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Center for Hermeneutics, Phenomenology, and Practical Philosophy

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After Husserl: Phenomenological Foundations of Mathematics

Guest Editor: Iulian Apostolescu

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Editorial

After Husserl: Phenomenological Foundations of Mathematics

First and foremost, I would like to express my gratitude to all contributors for all of the hard work they put into writing their pieces of original work to show the lasting vibrancy and vitality of Edmund Husserl's ideas and to encourage further investigations. From the outset it was my intention to open up a way of research on different interpretative perspectives on Edmund Husserl and his interlocutors (Gottlob Frege, Oskar Becker, David Hilbert, Hermann Weyl, J. Klein, L.E.J. Brouwer, Kurt Gödel, Bourbaki, Marc Richir), which does not end up either in the naive celebration of Husserl's investigations of *das Mathematisierendes Dasein* or in its brute rejection.

Professor Ilie Pârvu, my doctoral supervisor and an exemplary philosopher, was an inspiration: his erudition and analytical creativity spurred me on. This special issue would have been a very different project without his philosophical support. Also, I benefitted greatly from conversations on Oskar Becker's *opus magnum* from 1927, *Mathematical Existence* (*Mathematische Existenz*), and David Hilbert's 'transcendentalism' exposed in his *metamathematics*, with Constantin C. Brîncuş, with whom I worked in the project "The Phenomenological Analysis of Axiomatic Mathematics."

Brigitte Parakenings, the archivist at the Philosophical Archive of the University of Konstanz, helped me enormously with archival materials from The Oskar Becker collection.

I am very grateful to George Bondor for his constructive suggestions and excellent editorial support. Thanks go also to the representatives and the production team of *Meta: Research in Hermeneutics, Phenomenology, and Practical Philosophy*.

Guest Editor: Iulian Apostolescu
(University of Bucharest & *Phenomenological Reviews*)
December 15th, 2019
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How Does Mathematics Get into Science, and Why? A Husserlian Perspective

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Abstract

In this paper, I present, first, a categorization of the many uses mathematics has in science as a methodological tool. I identify four: representative, instrumental, predictive and heuristic. I introduce the issue in a historical context, discussing it more systematically afterwards. My approach is Husserlian thoroughly, which means that I hold the following views: 1) real nature is perceptual nature, constituted out of the hyletic material of the senses by the action of built-in psychophysical proto-intentional systems; 2) mathematized nature is an intentional construct devised for methodological purposes; it instantiates idealizations of formal-abstract structures of perceptual nature, but can also incorporate non-representing (imaginary) elements; 3) mathematics serves science by offering contexts of representation of perceptual reality and instruments of theoretical investigation of mathematical substitutes of reality. I conclude by contrasting my approach with Husserl's own.

Keywords: Phenomenology, Husserl, mathematics in science, idealization, intentionality

A philosophy is alive when it inspires, when it offers instruments with which to think. Particularly about questions its creator may have tackled, but not exhaustively or with the same interest one may have. When a philosophy is good only for exegetical rumination, incapable of addressing present-day problems, it became a museum piece.

Husserl is a philosopher still very much alive. His philosophy offers a wide perspective from which to consider a variety of contemporary philosophical problems. The applicability of mathematics in physical science, which I will tackle here, is one of them. My treatment of the problem will sometimes be very close to Husserl's own, but will sometimes diverge into domains that Husserl himself did not tread. However, I will never abandon a Husserlian perspective.

At the end of the paper I will point out what in my treatment can already be found in Husserl's published work and how I think one can fully explain and justify the applicability of mathematics in science (and how inefficient mathematics is as guide to metaphysics) from a perspective that is Husserlian in spirit but that Husserl himself did not completely embrace, not for narrowness of sight but to remain faithful to some basic tenets of his epistemology and the call to personal responsibility that lies at the center of his philosophy.

1. The scientific revolution of the 17th century was marked by two apparently conflicting tendencies; one, more accurate observation and experimentation, another, mathematization. But whereas observation and experimentation are modalities of perception, mathematization is the negation of it. To mathematize is to place physical reality beyond the possibility of perception, to make it perceptually inaccessible. There are then two essentially distinct conceptions of realities in modern science, the perceptual and the mathematical.

Which is *real* reality? How do they relate to one another? If reality is that which we can, at least in principle if not actually, perceive, what does mathematics, whose domains are not perceptually accessible, have to do with it? But if nature is, at its inner core, mathematical, what is the role of perception in empirical science, particularly in the validation of physical theories?[1] Which role does mathematics play in modern science and how did it come to play this role? Can it be logically-epistemologically justified?

Mathematics entered the domains of natural science cautiously in the beginning, with the application of geometry in the study of the motion of bodies in physical space, but progressively ever more intrusively to the point of becoming indispensable to and overwhelmingly present in modern science. The use of geometry as an instrument of representation and investigation of the kinematics and dynamics of bodies in space and of algebra as a means of expressing and dealing with quantitative relations among physical magnitudes pose, apparently, no mystery: after all, bodies, the space and trajectories in space have geometrical properties, and evaluating and comparing quantities are practices firmly established in our pre-scientific life. But, obviously, neither geometrical figures nor mathematical numbers are objects of perception. There is a gap to be filled between, on the one side, perception and the world of our common practices and, on the other, mathematical entities and mathematical reasoning. Things get more complicated with the development of science, when mathematical methods and objects with no immediate correspondents in perceptual reality or our common practices became ever more important and mathematics acquired other uses than the purely representational.

But before attempting to understand and ultimately justify the many uses of mathematics in science one must identify them. I want to do this by following, if only superficially, the historical development of modern science, paying attention to those moments when mathematics conquered extra territory and extended its range of scientific applicability.

Identification will be followed by explanation and, ultimately, justification. Understanding how mathematics can in so many ways be useful in science and justifying mathematization from a logical, epistemological and methodological perspective are philosophical tasks to which Husserl has greatly contributed, although not to the extent I think he could.

His last published work, the influential *The Crisis of European Sciences and Transcendental Phenomenology* (henceforth *Crisis*), is basically a piece of propaganda for transcendental phenomenology as, among other things, the correct way of restoring meaning to fossilized scientific practices, mathematization particularly. By going back to the enthronization of mathematical methodology in natural science, as a genetic phenomenologist, not a historian, Husserl was able to detect how mathematization came to be, its goals and what he believed to be its limitations. He managed also to uncover the many layers of intentional action that went into the establishment of the method but were, he thought, "forgotten" by tradition with the consequent endorsement of a wrong interpretation of both the concept of nature and the mathematical method of scientific investigation of nature. It is mainly this misinterpretation that Husserl criticizes, not the methodology per se, although some questions can be raised as to the extent to which he was comfortable with the full range of mathematical techniques in science.

Much has been published on *Crisis* and Husserl's analyses of "Galilean" science but my goal here is not to contribute to this literature. I use Husserl to my purposes. I believe that naturalism, which holds that the role of science is to describe what she finds in nature without in any way contributing to the constitution of nature itself, coupled with a wrong interpretation of the concept of nature, as pointed out by Husserl, obliterates any honest attempt at understanding the many uses of mathematics in science. But although I find Husserl's analyses of the intentional constitution of nature in modern science, together with his explanation and justification of some of the uses of mathematics as a scientific methodology, correct, I also find them incomplete. Ironically, however, it is Husserl himself who offers the key to understanding the method: purely mathematical techniques and materially meaningless symbolic manipulations can, as scientific strategies, be *logically and methodologically* justified in formal ontology.[2]

By investigating the formal-logical relations between formal theories and formal domains, formal ontology can tell us when truths of one domain or theory (for example, purely formal-mathematical extensions of mathematical representations of perceptual reality) are true in other domains and theories (for example, mathematical representations of perceptual reality themselves), thus safeguarding purely symbolic-mathematical means of scientific investigation.

But let's first better identify the diverse ways in which mathematics can be used as a methodological tool in science by following, as I said, a historical line.

2. Classical geometry has always been, from its beginnings with Thales and Pythagoras to its mature developments with Euclid and beyond, a science of perfect forms and their mutual relations, things that are not exactly of this world.

In the Platonic interpretation, geometry deals with ideal archetypes inhabiting a world of their own that this world where we live can at best only imperfectly instantiate. We can ascend to this *topos ouranos* only through reason, not the senses. In the Aristotelian interpretation, on the contrary, the ideal forms of geometry are idealizations of actual or possible abstract aspects of this world that, however, as the idealization they are, are not of this world either. In this interpretation, to ascend to the geometrical realm, perception must be complemented with abstraction and idealization, the exactification of the perceivable. Geometry was by then already far removed from its origins in land surveying.[3]

Despite is astronomical applicability, geometry had in antiquity no place in the science of the real world of our direct perceptual experience. The shapes of the world were not supposed to be, strictly speaking, geometrical; the quantitative aspects of reality were not supposed to be, strictly speaking, arithmetical. Before the Galilean revolution, empirical reality was *perceptual* reality, not an idealized copy of it where arithmetic and geometry *proper* had a place. For the ancients, the perceptual world could be measured, and bodies had perceivable shapes, but *perceivable* measures and shapes were *not* supposed to be approximations to a perceptually inaccessible core of mathematical exactitude lying deep *in* the world itself.

This radically changed with the development of modern science by Galileo, Descartes and Newton, among lesser emblematic names. It all began with Galileo's geometrization of the perceptual world, not as a mere methodological devise, but as the uncovering of a geometric reality *within* the world. Geometry entered natural science by means of a radical reshaping of the concept of nature, no longer what we perceive with our senses, but a perfectly geometrical reality buried deeply inside it, inaccessible to perception, only reason. *Nature*, real nature, *became transcendent* and immanent nature, only an essentially imperfect image of it available to the senses. Disguising an invention as a discovery is the inaugural act of modern science. For Husserl, this was the origin of the "crisis" of science. By taking a *product* for a *given*, science alienated itself from the constituting presuppositions on which it sits, unaware of its true nature and the scope and range of validity of its methods, among which Galileo's creation, mathematization.

Let's take a closer look at Galileo's procedures in geometrizing the natural world. Two operations are fundamental: abstraction and idealization. By abstraction I mean a refocusing of intentional consciousness that brings certain aspects of physical reality to attention in detriment of others, for example, shape instead of substance (form instead of matter) and arithmetic proportions instead of causal relations (at least in Galileo's typically kinematic treatment of motion). By idealization I mean mathematical exactification, for example, taking the shape of physical bodies as proper geometrical forms, *instances* of geometrical ideas.

