineated. By contrast, interpreted mathematics meant for Hilbert only finitistic mathematics and for the early Husserl even more restrictively only a small fragment of finitary mathematics. In contrast to Hilbert, Becker assumed moreover no longer that the formal mathematics needed in natural science be shown to be consistent and could thus be justified by means of intuitively interpreted mathematics.

From this there arise for Becker philosophical consequences for Heidegger's historical-hermeneutic view. The historical-hermeneutic method claims to be able to understand everything. But the totalizing claim of this mode of thought is shown to fail "wherever nature confronts it." (Gr., 170) In the natural world we find ourselves confronted with phenomena which we can only describe with abstract formulas and which remain therefore hermeneutically impenetrable. It is precisely where hermeneutic thinking fails that the mathematical mode of thought leads further. From this it is evident that human beings are not merely historically "existing" beings, as Heidegger thought.

> Existential analysis is completely justified in its own domain which cannot be circumscribed from outside. But there exist at the same time other powers which are inseparably intertwined with existing Dasein. An 'understanding' of these powers is however impossible; they resist altogether the existential hermeneutic, phenomenological analysis. (DD, 92)

The boundary between these two domains runs through mathematics itself. Within the space of human experience the

intuitionistic and constructivist conception of mathematics is undoubtedly phenomenologically correct. But the application of mathematics in natural science manifests a further moment which points to the limits of the constructivist conception. This does not mean that the mythological views of a Platonistic realism are after all justified, but only that in the interplay of history and nature neither the Platonistic nor the constructivist conception can finally triumph.

Looking back from these thoughts to what I learned from Becker's classes at the beginning of my academic career, I begin to see now also why he might have been so fascinated at this time by Heidegger's essay on the principle of reason. Like Heidegger he seems to have ended up with a view that leaves the world no longer entirely comprehensible to us. We can adequately describe it in our formulas but can no longer assign an intuitive meaning to our formulas. This sense of an aporia becomes perhaps most obvious whenever we try to sort out the paradoxes of quantum physics. The formulas fit, but every attempt to interpret them in ordinary words seems to lead to an impasse. Hence, Heidegger's thought finds an unexpected resonance in us. Being is, indeed, the abyss.

One may or may not find these considerations compelling. I have laid them out here in some detail to contrast them to a second line of thought in Becker's work, one particularly noticeable in his later writings, which proceeds more and more from the second of the two principles that Becker lays down as fundamental to existential philosophy. Whereas the line of thought explored so far proceeds most directly from the anti-Platonic assumption that the real is the temporal and which therefore seeks to construct both logic and mathematics in temporal terms, this new line of thought takes its departure from the idea that human understanding is inherently historical in character.

This assumption is already present in Becker's 1927 essay though its consequences are not fully explored till later on. In line with the historicist principle he writes in 1927 that the work of the mathematician itself must become a theme for phenomenological interpretation. We must consciously and

philosophically recognize the obvious but often overlooked fact that mathematics is "a human science". He writes at the time:

> The contrast: intuitionism-formalism is rooted in the fundamental philosophical opposition between the anthropological and the "absolute" conception of knowledge (science) and finally of life itself (as the ultimate reality). (Becker 1927, 625)

If we take the anthropological view, we must interpret current work in mathematics as the historical outcome of a prolonged process of mathematical construction. How Becker means to apply this thought, is already apparent in his earliest contribution to the philosophy of mathematics, his 1923 essay on the foundations of geometry. The work contains an extensive critique of Hilbert's formal-axiomatic treatment of geometry. Becker grants that it is, of course, possible to operate with formal axioms without saying anything about what they might mean. But formal geometry presupposes historically and systematically an interpreted geometrical science which refers to our intuitive experience of space. Hilbert's work can only be understood as the endpoint in a process that began in the intuitive geometry of the Greeks, proceeded via the axiomatization of Euclidean geometry, and through the development of non-Euclidean geometries in the nineteenth century to the contemporary formalist view-point.

Hilbert would, of course, not deny that modern geometry has its historical origin in the intuitive geometry of the ancients, but he would consider this fact irrelevant to the determination of what geometry is today. We can call Becker's alternative view a genealogical one, since like Nietzsche's genealogical investigations it assumes that the philosophical understanding of some subject-matter involves a tracing of its historical genealogy.