At a certain point in the Second Day of the *Dialogue Concerning the Two Chief World's Systems* (Galilei 1970), Simplicius (the spokesman of scientific conservatism) criticizes Salviati (Galileo's alter ego) for supposing that a sphere touches a plane in a single point. Spheres, Simplicius reasons, can be quite heavy and would deform a plane on which they are placed, thus escaping the idealized situation. In his answer, Salviati first recalls that in financial transactions one calculates with numbers independently of the matter coins are made of or the merchandise being sold or bought, and then explains that "[…] when the geometer philosopher wants to see concretely the effects proved abstractly, he must eliminate the interference of matter; if this is done, I assure you that things will be as exact as in arithmetical calculations […]" (ibid., 265ss). This exemplifies both abstraction and idealization, abstracting form in detriment of matter and idealizing real physical spheres and planes as ideal geometrical spheres and planes.

Clearly, Salviati's world is a mathematical world in the *proper* sense; however, he does not take geometrical idealization as an operation of *substitution* of reality by irreality, of what does not exist for what does, but as an operation of *prospection* of reality itself, as the unveiling of reality's inner, most fundamental reality.

Galileo follows Euclid's *Elements* very closely. For example, in discussing free-fall, velocity is *not* defined as the ratio between the space some mobile travels and the time spent in the journey, for this would go against Greek principle that ratios only make sense between homogeneous magnitudes. In a sense, for Galileo, numbers are not yet pure (ibid., 27). In discussing free-fall along inclined planes, Galileo states that two mobiles have the same speed when they travel equal spaces in equal times. Of course, one can define inequalities of velocities analogously. Notice that here Galileo reduces linear continuum physical magnitudes to *geometrical* line segments and deals arithmetically with ratios of line segments representing homogeneous magnitudes in conformity with Eudoxus' theory of proportions as presented in Book V of Euclid's *Elements*.

It is often said that pre-Galilean physics is essentially non-mathematical, and that experience and observation were not as important for the ancients as they were for Galileo and his followers. This is at best an exaggeration. From Thales, the half-mythical creator of philosophy and mathematics, to Galileo, science was a mixture of observation, sometimes very accurate observation that could also be of a quantitative nature, induction and explanation, often based on clever analogies. The physics of Aristotle, for example, although not mathematical in the same way of Newton's physics, is also concerned with quantities and quantitative relations. In *On the Heavens*, for example, Aristotle explicitly states a quantitative law relating the times two bodies take to cover the same distance in free fall: they are, supposedly, in the inverse relation of their weights; a body twice as heavy takes half the time. This is the law that Galileo claims, in the *Dialogues*, to have *empirically verified* to be false: bodies in free fall, he claims, cover the same distance in the same time independently of their weights if we do not take the resistance of the medium into consideration (a disproof of Aristotle's law by reduction is also provided).[4]

Regardless of whether he indeed verified, or *could* have verified this principle empirically, Galileo's criticism helped to establish the myth that Aristotle was not a good observer. The truth, of course, is that Aristotle's principle is approximately correct for many cases of bodies free-falling in viscous media, the cases that Aristotle probably observed with more attention.

There are, however, aspects of Galilean science that are completely strange to Aristotle. The concept of quantity of the latter is that of practical life and quantitative relations were certainly not supposed to be any exacter than those of ordinary mundane transactions. In Galileo, on the other hand, quantities are *exact*, represented by geometrical segments with which he could operate *geometrically*. Also, whereas Aristotle had only logic to derive the consequences of general principles, Galileo had geometry, a hugely more efficient instrument.

The object of Aristotle's science is the physical world as actually given to the senses, whereas that of Galileo is an *idealized version* of it where mathematics proper finds a place. Aristotle's quantitative relations are not, strictly speaking, numerical – nor could they have been – and are supposed to be valid in the perceptual world. Galileo's are numerical relations proper that are, however, strictly valid only in an idealized world.

Descartes made two major contribution to modern science. One was the "invention" of space or, better, the modern scientific conception of physical space. By establishing that the essence of physical bodies is their extension, not their matter, Descartes established that reality is ultimately abstract, thus providing the key for the identification of real physical space with geometrical space.

His second contribution was the creation of analytic geometry, where geometrical constructions can be carried out symbolically by algebraic means. Although not the first time that symbolic manipulations entered the realm of mathematics, for Descartes was preceded in it by the Italian algebraists of the Renaissance who invented "imaginary" numbers, the creation of analytic geometry opened the doors to the symbolic in science, mathematics *as well as* physics. This huge methodological step forward, however, is more clearly detectable in Newton.

The mathematical methods of Newton are substantially more sophisticated than those of Galileo. And this is already evident in the first proposition of his *Principia*: the areas, which revolving bodies describe by radii drawn to an immovable center of force do lie in the same immovable planes, and are proportional to the times in which they are described.

The proof of this assertion depends on Galileo's principle of decomposition of velocities (the motion of the body is the resultant of an inertial tangential motion and an accelerated centripetal motion), the geometrical theorem that triangles with equal bases and equal heights have equal areas independently of the size of the two other sides, and the mathematical novelty, the process that we would call today of "taking the limit", originally a creation of Archimedes. As physics is concerned, however, Newton's truly original contribution was that of force.

In our pre-scientific experience, there are at least two ways in which a body can move, by itself, due to some internal cause, or by being pushed, pulled or otherwise acted upon by another body that usually is in contact with it. The ancients attributed a "soul" to things that could move by themselves, people or animals, and did not seem to see any mystery in a body somehow inheriting the movement of another body that touches it. But there are situations in which inanimate bodies move without being touched by other bodies, for example, smoke that flies up and a rock that falls. Aristotle saw both types as natural motions driven by an internal disposition to regain a natural place, inertial motion as we would say today, involving final but no efficient causes.

But there is one unnatural type of motion without direct contact that the ancients knew well, magnetic-induced motion. Naturally, they tried to explain it in terms of causes that were familiar to them, direct action and souls. Aristotle says that to explain magnetism Thales attributed souls even to inanimate bodies ("all things are full of gods"). Others abandoned animistic for materialistic explanations like effluvia, emanations from the magnetic stone that somehow, by some clever mechanism, "pulled" the pieces of iron.

Inanimate bodies acting upon one another at a distance without any intervening mechanism was out of question. Aristotle criticizes both animistic and materialistic explanations then available for magnetism, thus running out of possibilities for explaining the phenomenon. Since it does not fit well into his theory of motion, which required direct contact for efficient causation, Aristotle passes in silence over the phenomenon of magnetic induced motion.

Newton's stroke of genius was to introduce a notion, that of force, as a completely general cause of non-inertial motion, that is, non-uniform motion in inertial frames. After guaranteeing in his first law of motion the existence of frames of reference in which bodies would be at rest or in uniform rectilinear motion if sufficiently far from other bodies (in empty space particularly) he introduces by a definition, the second law, the notion of force as *that which causes acceleration*, *whatever this may be*. In the words of Newton:

Def. IV: An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.

This force consists in the action only; it remains no longer in the body, when the action is over. For a body maintains every new state it acquires, by its *vis inertiae* only.

Modern science contradicts Aristotle's theory of motion twice: Galileo's principle of inertia had already eliminated the need of constant action for the preservation of motion, Newton's concept of force eliminates the need of direct contact for changing the state of motion.

Now, and this is very relevant in the story I'm telling, Newton does not care to tell us *what* forces are and *how* they act, by internal disposition, contact with other bodies, or some intervening mechanism.

"For I here design only to give a *mathematical* notion of those forces, without considering their physical causes and seats (my emphasis)", he says. One does not know what forces *are*, we only know what they *do* in quantitative terms: forces "cause" acceleration depending on an intrinsic property of the bodies upon which they act, their "mass", which measures the body's willingness to have their state of motion altered. In the very last scholium of the book, he says:

"But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called a hypothesis, and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy".

Clearly, only what has *quantitative* expression has a place in Newton's non-speculative "experimental philosophy". Only by abandoning the world of material beings and real causal relations and moving to a mathematically purified context where only quantitative relations mattered, Newton could introduce a purely mathematical notion of force with which to unveil the mathematically precise *mathematical* structure of the mechanism that makes the world go around.

There are causal explanations in Newton's science, but incomplete. We know that gravitational forces keep the world together, celestial bodies spinning around each other, and the seas in regular tidal motion, but we ignore what these forces are and how they act. We only know that forces impart acceleration, which allow us to find out the future and past trajectories of bodies and then, in principle, their precise position in space at any point in time, and this is enough.

In mathematized physical science, which Newton's mechanics exemplifies magnificently, only quantifiable magnitudes and quantitative relations matter. New notions are admissible provided they are quantifiable, and one knows how they relate quantitatively to familiar notions. Definitions take the form of algebraic identities telling how *definiendum* and *definiens* relate quantitatively to one another. Properties that are not at prima facie quantitative are admissible only if they can be given quantifiable representatives. For example, the principle of inertia states that in inertial frames rest and rectilinear uniform motion are "natural" states, which can only be altered by the action of "forces". But bodies have a "natural" tendency to preserve their state of motion. *What* this tendency consists of is immaterial provided we know how to measure it. The degree of "laziness" of bodies, their unwillingness to alter their state of motion, finds its quantifiable representative in the concept of inertial mass, which can be precisely expressed numerically in terms of some standard mass. The mass of the body increases with the force necessary to impart upon it a given acceleration.

Other notions such as momentum (linear and angular), work, energy, etc., follow the same pattern. A particularly important methodological strategy can now be introduced, namely, the search for *invariants*, magnitudes that are conserved, for instance, energy or momentum, thus allowing the establishment of principles that have both explicative and heuristic virtues. Centuries after Newton, the neutrino was conjectured to exist only to guarantee the conservation of energy and other relevant quantities in certain nuclear reactions.

The geometrization of motion became possible after we learned how the position and trajectory of bodies depended on kinematic and dynamic variables and how the former depended quantitatively on the latter. In its time, Newton's *Principia* was the best expression of this new knowledge, the magnificent synthesis of geometry, kinematics and dynamics heralding a new science: mathematical physics.

Geometry was by then almost synonymous with mathematics, but that was about to change. The analytic geometry of Descartes and Fermat, where geometrical constructions are replaced by algebraic manipulations, opened the doors to *symbolic reasoning* in mathematics. It would soon be adopted in physics too, proving to be in both domains an immensely rich and useful methodological strategy. By symbolic reasoning I mean reasoning by manipulating symbols according to rules without necessarily taking into consideration what the symbols mean or whether they have meaning (symbols usually stand for something, but not necessarily).

Despite its practical utility and logical justification, Husserl believed that this strategy presented risks. Symbolic reasoning, he thought, cuts us from the things themselves, we are no longer directly concerned with intuitive contents, only with intuitively empty symbols that, in the best possible case, stand for things that can only *in principle* be intuited. By forsaking intuitiveness, that is, perceivability, science became mechanical and ritualized. This was, Husserl thought, the beginning of the crisis of science that would eventually develop into a crisis of culture in general characterized by loss of meaning and direct personal responsibility.

The problem gets considerably worse if symbols exist in the symbolic machinery of science that have no representational value. In this case, intuitiveness not only recedes into the background but is completely lost. Here a tension appears between undesirable symbolic alienation and desirable methodological effectiveness. Meaningless symbolic manipulation had at least since the 16th century proven its value as a methodological mathematical tool with the introduction of "imaginary" numbers in the theory of algebraic equations. There was no a priori reason why symbolic reasoning should not be methodologically relevant in mathematical physics as well. Indeed, time has shown that this strategy is not only useful but central in scientific practice. Although non-representing symbols have no *representational* value, they can still play *instrumental*, *predictive* and *heuristic* roles in science.

This considered, is Husserl's criticism of "symbolic alienation" still relevant or should it be dismissed as old-fashioned and misguided conservatism? I believe Husserl's cautious admonitions still resonate, although not as a critique of scientific methodology itself, which he never meant them to be, but of the absurd interpretation given to it. Husserl urges us to uncover, recover and vivify the sedimented meaning of our scientific practices. By tracing the intentional genesis of both the modern conception of physical reality and the methodological strategies devised for investigating it we can shun absurd accounts of the adequacy of these methods. As we will see, Husserl's philosophy itself opens a possible route for explaining the efficacy of mathematization in science, even when involving meaningless symbolic reasoning, and justifying it on logical and epistemological grounds.

There are essentially two ways of accounting for physical action, action at a distance and action through a medium that "transmits" the action. Classical Newtonian theory of gravitation favored action at a distance. Classical electromagnetism, with Faraday and Maxwell, preferred action through a medium, whose nature, however, changed dramatically throughout history, from something *real* to something *mathematical*. At first, electromagnetic action was supposed to be transmitted through a *material* medium that supported the electromagnetic field, the never detected ether, an elusive substance known only "indirectly" through its quantifiable properties. Later, the medium became space itself, but with the property of acting and reacting to the presence of electric charges and currents. More recently, with quantum field theory, the role of carriers of electromagnetic action was taken by photons. In relativistic theory of gravitation, at least until it is properly quantized, the medium is still space, or rather spacetime, whose *geometry*, however, depends on the distribution of matter. Bodies act on the geometry of space, which in turn determines how bodies move. This relation between matter and geometry, the world and mathematics is illustrated vividly in Einstein's field equation, mathematical entities on one side of the equation representing physical entities, those on the other standing for geometrical entities. More than a formal *equivalent* of gravitational action, the geometry of space-time *is* gravity and *explains* its action. As we see, besides mathematizing reality, mathematization also often reifies mathematics. Mathematics does not simply *represent* reality, mathematics is *part of* reality. Some go as far as saying that reality is *nothing but* mathematics.

Mathematics is most active in quantum mechanics, playing therein all the roles it can play.

De Broglie conjectured that electrons and similar particles that carry energy E and momentum p are associated with waves with frequency υ and length λ such that $E = h\upsilon$ and $p = h/\lambda$ (introducing the wave number $k = 2\pi/\lambda$ and the angular frequency $\omega = 2\pi\upsilon$, $E = (h/2\pi)\omega = \hbar\omega$ and $p = \hbar k$). Here, mathematics is a *language* where to express physical correlations in idealized form, playing thus a *representational* role.

Schrödinger imposed upon himself the task of finding the equation of such a wave (Schrödinger 1926). It is not very clear how precisely he did it, the heuristic strategy he used, but for my purposes this is not important, the following "derivation" suffices. Consider first a particle with definite energy; its associated wave has, then, definite frequency and can be represented as a sinusoidal wave: $\psi(t) = \sin(\omega t)$. Helmholtz's equation applies: $\Delta \psi + k^2\psi = 0$, where Δ is the Laplacian operator. Substituting one gets $\Delta \psi + (p/\hbar)^2\psi = 0$. But $p^2 = 2mK$, where m is the mass of the particle and K its kinetic energy. Therefore, $\hbar^2/2m\, \Delta \psi + (E - V).\psi = 0$, where E is the total energy and V the potential energy of the particle. This was the original Schrödinger equation, usually written thus: $-\hbar^2/2m\, \Delta \psi + V\psi = E\psi$. (1)

Consider now the evolution of the state function of the particle. *Supposing* that $\psi(t)$ is a *complete* characterization of the state of the system at time t, the evolution equation must involve *only* the *first derivative* of ψ. But, under this hypothesis, ψ cannot be a sinusoidal function, it must have the more general form $\psi(t) = \exp(i\omega t)$. From this we get: $i\hbar\psi' = E\psi = -\hbar^2/2m\, \Delta \psi + V.\psi$, which is the time dependent Schrödinger equation. Here, we see, first, how mathematics succeeds in providing an adequate context of *representation* – complex analyses – where both the form of the wave (*given the formal restriction* that the expression for the wave at one instant must determine its expression at any future instant) and *the presupposition itself* (the wave equation must be of first order in the time variable) can be expressed. And, second, how by using mathematics *instrumentally*, i.e. as *a context of derivation*, one succeeds in writing the evolution equation, the time dependent Schrödinger equation. Now, still using mathematics instrumentally one can, ideally, solve the equation and obtain the function $\psi(t)$ from which one can derive knowledge about the system that allows us to make predictions about it. Here, mathematics plays its *predictive* role (of course, these roles are not independent one of the others). The success or failure of these predictions determine the success or failure of the whole theoretical schema.

Historically, the development of quantum mechanics was heavily conditioned by the hydrogen atom (one proton and one electron) and its spectrum, characterized by lines of emission of radiation with different well-defined frequencies. If a wave function ψ is supposed to characterize the state of the H atom, it is natural to see its states of definite energy as eigenvalues of an equation of the type $\hat{E}\psi = E\psi$ (2), where $\hat{E}$ is some Hermitian "energy operator", with state-functions associated with the states of definite energy as corresponding eigenvectors. ψ must, then, be a vector in a complex vector space. The formalism of linear algebra is then summoned as the adequate to express quantum mechanics (matrix mechanics).

Now, comparing (1) and (2) we see that $\hat{E} = (-\hbar^2/2m\,\Delta + V)$ and, therefore, the operator associated with the x-component of momentum must be = $-i\hbar\partial p/\partial x$, etc. Again, as we see, complex numbers cannot be avoided. This gives us a "recipe" of how to write the Schrödinger equation for a quantum system: write the Hamiltonian, "quantize" using the correspondence above and substitute in (2).

The solutions of the corresponding equation for the H atom fit well the experimental data, i.e. the known frequencies of spectral lines, showing the correction of the approach to the problem. Here, again, mathematics plays its predictive role and *because* it plays this role the entire symbolic apparatus can be put to test.

Notice that not all terms of the mathematical language plays a representational role. The symbols i and ψ, or those for the operators, for example, are not denotative, there is nothing in perceptual experience that corresponds to them directly. The situation is like that of language, in which some terms, such as names, denote but others, such as prepositions or conjunctions, do not, being only elements of internal articulation of the discourse.

Let's consider now a particularly relevant instance of the *heuristic* role of mathematics in quantum mechanics, the mathematical "prediction" of the positron and antimatter in general.

By substituting the energy and momentum operators in the relativistic equation for the energy of a free particle, Oscar Klein and Walter Gordon succeeded in obtaining, in 1928, a relativistic version of Schrödinger's equation (the Klein-Gordon equation). The equation can be generalized to particles under the action of a potential. However, it did not agree to content with the experimental data when applied to the hydrogen atom. A characteristic feature of Klein-Gordon equation is that it is of second-order, the momentum operators are squared. Dirac considered that if the wave function provided indeed a complete characterization of a quantum system, then its value at some instant should be sufficient to determine its behavior in the future, and then the wave equation should be of first-order in the *time* variable. Now, he argued, since time and space variables are symmetric in relativity theory, the wave equation must be of first-order in *all* four spacetime variables. Based on these *formal considerations*, that the true quantum-relativistic equation of a particle must be of first-order on all variables, he arrived at a wave equation whose solutions, the wave functions characterizing the particle, are four-component spinors, each component obeying the Klein-Gordon equation.

Again, the formal restrictions Dirac imposed on his equation were the mathematical translation of one *desideratum*, namely, that the wave function should contain a complete characterization of the system, and the established physical *fact* that space and time are *formally* symmetrical in relativity theory. Mathematics functions here as a context of representation where both the desideratum and the physical fact are expressible.

Now, in accordance with Dirac's equation, the wave function of, say, an electron, has four components, each characterizing a possible state of the particle. Two of them, corresponding to states of positive energy, have natural interpretations, corresponding to two different possibilities for the spin of the electron, but the two remaining components, corresponding to states of *negative* energy, had by then no available physical interpretation.

Dirac had *two* alternatives, either to dismiss the negative-energy solutions as senseless mathematical sub-products of the formalism, or, more interestingly, look for some physical interpretation for them. But we should be careful here, there is nothing in the formalism itself pressing for the latter alternative and, even more importantly, nothing indicating *what* these physical things could be. Dirac was completely free to *guess* what the "imaginary" solutions corresponded to, *provided* it had the required *formal* properties. Dirac guessed that these things were undetected electrons forming a "sea" of electrons (Dirac sea) trapped in all possible states of negative energy.

Now, once one of these electrons moved to a state of positive energy, it would leave behind a "hole" that would behave *formally* as a positively charged electron with positive energy. This is how far mathematics can go, *whether* there is something in physical reality corresponding to this "hole" and *what* it is, mathematics is completely silent about. Dirac, however, was free to conjecture that there *may* exist in nature particles just like the electron, but positively charged, whose states are given by the two components of the spinor that corresponded formerly to states of negative energy of the electron. A few years later (1932), Carl Anderson discovered the positive electron, the positron, showing that Dirac had *guessed* right. *But that remained a happy guess, not a prediction*. Of course, the guessing was from the start subjected to *formal constraints*, which is *all that mathematics can provide as a heuristic instrument*. To claim that Anderson's particle *is* Dirac's "hole" is to claim more than what the facts allow. Anderson's positron has only the *formal* properties of Dirac's "hole"; there was no a priori guarantee that these "holes" were real nor that they would manifest themselves as positrons, since they *could* in principle materialize as *anything* with the right formal properties. *In his heuristic role, mathematics unveils formal possibilities*. It is all it can do, but it is already a lot.[5]

3. This *vol d'oiseau* over the history of science illustrates what I believe to be the main uses of mathematics in science, representational, instrumental, predictive and heuristic. Summarizing:

1) *Representational*. In this role, mathematics offers contexts where certain formal-abstract aspects of physical reality are instantiated in an idealized manner.[6] Mathematics represents to the extent that it provides contexts of instantiation (materialization) of idealized formal-abstract (structural) aspects of physical reality.[7] One is often interested in mathematically representing only restrict structural aspects

of reality, not the whole of it, and in general in such ways that not all mathematical entities and situations in the context of representation are themselves representational.[8]

The single most important strategy in the process of mathematizing perceptual reality[9] is *quantification*. The world is never the same, it changes in many ways; to quantify is to express the *quantitative* variability of the world as mathematical variables over numerical domains. For example, bodies have different volumes and the volume of a body may change in time (by volume one can simply understand the amount of space delimited by the body's surface). We can compare volumes perceptually and convince ourselves that bodies have always more, less or the same volume as other bodies. These are perceptual facts (effectively perceived or potentially perceivable), but perceptual quantitative relations are not yet mathematical; at best, they are proto-mathematical.