According to Becker's genealogical story, we discover that though mathematics is initially a demonstrative and intuitive undertaking, formally analytic modes of mathematical thinking can already be found in antiquity. But they become dominant only in modernity and the strictly formalist view is only a product of the late nineteenth century. Becker asks now what kind of care and meaning is hidden in this mathematical formalism. And he concludes that it is the "care for the

unlimited continuation of deductions." To put it differently: the business of deduction is to be secured without regard to content and factual problems which are at stake or, at least, might be at stake. (628*ff*.) To say it more neutrally: mathematical formalism is, for Becker, the product of a professionalization and specialization in the field of mathematics in which foundational philosophical problems and the problem of the uses of mathematics are increasingly bracketed out and in which therefore mathematics becomes increasingly more incapable of understanding itself philosophically. The result is that mathematics is now understood as a purely formal operating with un-interpreted symbols.

There is another genealogical line that Becker pursues in his historical reflection. It is the discovery that the modern opposition between a realist and a constructivist conception of mathematics goes back at least to the conflict between Plato and Aristotle. For Plato and the Pythagorean tradition that is linked to him, mathematical reality is autonomous and characterized by its own laws. On Aristotle's view, mathematical objects exist only as products of a process of abstraction. For Plato the infinite is something given, for Aristotle it is only something potential. For Plato the mathematical is above and outside time, the Aristotelian insight that mathematical thinking has to proceed by abstracting and idealizing introduces a human, subjective, and hence temporal element into mathematics. The two views characterize archetypal positions which return over and over again in the history of mathematics. "In contrast to a common view... one must put Plato together with Leibniz and Aristotle with Kant as far the philosophy of mathematics is concerned. Plato and Leibniz start evidently of as mathematical mystics. They are both 'Pythagoreans'; they both end up ascribing to mathematics a decisive role in the construction of the world. They both assume it to represent the metaphysical and ontological structure of the world... Aristotle and Kant are, by contrast, critics, sober opponents of all mythical reminiscences and excesses... They both strip mathematics of its mysterious character. The mathematician remains a finite human; any

concept of the infinite that goes beyond the phenomenologically accessible is strictly rejected." (*747ff.*)

According to Becker, Hilbert's attempt to rescue classical mathematics as a whole remains ultimately committed to the Platonic-Leibnizian form of thought whereas intuitionism is the natural descendent of Aristotelian and Kantian thinking. This is hidden from view only because Hilbert casts his Platonism in the form of a completely formalized mathematics.

What is important here for us is Becker's realization that both the constructivist and the non-constructivist view in mathematics have their own long genealogy within the history of mathematics. The history of mathematics shows us how these views have, over time, been constructed and reconstructed. There emerges thus from these reflections a new concept of construction, that is, the concept of the construction of mathematical theories in historical time. In this history both constructivist and non-constructivist forms of mathematics have been produced; they are both, so to say, possible constructions that have been carried out. In his late work Becker begins therefore to speak of mathematics more generally as a construction of possibilities. "The world of mathematics ties together the possible," he writes. The realm of mathematics is that of "possible worlds." (Gr., 66*ff.*) But Becker is in no way a modal realist. He does not believe that these possible worlds are real. They are rather to be thought of in terms of our original constructivist and anthropological conception as results of intuitive human constructions.

From the possibilities that mathematics invents the mathematical physicist selects fitting models for the characterization of the empirical world. We can clarify this idea by reference to non-Euclidean geometry. Mathematicians describe a multiplicity of possible spaces. But it would be absurd to assume that these all coexist in some hyper-space. Geometry is rather the construction of these possibilities. Natural science, in turn, builds with these "all kinds of models *in abstracto* of which it knows, however, that each of them can represent only some features of the observed phenomena." (Gr. 66)

In this historical and modal conception of mathematics a new attitude towards the conflict between mathematical

realism and mathematical constructivism becomes apparent. These two positions can only be considered as constructions of alternative mathematical possibilities. Where Becker had originally assigned pre-eminence to intuitionist constructivism, his new view allows for a neutral distance between the mathematical possibilities. The difference between them, he now allows, may concern the range of their proper applicability. Constructive mathematics may prove to be the appropriate model for the characterization of the space of human experience, whereas classical mathematics may prove indispensable for the scientific description of the world.

This view allows for the redemption of the constructivist view at a different and higher level. For it treats all mathematical structures as constructed, whether they are those espoused by mathematical realists or those put forward by traditional constructivists. But the method of construction to which this new view refers to is not that of piece-by-piece construction of the intuitionist constructivists. The question is now rather what kind of historical events generate mathematical structures and these may be of various sorts. They are certainly not step-by-step, number-by-number, free choice by free choice constructions. The problem with intuitionistic constructivism is now seen to be that it has operated all along with a purely abstract, dogmatic, and formalistic notion of temporal construction.