Quantifying the notion of volume amounts to expressing in numerical terms, in principle if not actually, the volume of *any given body* in terms of the volume of a standard body taken as reference, the unit. A *magnitude* is any physical entity that can be quantified, for instance, the volume of bodies; mathematically, a magnitude is a numerical variable ranging over the domain of all its possible values; i.e. all the numerical values representing the quantitative variability of the entity in question with respect to the relevant unit.[10]

Magnitudes can have determinations beyond the purely quantitative, such as direction in space (velocity, acceleration, and forces, for example), or be combined in mathematical entities more complex than pure numbers (e.g. tensors). There are no limits to how mathematics can build complex entities from numbers to serve representational and instrumental purposes (see next section), complex numbers, numerical functions of many variables, vectors, tensors, fields, etc.

It is important to emphasize that numbers, insofar as they represent quantitative relations[11], do *not* express them *as they are or can be experienced*; numbers express only *non-experienciable idealizations* of quantitative relations as they are perceived. The more and the less of perceptual reality can only be given a number by being idealized beyond the possibility of perception. In short, to quantify, i.e. to express quantitative relations *numerically*, is already to idealize.

Now, when sufficiently many physical magnitudes are quantified, for example, volume and temperature, one can express mathematically by formulas, i.e. algebraic correlations among the variables standing for these magnitudes, how one perceives (or conjectures) they to be related; how, for example, volume changes with the change of temperature. But importantly, formulas not only express what is effectively perceived, but also what is perceivable but not yet perceived, thus offering a sort of *anticipation of perceptual experience* (allowing *predictions* to be made).

Structural aspects of physical reality other than the quantitative as, for example, spatial structure, can be mathematically represented by *substituting* perceptual space by an *isomorphic* numerical copy where spatial properties are expressible by numerical functions. We first label the points of space with *n*-tuples of numbers, their *coordinates* (*n* being the dimension of the space). This numerical labeling is not completely arbitrary for it must express the topological continuity of space (in Riemannian contexts) or its metric (in less general Euclidean contexts). Although the labels themselves do not express quantity, geometrical properties of space such as metric or curvature can be represented by numerical functions of the coordinates. The symmetries of perceptual space can also be represented by mathematical constructs in the numerical domain representing it.

General principles and laws that we believe (perceive, conjecture) to rule over the behavior of reality can also be given mathematical expression. For example, the law of universal gravitation or principles of invariance such as the principle of conservation of energy or angular momentum (infinitesimal calculus offers the ideal context where to express these and other conservation principles). There are many such principles in physics, playing predictive, explicative, and heuristic roles, conservation of energy being probably the most important. Variational principles such as the principle of least action, for example, are also central: a certain function of given physical magnitudes, the action, is supposed to be always either minimal or maximal in physically real processes. Again, only in calculus such a principle can be adequately expressed.

Mathematical formulas, principles and laws express *mathematical* facts, representing in mathematically idealized form relations among physical entities and regularities in physical reality that cannot be adequately represented in any other way. Mathematical avatars of physical reality are not, even in principle, experienciable, but are assumed to be in principle perceptually approachable to any given degree of accuracy. Instead of a photographic copy of reality, mathematics provides an X-ray that captures only structure, but with an infinite degree of precision. Precision, however, that is not in reality itself, only in the way the X-ray machine operates.

2) *Instrumental*. The first and most crucial step in the mathematization of physical (perceptual) reality is the representation of certain of its structural aspects mathematically, i.e. *as* aspects of convenient mathematical manifolds that *substitute* reality. Once these manifolds are in place, their *mathematical* theories can be developed. Now, mathematics takes the lead; its task, to investigate by mathematical means the *mathematical representatives* of (formal-abstract aspects of) the physical world, to bring to light subjacent relations, organizing principles, hitherto unperceived correlations, in short, any structural aspect of reality representable in the mathematical context in question. Thus, mathematics becomes *instrumental*.

As such, mathematics is free to introduce terms that may or may not have representational value, provided they play a role in the internal organization of the theory. A good example is Schrödinger's wave function, not itself representative of anything real (only the square of its module, a real-valued function, represents something "real", a density of probability distribution).

Additional terms may, of course, also represent, but not necessarily. The scientist must always be alert to the possibility of "imaginary" terms representing *something* in physical reality, in which case mathematics plays a *heuristic* role. When mathematical manipulations disclose hitherto unknown facts involving only *representing* terms, mathematics plays a *predictive* role.

3) *Predictive*. Mathematics predicts when it discloses mathematical facts that *must* correspond to facts in represented reality *if* the mathematical theory in question is a reliable representation of reality. If, for instance, the formula relating variation of volume to variation of temperature gives, after the required algebraic manipulations, a volume corresponding to a given temperature, then, if the volume is measured at that temperature and the formula is correct, one *must* get that volume.

Therefore, the fact that predictions can in principle be falsified by observable facts opens a road for theories to be empirically tested. Of course, no test is definitive; theories are always pouring out predictions that must be put to test. If the prediction proves to be correct, the theory passes the test. But tests never end, and final confirmation is forever postponed. Empirical verification can falsify a theory but never verify it once and for all.

Once a prediction is proven incorrect, the whole theory, its presuppositions, even its mathematical and logical setting is under threat. Fixing it requires ingenuity and is usually done in the most conservative way. There are no predetermined rules.

Now, something interesting can happen. As said before, to play its instrumental role to content, mathematics can introduce any additional terms it finds necessary to develop the theory. There will, of course, be "predictions" involving terms that have no correspondent in represented reality. Of course, these are not really predictions but, rather, meaningless sub-products of the mathematical machinery. They, however, allow mathematics to play maybe its most puzzling role, the heuristic.

4) *Heuristic*. Mathematics plays a heuristic role when it suggests that there *may* be things in physical reality, though *not for sure*, that regardless of what they are, *mathematics cannot say*, have certain formal-structural properties that the mathematical formalism seems to reveal, *but that could also be meaningless non-sense*.

There are diverse ways in which mathematics can help us in the risky business of guessing how reality may be like. Non-denoting terms that play only an instrumental role in the formalism may correspond to hitherto unknown physical entities. Their efficacy in the derivation of observable consequences may be an indication of their physical reality. Hidden aspects of reality may announce their existence, *formally* if not materially, by the way they relate *formally* to known reality.

Formal "predictions" can also be derived from principles or presuppositions built into the formalism. For example, the possible existence of the neutrino or conduction currents conjectured as consequences of the principles of conservation of, respectively, energy and electric charge. Or the "prediction" of anti-matter. The *formal* properties that Dirac imposed on his quantum-relativistic equation expressed both necessary relativistic constraints and a well-established methodology of quantization. The fact that his formalism could naturally account for the spin of electrons indicated that spin is essentially a relativistic property. That it also opened a *formal possibility* for the existence of positrons showed that *something formally analogous* to positrons might also be a necessary consequence of the junction of these two desiderata, relativistic equivalence of space and time (and Lorentz invariance) and standard quantization techniques (quantum equation derived from the expression for energy by substituting standard operators for variables). In short, the "prediction" of anti-matter did not come out of the blue sky merely as a sub-product of meaningless mathematical manipulations; symbolic manipulations were only an *instrument* for deriving the necessary formal consequences of the formal expressions of *physical presuppositions* and a *methodological orthodoxy*.

4. The centrality of the representational role of mathematics in science requires closer attention: how and in what sense can mathematics represent reality? How can a mathematical world emerge from the perceptual world? Husserl in §9 of *Crisis* provides I believe, the answer: a mathematical world is constituted from the perceptual world by intentional action; the former represents the latter by replacing it as the object of science.

The life word: a world imposes itself upon us that we must either understand and control or die. The outside world has many dimensions, the physical is one of them. Our access to the physical world is essentially sensorial-perceptual; the sensorial is the purely material hyletic, the perceptual, the sensorial matter endowed with form, either given directly with the sensations or imposed upon them by the perceptual system itself: to perceive is already to constitute, at a pre-logical, pre-categorial, proto-intentional level.

The perceptual world is the physical world of our pre-scientific life, the physical world of the lifeworld; it is a real, concrete, materially filled, finite world. Our knowledge of the perceptual physical world is basically inductive, the result of non-scientific perception-based attempts at understanding and disclosing patterns and regularities in it that allow us to control it to some extent and survive. Science, on the other hand, is a rational endeavor whose most basic task is to improve our capacity of making inferences about the perceptual world[12], but that is not itself a practice of the lifeworld. Mathematization is a *further* methodological development of scientific practices.

The perceptual world is made of objects, processes, properties, correlations that display proto-mathematical (not yet properly mathematical) aspects. Bodies have form, although not geometrical form; they occupy position in space and maintain with one another spatial and kinematic relations that, however, are not properly speaking geometrical or chronometrical. Bodies have intensive and extensive properties such as height, length, volume and size (extensive), temperature, color and hardness (intensive) that can be measured by means available in the lifeworld, meters, balances and thermometers, but not with mathematical precision. Bodies can be compared as to the hue, luminosity and intensity of their color or their degree of hardness, but with some degree of fluidity and subjectivity. In the life-world, there always is room for arguing about which body is redder or harder. Bodies act upon one another, but not by means of precisely quantifiable mathematical agents.

Quantitative determinations of the perceptual world are, from a mathematical perspective, essentially imprecise, but cannot be otherwise as far as perception is the privileged means of access to reality and objectivity is only another name for intersubjectivity. Mathematization is an attempt at overcoming subjective perspectives and perceptual "imprecisions". However, since *only* the formal-abstract aspects of perceptual reality can be a matter of intersubjective agreement, for *form* only is *objectively real*, the cost of objectivity is the loss of essentially private perceptual materiality. The materially empty skeleton of perceptual reality can then be mathematically rectified, i.e. exactified, so mathematics can come in with full force as a methodological device.

The perceptual word, a dimension of the pre-scientific lifeworld where we live our daily lives, as is obvious, is not merely *given*, we do not stumble upon it readymade. Although there is, of course, a given of the world, it must interact with our perceptual systems for a perceptual world to be *constituted*. The perceptual world is a *product*, although not of a conscious and fully intentional process; it is the given of the world endowed with *perceptual sense*. To perceive is to make sense of the senses; perception is already a cognitive process of a fully active, although not fully conscious subject. The proto-mathematical aspects of the perceptual world are then as much a given as a contribution of the perceiving subject, not because it so wants, but because it cannot help it; the perceiver cannot choose to perceive differently from how it perceives because it cannot choose to be different from how it is. The way *we* perceive the world tells as much about the world as about us. The proto-mathematical aspects of the world are not necessarily out there; they can be, at least in part, a contribution of ours.

Our daily lives are directed to essentially practical ends, our own survival and the survival of our species being the supreme good. A certain capability of understanding, controlling and predicting the behavior of the world as perceived is an important instrument of survival to which certain practices of the lifeworld contribute, such as measuring, counting or comparing. This is not yet science but science will gradually emerge from these practices as a way of improving our ability to understand, predict and act on the world.[13] Mathematization is only its most radical development, one that goes beyond the perceptual world to better investigate *some* of its aspects – its proto-mathematical aspects, in particular – those precisely that can be mathematically idealized.

The mathematical-physical world. The mathematical-physical world is abstract (ontologically dependent), idealized (exactified), formal (materially empty), non-perceivable, and infinite; our knowledge of it is mathematical and deductive, based on theories, principles, and laws (for they only can theoretically master an infinite domain[14]).

The mathematical-physical world is *not* an independently existing entity, but a higher-level *intentional construct*; it is the mathematization of the perceptual world, requiring for its constitution many layers of intentional action. Abstraction or, more precisely, formal abstraction is one of them. By stripping the perceptual world of its material, intuitive, private content, it reduces it to its objectifiable abstract *form*. Spatial shapes regardless of material support and quantitative relations regardless of what is related, for example.[15] Quantification is an instance of formal abstraction; once a magnitude is quantified it is reduced to a numerical variable. The mutual dependence of magnitudes in the perceptual world is, when mathematized, restricted to their quantitative aspects, expressible by numerical correlations, formulas and equations.

The spatial form of bodies, despite abstract, are still *real* forms in perceptual space. Their geometrization requires a further step, idealization, by which real forms are *exactified* as geometrical forms proper. To idealize something (a spatial form, a quantitative relation) is to take it as an instance of an idea (a geometrical form, a number).[16] By so doing one can investigate the spatial properties of the real, perceptual world geometrically. Let's consider an example. Suppose one wants to know whether a rigid object of the physical world, say a right-hand glove, can occupy a predetermined place in space, say, that of a similar left-hand glove, by simply moving in space without changing its form. Since the problem involves only the spatial form, not the material content of gloves, we can consider it abstractly and, by idealization, geometrically: are the right-hand glove *form* and the left-hand glove *form* equal under rigid motions in space? The *geometrical* problem can now be tackled and the negative answer obtained. The standard approach is *algebraic*, in terms of groups, because the problem has more to do with group-theoretical properties of motions in space than space itself. This only solves the initial *real* problem because abstraction and idealization can somehow be undone. Abstraction does not do away with matter altogether, it only puts it in the background as a standard semantics for filling (giving material content to) real forms mathematically idealized. By going back from mathematical to real forms, in the direction opposite of mathematization, and filling these real forms with their original material content, one gets from the mathematical a *physical impossibility*. By being purely formal, the impossibility of the body in question being placed in the place in question can be disclosed by a formal-mathematical investigation.

Another crucial step in the process of mathematization is the sorting out of qualities into primary and secondary. Primary qualities are objective and mathematizable; secondary qualities are subjective and intrinsically perceptual. Today, we take as an evidence that qualities such as taste, color and texture are secondary, residing in consciousness rather than in the object, but that spatial form is primary, residing in the object itself. But every evidence one may adduce for the subjective character of, say, color (dependence on subjective states, conditions of observation and the like), holds good for spatial form too. The privilege of primary properties is based on its willingness to be mathematized. If, say, color, could be as easily quantified in terms of *objective* standards of hue, saturation and brightness there would no doubt exist a mathematical theory of colors independently of electromagnetic theory (where color is reduced to a quantifiable magnitude, frequency of luminiferous radiation).

Instead of bodies with all the properties one perceives them to have, the “bodies” of the mathematical-perceptual world are essentially clusters of objectively measurable abstract qualities, *geometric extensions* endowed with *mass*, which measures its capacity to resist changes of state of motion, *electrical charge*, *electrical conductivity*, *thermal capacity*, *velocity*, *position*, *acceleration*, etc., all essentially numbers or higher-order mathematical objects such as vectors.[17] The mathematical-perceptual world can be further enlarged with entities that have no *direct* correspondent in perception, such as fields, forces, potentials, etc. At this point, mathematical substitutes of perceptual reality are no longer exclusively representational; they became methodological tools with which to explore and probe perceptual reality, mathematical manifolds like any others to be investigated by mathematical means like all others. Mathematical theories of the mathematical-physical world must eventually be confronted with perceptual reality, but not *directly*, with *raw* perception, but *indirectly*, with a *mathematically purified* version of perception.

The state of the world is at any point in time characterized by the values of relevant variables and how they correlate to one another. Both variables and correlations can change with the flow of time, hopefully in a lawful manner. For, infinite as it is, the world is *supposed* to be submitted to strict causal laws that can be adequately expressed only mathematically.[18]

The contrast with the perceptual reality of the life-world is striking. The perceptual world is a materially filled, sensorial world, a world of colors, scents, sounds, textures, and tastes. It is also a finite world, although open to a potentially infinite horizon of possibilities. Perceptual space is a sensorial, not mathematical space, and chains of causalities in the perceptual world can only be expressed in morphological (descriptive), not mathematical terms.

The constitution of a mathematical representant of the life-world is, as we have seen, a complex intentional process. It involves formal abstraction and idealization, but also selection. Of all the qualities that make the life-world, only those that can be objectively mathematized, the primary qualities, are considered worthy of inhabiting the mathematized physical world. The others, the secondary qualities, are dismissed as essentially subjective unless they can be causally related to mathematizable qualities that can, then, take their places as true objectively real qualities. A lot happens in perceptual physical reality, the only truly *real*, that do not find a way into mathematical-physics.[19]

Moving back from mathematical representations of the perceptual world to the perceptual world itself is not always so straightforward, even when all mathematical entities represent something in principle perceivable. Idealizations are sometimes so dramatic that mathematical representations are almost useless to handle concrete situations. Approximate techniques must then be devised – for example, linearization – and sometimes sheer brute force is preferable to sophisticate mathematical models.[20]

5. To conclude, let's see how much of what was said can be traced back to Husserl himself. One thing he clearly saw, that the mathematical representation of perceptual reality, the first and most fundamental use of mathematics in science, on which all others depend, is a *methodological tool*, *not* the uncovering of the innermost aspect of physical reality.[21] Reality, *as it appears to us*, is *perceptual* reality, which we manage to constitute out of the sensorial hyletic material.[22] *Mathematical-perceptual reality is a methodological construct, not the unperceivable mathematical core of perceptual reality*.[23] Mathematical structures are not *in* reality, they only *represent*, for *strictly methodological purposes*, formal-structural aspects of reality in highly idealized form. To forget this is to cloud the applicability of mathematics in science in mystery, making it utterly incomprehensible.

Husserl was also aware of the predictive role of mathematics; in fact, it seems that this was for him the sole role mathematics plays in science: to provide more refined anticipations of experience than those allowed to perception. All this is clearly stated in *Crisis* (Husserl 1970, §9h):

"Mathematics and the mathematical science of Nature', or still the *dressing with symbols* of symbolic-mathematical theories, contains all that that, for the expert and the cultivated men, replaces (as the objectively real and true Nature) the life-world, substituting it. It is this covering of ideas that makes us take for the true Being what is only a method – a method that is there to correct, in an infinite progression, by "scientific" anticipations, the "rough" anticipations that are originally the only that are possible in the realm of the effectively (really and possibly) experienced in the life-world. It is this covering of ideas that renders the authentic sense of the method, formulas and theories incomprehensible, and that, in the naiveté of the method at its birth, was never understood".

The expression "anticipation of experience" is somewhat ambiguous. Clearly, when calculating the unknown value of some magnitude in terms of others that are known by using mathematical formulas one is making a prediction, i.e. an anticipation of experience. But two things can happen here; one, all variables in the formula have known denotations. For example, computing the value of the pressure of a gas knowing its temperature and volume using the law of perfect gases. Another, some variables do not have interpretation (a semantic content), appearing in the formula only as mathematical contributions. For example, "electrical resistance" as something "in the body" accounting for the expression of the linear dependence between voltage and current intensity in *that* body. In this case, mathematics does not "predict" simply the *value* of a *known* magnitude but the *existence* of a *new* relevant magnitude, thus playing a heuristic role. Both cases can count as anticipations of experience and Husserl may have had both in mind, thus including the heuristic as part of the predictive role of mathematics, but this cannot be asserted with certainty.

Husserl also realized that mathematics has an instrumental role in science, but here he becomes a bit too conservative. He thought that with the introduction of non-representing elements in mathematical context of representation science risked meaninglessness and losing itself in symbolic alienation, away from the *possibility* of perception, the ground where knowledge must be rooted. Physical knowledge can be symbolic, he thinks, but only insofar as symbols *already have or can be given a meaning in the perceptual world*, and for strictly practical reasons. To indulge in *essentially* meaningless symbolic manipulations, involving symbols that could not, even in principle, be referred to perception, was for him to open the doors to possible falsities. Even though mathematical-physical theories involving in an essential, non-eliminable way meaningless symbols may be consistent, if these theories consistently extent theories without meaningless symbols, Husserl claims, their observable consequences need not to be true.[24]

However, as I believed to have shown, purely formal (non-representing) extensions of representing mathematical theories are fundamental for mathematics to cover the full spectrum of its possible scientific applications, more notably the predictive and the heuristic, and any restriction in this direction is undesirable, no matter how counterintuitive. Husserl seems to oppose such liberalism. The fundamental place he reserves for intuition, perception particularly, in his epistemology seems to block the essential (non-eliminable) use of non-intuitive, symbolic methods of knowledge.