On Becker's ultimate view, or at least on the view I extract from his words, the nature of mathematical construction can only be understood by attending to the actual genealogy of mathematical thought. The history of mathematics becomes here the key to the philosophy of mathematics, the history of logic the key to the philosophical understanding of logic, and the history of analytic philosophy the key to a philosophical understanding of analytic philosophy. This is a lesson I have learned from Becker's work; one which I have tried to apply over the years in writing about the foundations of mathematics, of logic, and of analytic philosophy. This view does not mean to reduce philosophical inquiry to historical description, but sees an understanding of the historical process

as indispensable to the philosophical interpretation of the meaning of the end results of that process.

This is probably not the moment to ask what general characterization we can give of mathematical construction, if we see it as a process in real time. A number of things are clear, though, very quickly. That process is not one that proceeds step by step. The same ground is covered again and again, concepts are clarified and made more precise as we go, distinctions are made which were formerly not seen, assumptions are withdrawn that were previously made. The construction of mathematical possibilities proceeds in some ways not unlike that of other human creations such as, for instance, literary fictions. There we invent figures by specifying in general some properties they are meant to have but leaving other necessary properties of real figures undetermined. We do so by laying down in general who Lady Macbeth is, not by identifying one particular fictional object. The same is true of our mathematical objects. We specify them by certain characteristics and leave others undetermined at least till a later moment. We lay down that the natural numbers are to form a progression but leave unspecified whether they are to be sets and if so, what sets they are. Mathematical objects have no determinate identity, they are rather figures in a design and that design is usually described only in general terms. Frege once wrote:

> The historical approach with its aim of detecting how things begin and of arriving from these origins at a knowledge of their nature, is certainly perfectly legitimate; but it has also its limitations. If everything were in continual flux, and nothing maintained itself for all time, there would no longer be any possibility of getting to know anything about the world and everything would be plunged in confusion... What is known as the history of concepts is really either a history of our knowledge of concepts or of the meaning of words. Often it is only after immense intellectual effort, which may have continued over centuries, that humanity at last succeeds in achieving knowledge of a concept in it pure form, in stripping off the irrelevant accretions which veil it from the eyes of the mind. (FA, VII)

I see it as Becker's merit to have shown how we can avoid such a mythological account without having to abandon large parts of our logical and mathematical thinking, that we can take a constructive view of mathematics without being

forced into the procrustean bed of a mathematical constructivism. And this achievement, I believe, has been made possible precisely because throughout his work has held steadfastly on to the existential insights that we are thoroughly temporal and historical beings.

Oskar Becker died shortly after I began my studies and so I moved on to the University of Munich and then to Oxford where I learned that it was quite illegitimate to think about both logic and Heidegger. The gap between so-called analytic and so-called Continental philosophy is, however, not an inevitable one. We should remind ourselves first of all that Husserl initially studied mathematics and remained throughout his life concerned with the foundations of logic and mathematics, that Heidegger, too, had an early interest in logic and the philosophy of mathematics and that he contemplated initially to write his dissertation on the philosophy of mathematics, but then wrote it and his subsequent *Habilitationsschrift* instead on questions in philosophical logic, and that he reviewed the development of modern logic in a series of early essays which show him to be fully familiar with Frege's writings and with Russell and Whitehead's *Principia Mathematica*. It was only much later, after the publication of Being and Time and with his 1929 essay "What is Metaphysics?" that he turned critically against logic. In that essay he announced that the most important and ultimate concern of metaphysics was with nothingness and that of this nothing one could not even say that it exists but only that it nihilates, something that could not be grasped with the means of logic. Shortly afterwards, Rudolf Carnap was to ridicule these remarks as characteristic of metaphysical nonsense and with this the trenches were opened for the war between the "analytic" and the "non-analytic" movement in philosophy and those, like Becker, who refused to take sides in this confrontation were quickly shunted to the sidelines and then forgotten.

Recalling Oskar Becker today may be justified by the very different situation in which we now find ourselves. The confrontation between analytic and Continental philosophy has ceased to be an exciting battle; it has become instead now for the most part a long, dreary, uninformed trench warfare. As a

result some philosophers have sought to think across these hardened lines. There is a sense that something is to be gained from such an exchange and it is precisely in this context that Becker becomes once again of interest.

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