Nonetheless, he poses no such restrictions to formal mathematics as a logical propaedeutic to knowledge. At its highest level, Husserl claims, formal logic has the task of investigating (on the formal apophantic side) formal systems in general, even invented ones, their corresponding formal domains (on the formal ontological side), and their mutual relations: theories to theories, domains to domains (a sort of universal algebra) and, as should be obvious but not explicitly mentioned by Husserl as far as I know, theories to domains (a sort of model theory). By investigating how truths migrate from theories to theories and from domains to domains through the formal-logical relations theories entertain with theories and domains with domains, we can hope, it seems, to find out conditions that *guarantee*, at least in some cases, that (in principle) perceivable consequences of theories with imaginary symbols consistently extending purely perceptual theories are actually *true*, thus providing logical-epistemological justification for the use of theories with non-eliminable "imaginary" components in science.

Husserl's dismissal of imaginaries as possible sources of falsities in science as explicitly stated in his talks of 1901 in Göttingen and apparently implicitly admitted in *Crisis*, where it is also considered as a form of alienation, does not have, then, a strictly *formal-logical* motivation. Such a prejudice seems to emanate from an *epistemological* constraint: our knowledge of the perceptual world, the only real world, can rely on mathematical methodologies, provided, however, that we can always "undo" abstractive and idealizing acts and move back from mathematics to perception. Mathematical representability must always be a two-way street. The primacy of intuition in Husserl's epistemology holds firm to the very end and the call to personal responsibility still resonates in *Crisis*, his last word on the matter.

NOTES

[1] I will not address this question here but, according to Husserl, mathematical theories of perceptual reality (whether they involve imaginary entities or not, see below) are not confronted with raw perception directly, but with mathematical *substitutes* of perceptual reality. Therefore, the mathematical character of reality, as a presupposition about the essence of reality, is never put to *empirical* test. This presupposition (or hypothesis, as Husserl calls it) must, then, be either a metaphysical presupposition, as tacitly admitted in general, or, according to Husserl, a *transcendental presupposition* that goes with the intentional constitution of physical reality as an object of mathematical-physics (see Husserl 1970, §9e).
[2] However, as I will argue below, Husserl probably did not see such a justification as *epistemologically* valid.
[3] Husserl sees in Greek geometry already a change of the original meaning of geometry.
[4] Free fall is an important topic in *Dialogo*, where a law is stated according to which the final velocity of a body falling freely along an inclined plane depends only on its height, not inclination. From the definition of equality of velocities, Galileo arrives at the law that the times spent by two bodies falling freely along inclined planes of same height and different inclinations is in the same ratio as the lengths traveled.
[5] Here is a much simpler example of how mathematics can reveal, once physical nature is mathematized, *quantitively, but not qualitatively*, hitherto unknown aspects of reality: experience shows that the intensity of electric current depends linearly on the difference of electric potential (Ohm's law), but that this relation varies with the object considered. This suggests that objects have different "electrical resistances", the *nature* of which we may not understand, but whose *intensity* one can measure. From this notion one can define another, electrical resistivity, which depends only on the material the object is made. Theories can be devised to give these magnitudes a qualitative content (atomic theory, for example), but in these efforts mathematics can only help by imposing *formal* (in this example, quantitative) *constraints*.
[6] Formal-abstract aspects of reality are "structural" in the sense that they have to do with *how* things relate to one another independently of *what* they are and the nature of these relations. Formal abstraction dematerializes, leaving behind only abstract form (structure).

[7] Formal aspects of reality are *abstract* in the sense of being *ontologically dependent*: *how* things are related depends ontologically on there being things related in some way. Structures are abstract aspects of structured system of things, themselves somehow given or only theoretically characterized. By being *ideated* (turned into ideas) structures admit different instantiations. *Idealization* is not the same as *ideation*; idealization is a limit process; for example, a physical body and its trajectory in space taken, respectively, as a material geometrical *point* and a geometrical *line* or the quantitative relation of the mass of a body with respect to that of a standard body as given by a *single* well-determined real number.

[8] Structural aspects of reality are, in a sense, ways of seeing, perspectives.

[9] Perceptual reality is only another name of physical reality; physical reality is in principle always perceivable, directly or indirectly.

[10] Magnitudes are countable when expressible by natural numbers and continuous when expressible by real numbers. Numbers, either natural or real, are ideal formal-abstract relations with which one can express in idealized form quantitative relations of physical reality. So-called complex and more general number concepts are not, strictly speaking, numbers, but number-like entities behaving only operationally like numbers.

[11] Sometimes numerical structures are used to represent aspects of physical reality without numbers expressing quantity but functioning instead as tags.

[12] Of course, explanation usually goes with predictive power, but I believe that science is fundamentally an instrument to predict and control. The history of modern science seems to show, or so I think, that putting order in the phenomenal world and being capable of making reliable predictions about it are usually preferred to having an explanation of its behavior when these tasks cannot be simultaneously fulfilled.

[13] Mathematics itself is rooted in and born out of practices of the lifeworld (see da Silva 2017).

[14] It is *because* scientific theories have infinite domains that they must, according to Husserl, be ideally *definite*, that is, finitely axiomatizable and syntactically complete.

[15] Abstraction, of course, is an intentional, not a real process, either physical, which would be absurd, or mental. Abstraction is a refocusing of intentionality.

[16] To ideate, on the other hand, is posit an *idea* from its instances by free variation.

[17] After reducing the objects of the world to clusters of numbers it is easy to be misled into believing that they are *nothing but* numbers, a dramatic form of Pythagoreanism. This, however, is taking a product for a given, "forgetting" intentional action. Max Tegmark (Tegmark 2014) indulges lavishly in this mistake; from the purely mathematical character of mathematical-physical reality (a construct) he infers that transcendent reality (a given) is purely mathematical, taking a *method* of scientific inquiry for its *object*.

[18] One may wonder at the status of such presupposition, that the world is submitted to laws. As I see it, this is neither a hypothesis nor an empirical fact but, rather, a transcendental presupposition that goes with the intentional constitution of the world as an object of scientific inquiry.

[19] Therefore, as Husserl insisted, mathematical-physics cannot serve as the model for all sciences, not even for all the *natural* sciences. Phenomenology, for example, which intends to be essentially descriptive and intuitive, has not much to learn from it.
[20] "The point here is that when we carry out engineering in different circumstances, the way we perform mathematics changes. Often the reality is that when analytical methods become too complex, we simply resort to empirical models and simulations" (Abbot 2017, section III).
[21] The methodological effectiveness of mathematical representations of perceptual reality can easily mislead the scientist into believing that *transcendent* reality existing out there, *prior* to being projected into the consciousness of the perceiving subject as *perceptual* reality, is *itself* a mathematical manifold. Otherwise, he thinks, the effectiveness of mathematics in science becomes a mystery. It takes a transcendental-idealist perspective to solve this "mystery" and move mathematics from metaphysics to methodology, where it belongs.
[22] The constitution of perceptual reality, as we have seen, is not properly intentional since it is not a fully conscious process. Perceptual reality is constituted as transcendent reality "filters through" the senses and the sensorial impressions are "interpreted" by proto-intentional psychophysical systems that give them perceptual sense.
[23] Husserl is meridionally clear about this, the mathematical world is not a given, but a rational construct (see Husserl 1970, §9b)
[24] Although Husserl is not so explicit in *Crisis*, he leaves no margin for doubt as to the role of the "imaginary" in science, or at least in mathematics, in a couple of talks he delivered in Göttingen in 1901 (Husserl 1970, pp. 430-51): maybe to facilitate calculations that must, however, at least in principle, be possible without them. Non-denoting terms cannot have an *essential* role in science (see da Silva 2010). He never disowned such a view, which appears in later texts as well, and there is no reason why he would not have extended it to the whole of science, not only contentual, interpreted mathematics (*formal* mathematics, on the other hand, as a chapter of formal ontology is another matter; it is, in a sense, "imaginary" mathematics and need not care about interpretations).

REFERENCES

Abbot, Derek. 2017. "The Reasonable Ineffectiveness of Mathematics". In *The Best Writing in Mathematics 2016*, edited by M. Pitici, 32-47. Princeton (NJ): Princeton University Press.

da Silva, Jairo J. 2010. "Beyond Leibniz: Husserl's Vindication of Symbolic Knowledge". In *Phenomenology and Mathematics*, edited by M. Hartimo, 123-145. Dordrecht: Springer.

_____________. 2017. *Mathematics and its Application – A Transcendental-Idealist Perspective*. Synthese Library v. 385. Cham (Switzerland): Springer,

Galilei, Galileo. 1970. *Dialogo sopra I due massimi sistemi del mondo*. Turin: Enaudi.

Husserl, Edmund. 1970. *The Crisis of European Sciences and Transcendental Phenomenology*. Evanston (IL): Northwestern University Press.

________. 2003. *Philosophy of Arithmetic, Psychological and Logical Investigations, with Supplementary Texts from 1887-1901*. Translated by Dallas Willard. In *Edmund Husserl Collected Works*, edited by Rudolf Bernet, vol. X. Dordrecht: Springer, 2003.

Schrödinger, Erwin. 1926. "An Undulatory Theory of the Mechanics of Atoms and Molecules". *Phys. Rev.* 28(6): 1049-1070.

Tegmark, Max. 2014. *Our Mathematical Universe – My Quest for the Ultimate Nature of Reality*. New York: Knopf.

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Gödel – Husserl – Platonism

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Abstract

We know that in 1959, Husserl started to replace Leibniz as Gödel's favorite philosopher. We think we know what caused Gödel to shift his theoretical alliance: he perceived a likeness between Husserlian phenomenology and his own goal of demonstrating mathematical realism, and he thought he could use Husserl's phenomenology to find new axioms for set theory. In this essay, we argue for the claim that insofar as their common goal of a robust realism is concerned, it was a marriage made in heaven or, better, it could have been a happy marriage, were only Gödel prepared to accept some qualifications to his Platonic gut intuitions.

Keywords: Gödel, Husserl, Platonism, mathematical objects, realism, idealism, abstract objects

Introduction

Although Platonism has been around since Plato, it is a fairly recent topic in the philosophy of mathematics. In its current meaning, it was first used by Paul Bernays in 1935; Bernays set the meaning of "Platonism" within its contemporary mathematical usage as the postulate advocating the consciousness-independent existence of mathematical objects. He wrote:

> Euclid postulates: One can connect two points with a straight line, while Hilbert states the axiom: Given any two points there exists a straight line on which both points lie. Here, "exists" refers to the system of straight lines. This example exhibits already the tendency (which we are talking about) to consider objects as cut loose from all bonds to the thinking subject. This tendency was emphasized in the philosophy of Plato; allow me therefore to call it "Platonism." (Bernays (1935, 53; our translation)[1]

What prompted Bernays' remarks were certain developments in late 19th and early 20th century mathematics; a hundred years earlier, in 1835 mathematicians didn't have a Platonism problem.

Traditional mathematics was defined as the science of measure and number (geometry and arithmetic), and both branches were closely tied to the scientific study of the natural world: geometry studied physical space, arithmetic and analysis physical magnitudes and their change over time. Consequently, Jean Le Rond d'Alembert classified mathematics as a "science of nature."[2] This started to change in the 18th century, when Leonard Euler (followed by Joseph-Louis Lagrange) made analysis more abstract by severing its treatment from the consideration of geometric curves, a development that eventually led to the ε-δ techniques of the 19th century by Augustin-Louis Cauchy and Karl Weierstrass, which in turn triggered a study of the real numbers and which of their properties makes analysis possible (see Gray 2008, chs 2.2, 3.2, and 4.4). The latter had become a pressing issue, since problems in various fields of mathematics, both pure and applied, required the study of classes of functions whose properties (metric, completeness, limits) were like those of the real numbers. During the same time period, investigations into the solvability of equations had led to the development of concepts such as group, ring, or field, which, in the hands of Richard Dedekind and Emmy Noether, led to the formation of modern, abstract algebra (see Kleiner 2007; Gray 2008, chs 2.3, 3.3, and 4.3). Furthermore, since the mid-19th century, counterintuitive conceptions such as non-Euclidean geometries or continuous-but-nowhere-differentiable functions were slowly established as rightful denizens of the mathematical world.[3] In short, by the beginning of the 20th century, abstract algebra and set theoretic methods had given rise to new concepts and new fields which, although used in physics (operator theory in quantum physics or non-Euclidean geometry in general relativity theory), could no longer be said to have been abstracted and refined from its study.[4]

Two types of reasoning in particular were contested among mathematicians. First, the law of excluded middle (LEM) (see Troelstra & van Dalen 1988, ch. 1). It says: For any meaningful statement *p*, either the statement *p* or its negation *non-p* must be true. Controversial was not the LEM itself but its unrestricted applicability to infinite totalities. When presented with a finite collection of objects, we can apply the LEM by checking all objects one by one to see whether they have a certain property—or not. But for an infinite collection, this is no longer possible to do. The significance is this: if we still admit the application of the LEM to an infinite collection, we can prove that objects with specific properties must exist (e.g., that an equation has a solution) *without being able actually to produce these objects*. Other instances of non-constructivity are certain axioms that postulate the existence of equally controversial, infinitary objects (e.g., power set, choice set).

Second, there are impredicative definitions (see Parsons 2002; Crosilla 2017). Let R denote the set of all real numbers. One property of R that emerges as absolutely critical for analysis is their completeness. It can be expressed in various, equivalent ways, but a very versatile formulation is to require that every non-empty subset S that is bounded above has a least one among its upper bounds (the so-called "supremum"). Now, a subset (like S) is not defined unless its superset (like R) is. But we just said that R can only be defined by a property that all suitable subsets S have. This seeming circularity in the definition of R is called its impredicativity; it thus lies at the very foundation and core of modern mathematics. David Hilbert was a staunch defender of the unrestricted use of LEM and of non-constructive and non-predicative definitions, while equally famous contemporaries like Henri Poincaré, Jan Brouwer, or Hermann Weyl had their reservations.

We take it as an assumption that, when it comes to modern mathematics, Kurt Gödel and Edmund Husserl knew what they were talking about. Gödel, as it is well-known, made seminal contributions to and left his mark on modern mathematics (see Dawson 1997). Husserl, a mathematician by training, enjoyed frequent personal interactions with key players in the dispute just sketched (see Schuhmann 1977). Husserl had studied, among others, with Weierstrass in Berlin and completed his doctoral dissertation with Weierstrass' student Leo Königsberger in Vienna (see Fraser 2019). Later in Halle, when his foreign degree had to be *nostrified* for the purpose of his habilitation, it was his friend Georg Cantor, the founder of set theory, who examined him.[5] Furthermore, we now know that he tried to keep current even later in life (see Hartimo 2017*b*). Hilbert, a preeminent mathematician of his generation, acted, on many occasions, as Husserl's advocate during the latter's time in Göttingen (see Hartimo 2017*a* and the literature cited) and even suggested, shortly before Husserl retired from the University of Freiburg, that he should return to Göttingen (Vongehr 2013, 17). Weyl, successor of Hilbert in Göttingen, openly adopted Husserlian views in his writings (see Da Silva 2017 and the literature cited). In short, we have reason to take what both Gödel and Husserl say seriously.

We now look at Gödel, Husserl, and Platonism, in that order. Our focus is fairly limited. We do not aim at a comprehensive study of either thinker but instead home in on a single question: After Husserl replaced Leibniz as Gödel's favorite philosopher in 1959 (see Wang 1987, 121), what kind of Platonism, if any, could Gödel have justified based on what he found in Husserl's works regarding the ontological status of mathematical objects?

1. Gödel's Views on the Mathematical Realm

We have a number of excellent, rigorous scholarly studies on our topic we can build upon.[6] This means, we can be brief and focus on what will be relevant to our discussion. Generally speaking, so that the reader knows where we are coming from, we agree with Parsons—that Gödel's position is characterized by both realism and intuition. (We will be more specific below.) Furthermore, we think that the narrative, as told by van Atten and Kennedy, is convincing (i.e., that Gödel turned to Husserl hoping to find more compelling arguments for his realism) but would give equal weight to Tieszen's observation (i.e., that Gödel hoped to find support for his program of finding new axioms for set theory based on a phenomenological meaning analysis of the concept set).

Gödel was reluctant to publish his philosophical views, because he felt that while his convictions were strong, his arguments were less so.[7] Another complicating factor was that he avoided clashes of opinions; he preferred being silent over saying something controversial (see Feferman 1984). Thus, during a time in which he believed himself to represent a minority opinion ("in view of widely held prejudices"), he was hesitant to express his philosophical views at all:

> "Of his philosophical interests, it appears that for many years he kept his ideas to himself both because he had not formulated them to his own satisfaction and because he had not found a sufficiently sympathetic audience." (Wang 1987, 123)

As a result, there are only two published pieces in which Gödel included tentative statements of his views on mathematical existence—Gödel (1990b), his contributions to the Schilpp volume on Russell, and Gödel (1947), a solicited popular exposition of Cantor's continuum problem, subsequently revised as Gödel (1964). But Gödel left more detailed remarks unpublished. These remarks can be found, among others, in Gödel's Gibbs-Lecture, his planned contribution to the Schilpp volume on Carnap, or the draft for a lecture probably prepared for a meeting of the American Philosophical Association.[8] Based on the little he had published, no one seemed to have taken offense at Gödel's views,[9] views that Parsons would later call "scandalizing." (Parsons 1995, 44, 45) It seems fair, therefore, to say that before the publication of his *Collected Works* (1986ff.), Gödel's views were widely known only thanks to Hao Wang's book-length studies, which portrayed Gödel as a philosopher.[10] From Wang, we first learned that Gödel considered himself "a conceptual and mathematical realist since about 1925"; that Russell considered him an "unadulterated Platonist"; or that he contributed his breakthrough work as a result of his philosophical views.[11]

Gödel never arrived at a definite articulation of his views, but we also do not know how much change they underwent, if any. Thus, for the purpose of this paper, we will characterize his views by a number of key features he mentions repeatedly over the course of 20 years (roughly, 1944–1964). We distinguish between passages written before and after 1960, the year after Gödel's Husserl studies had begun, and we will speak of a "mathematical realm," so as not to insert any prejudice into the question of whether it is a reality. We collect, for the reader, a sampling of Gödel's views on the mathematical realm, its existence, and its existence relative to the physical realm.

A. Pre-1960 quotations

A.1 A mathematical realm exists

There is an independent, objective mathematical realm. Independent means it exists "independently of our definitions and constructions" (1990b, 128); it exists "independently of our mental acts and decisions […311 | 312…] our free choice and creative acts" (1995c, 311f.); it is as "independent […] of our thinking as nature [is]" (2003c, 505). Objective means "we cannot create or change [it] but only perceive and describe [it]" (1995c, 320); its objectivity follows from being "entirely independent of any convention and free choice" (ibid.).

A.2 What the mathematical realm is

The mathematical realm consists in the "non-'tautological'" fabric of "relations between the concepts of mathematics" (1995c, 320) or the system of "properties of those concepts" (1995d, V, 360). These "concepts are composed of primitive ones" (ibid.), and even if their definition is "arbitrary, […] what can be asserted on [their basis is] objectively determined" (ibid., 359). The mathematical realm is "well-determined" and makes axioms or theorems "either true or false" (1990c, 181).

A.3 How the mathematical realm and physical realms compare

The mathematical realm "confronts […] our thinking as nature [does]" (2003c, 505) and "[its] assumption […] is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence." (1990b, 137)

A.4 Mathematical Truth

Mathematical knowledge is "purely conceptual knowledge" (1990c, 312) and its statements are "analytic" (1995d-III, 347), that is, "true already owing to the meaning of the terms occurring in it […] that is, the concepts they denote" (1995c, 320).

A.5 Mathematical knowledge

Russell's comparison of "logical evidence with sense perception […] has been largely justified […] and will be still more in the future" (1990b, 121); indeed, "the similarity between mathematical intuition and a physical sense is very striking […] [f]or the difference, as far as it is relevant here, consists solely in the fact that in the first case a relationship between a concept and a

> particular object is perceived, while in the second case it is a relationship between concepts" (1995*d*-V, 359). Mathematical truth can "directly be perceived" (1995*d*-III, 347) by an "additional sense [… i.e.,] mathematical intuition" (1995*d*-III, 353), that is, "by means of reason alone" (1995c, 312).

B. Post-1960 statements

B.1 Phenomenology

> "[T]here exists today the beginning of a science which claims to possess a systematic method for such clarification of meaning, and that is phenomenology founded by Husserl. […] But not only is there no objective reason for [its] rejection, but on the contrary one can present reasons in its favor." (1995e, 383)

B.2 The mathematical realm's independent existence

> "[S]omeone who considers mathematical objects to exist independently of our constructions and of our having an intuition of them individually, and who requires only that the general mathematical concepts must be sufficiently clear for us to be able to recognize their soundness and the truth of the axioms concerning them." (1990d, 258)

B.3 Knowledge of the mathematical and physical realm

> "[D]espite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception. […/] It should be noted that mathematical intuition need not be conceived as a faculty giving an immediate knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we form our ideas also of those objects on the basis of something else which *is* immediately given. […] Evidently the "given" underlying mathematics is closely related to the abstract elements contained in our empirical ideas." (1990d, 268)

> "[T]he question of the objective existence of the objects of mathematical intuition […] is an exact replica of the question of the objective existence of the outer world." (1990d, 268)

Gödel himself once remarked that if a Platonism were ill-defined, then it would not "satisfy any critical mind" (1995b, 50) and was finely attuned to nuances in meaning (see Benacerraf's observation reported in Moore (1990, 166) and to how much difference they may make (see Gödel's critical remark (1990b, 127) on Russell's sloppy formulation of the vicious-circle principle). We therefore think it is uncontroversial that Gödel would argue that what specific philosophical position the features we just collected amount to will entirely depend on the way we spell out the meaning of their key terms. This is what we will do in the last section: assign key terms in the quotations above a meaning informed by Husserl's philosophy.

2. Husserl

We should not expect that Husserl never changed his mind during the period of about 50 years (1887–1939) that he published (see Mohanty 1995). Actually, we would consider it a death knell to his philosophizing if he never did. In fact, Husserl's language changed, and it changed enough to cause a rebellion among his students (see Ingarden 1975). But if major changes in Husserl's philosophy had occurred, then they might affect how we understand certain texts. And indeed, there are two obvious candidates for such dramatic changes: the alleged recanting of what Frege called his psychologism with the publication of his *Logical Investigations* in 1900, and the so-called "transcendental turn" he took with publishing the *Ideas* in 1913. In light of recent scholarship (see Centrone 2010 and the literature cited), we believe that the preponderance of evidence suggests that Husserl's mature philosophy is a continuation of tendencies already visible in his earlier philosophy. This means that while Husserl continues to modify his theories, and while new elements are added through the progression of his thought, his responses to various critics do not constitute any radical reversals. We mention this, because it has some bearing on how we interpret certain bold statements that Husserl made in his *Logical Investigations*, which echo the Platonic language of Bolzano's *Theory of Science*.

Given our goal—what could Gödel have found in Husserl?—it makes sense to limit ourselves, wherever possible, to those of Husserl's works that Gödel owned.[12]

2.1 Objectivity: The Bare Account

How to wrestle objectivity from subjective acts, is the one question we see running through Husserl's entire *opus*. The question of objectivity arises within the realm of mathematical objects just because it seems obvious that mathematical entities are the creation of a human mind; to what extent, then, might their existence not be purely subjective? According to Husserl (FTL),

> "[people] never had the courage to confront head-on the embarrassing question how subjectivity can create entities that may count as ideal objects of an ideal 'world'." (Hua XVII, §100, 267; our translation)[13]

On one hand, it is the individual human being who does mathematics; and they do so within their own personal limitations (say, they are struggling with a proof everyone else finds very easy to understand) or by employing some outstanding ability they may have (e.g., finding a proof for a conjecture that had resisted the attempts of generations). On the other hand, mathematics consists of the lasting products of individual acts of consciousness and how the mathematical community collaborates to systematically forge from these individual contributions an accepted body of mathematical knowledge: fruitful definitions and axioms, communicated or published proofs, papers and theories, and, finally, canonical textbook knowledge.

Where does the objectivity of mathematics come from? Suppose we wander through a museum of modern art and you say: "Oh, look at that fancy chair!" In that situation, we use sense perception to achieve agreement that there is an object in our common field of vision that has certain specific properties. Thus, a shared external object serves as basis for objectivity. But how do we agree that it is a chair? We agree because we both know the meaning of the word "chair." Thus, a shared meaning serves as basis for objectivity. This much seems uncontroversial.

What is controversial and sets Husserl apart is how he proposed to analyze the two observations. Seeing something and knowing the meaning of a word are mental events, or, in the language of phenomenology (which prefers to remain agnostic with respect to the existence of a "mind"), they are intentional acts of consciousness. If, following Husserl (see LI V, §§1–8), we identify consciousness with conscious acts, or, to be more precise, with intentional acts,[14] then seeing an object or knowing a word's meaning is an intentional act. Furthermore, according to Husserl, that to which an intentional act is directed is always an object, whether that object is a chair, a word meaning, an isosceles triangle, or a unicorn fantasized under the influence of illicit substances. (An intentional act is only intentional insofar as it takes on an object; that is what intentionality is.) In other words, Husserl wants to identify what constitutes a given intentional act so that it will give rise to objectivity among all who perform it.[15] The mistake of psychologism was not, according to Husserl (see Hua XVII, §56, 160), that it looked at individual psychological acts, but that it failed to notice that consciousness transcends itself into the realm of objectivity.

Thus, objecthood is both the precondition of consciousness and the result of the constituting acts of consciousness. The object is that *of which* consciousness is conscious, without which there would be no intentional act to speak of, and it cannot be reduced to the agreement between various subjects with regard to any particular pre-given object. If we understand "intersubjectivity" to mean general cross-individual agreement and understand "objectivity" to mean independence from any individual subjectivity, then the terms are not synonymous, and neither entails the other. We may agree on something that objectively speaking is not the case (we mistake a beech for a birch) and disagree on something that objectively speaking is the case (we disagree on whether a series converges). More importantly, we would not *yet* be doing phenomenology. These terms acquire much more nuanced definitions over the course of Husserl's thinking (which we will go into later.)

2.2 Phenomenological analysis and **epoché**

Following Husserl, we want to look at intentional acts; but how do we do that? More often than not, we need the objects we study to appear not how they occur in the wild but primed: appropriately individuated ("cut at their joints") and subsequently cleaned, purified, sliced, stained, etc. The first step to phenomenological analysis is to objectivate the intentional act—to take it as the object for analysis. We have to consider a particular intentional act as if it were removed from the entire stream of consciousness, though it is immediately apparent that this is not how an intentional act is experienced. (It is, in actuality, part of the stream of consciousness; to remove it is to consider it as it does not appear in the wild.) First, we need a sound taxonomy of intentional acts; then we need to learn how to prime them.[16] While for our purposes we can ignore questions of taxonomy, we have to mention priming. Phenomenological priming is not required of the object but of the observer; for the main risk is for the researcher to contaminate the specimen.

The main contamination risk is caused by what Husserl calls the natural attitude; that is, the attitude of our everyday lives.[17] The natural attitude comes with many beliefs—for instance, that the tree I see really exists—that outdazzle and hence hinder a sober analysis of all the finer nuances of what is or is not actually given in consciousness. And already supposing that there are extra-mental objects (objects that transcend consciousness) that then enter my consciousness, is an assumption we cannot make. Whether we have (or do not have) a license to make this assumption could, however, be an outcome of the phenomenological analysis at a much later stage; initially, it is a complete no-go. We have to set aside considerations of the modes of existence of the objects of consciousness, which are to fall out of our analysis at some eventual stage. The suppression of the natural attitude is what Husserl calls "bracketing" or the *epoché*. It is the first in a sequence of steps, called the phenomenological reduction, which are meant to support an unobstructed and uncontaminated view of what is given to consciousness. We do not specifically *doubt* the existence of the world, when we bracket it; we put it in parentheses, for the moment. When I perform the *epoché*,

> "[it] shuts me off, *eo ipso,* from effecting any judgment, from taking any position predicatively toward being and being-thus and all the modalities of being which pertain to the spatiotemporal factual being of anything 'real'." (Husserl 1982, §27, 51)

That is not to say that we deny the reality of the universe. We set it aside, or bracket it off. In what follows, we just need the first step, the *epoché*, and will bracket things as needed.

An immediate and legitimate worry arises: for any phenomenological analysis, it seems to belong to a pre-scientific period of predominantly introspective psychology,[18] or, as Auguste Comte argued, is outright self-contradictory:

> We can indeed note that by some indomitable necessity the human mind can directly observe all phenomena, except one's own. For by whom should the observation be done? The thinking individual cannot split itself into two, one of which does the thinking while the other observes the thinking. How could an observation occur when, as in this case, the observed organ and the observing organ are identical? (Comte 1869, 30ff.; our translation)[19]

Here we assume that such Comtean concerns have been successfully alleviated (see Zahavi 2005); in fact, Husserl never saw introspection as a possible contradiction just for the fact that it happens all the time. (Only the least philosophical philosopher denies what is immediately apparent for the sake of making some technically attractive but altogether false claim.) Still, we would like to take this as an opportunity to clearly state that we take any phenomenological analysis to be as fallible as any other human endeavor.[20] We further believe that a phenomenological analysis need not be autonomous but may benefit and take cues from the neurosciences, nor is it immune from WEIRD distortions.[21] In short, we do not assume phenomenological analysis to enjoy any special first-person privileges familiar from a Cartesian philosophy of mind.[22]

With consciousness' capacity to be conscious of itself, among other objects, we can properly begin an analysis of intentional acts and their objects.

2.3 Intentional acts and their objects

Conscious acts are intentional insofar they are about something. For instance, I see *X*, or know *X*, or detest *X*, etc. The mark of the conscious is its intentionality, this *being about some X*. When we discuss an intentional act that is about some *X*, we call *X* the intentional object. This suggests a view prevalent in the philosophy of mind but with which Husserl would vehemently disagree—that we have, on one hand, intentional acts and, on the other hand, the intentional objects, and both as separate entities. This split is a familiar theme; we speak of propositional attitudes and propositions and assume that propositions (e.g., as Fregean thoughts) are entities that exist independently from any attitudes. This is wrong, however, in phenomenology (see Hua XIX, II.1 and V, §11).

An intentional act of our consciousness is never an empty act, like a container, which is then filled with an object (objectual reading). Likewise, it is not quite correct to say that the intentional act is *modified* by an object, like my sense of temperature is when I feel heat near a fireplace (adverbial reading). But to say that an act of consciousness is modified by its object is like saying a pain is *modified* by the accidental striking of a clumsy shin on a misplaced coffee table—not quite correct (because it implies that the pain exists prior to the strike, or that consciousness can exist that isn't consciousness *of* anything). It is difficult to use a SVP language to express the matter more correctly when employing the verb-phrase (adverbial reading) or the predicate phrase (objectual reading) fall short of the task. Can we put it into the subject? It might help to picture consciousness as a shapeshifting slime whose outer shape constantly undergoes change; then we can call its temporary stages intentional acts and their outer shapes their objects, but we would still fall short. However poor the analogy, it makes clear the Aristotelean hylomorphism that enters the phenomenological analysis. There are forms of consciousness that are instantiated in particular intentional acts, of which the material is their content—that of which we are conscious, i.e., the object. Were there no object of consciousness, there would be no act of consciousness. (The slime would not exist.) Consciousness is dependent on (or relative to) its object. Consciousness is always consciousness *of* something. That is to say, to speak of "consciousness" as if it were something independent is already to have abstracted it from how it appears to us. In the wild, the object of consciousness is not separate from the intentional act; we may consider it as separate, though it is not. The fact that we can consider consciousness and its object as separately existing makes it seem as if consciousness is an independently existing *thing*, a subject or predicate, when in fact the object is so integral to the act of consciousness that *without the world* for consciousness to be conscious of, there would be no consciousness.

In order to emphasize this point, it would be good to say something like: an intentional object is woven into the fabric of its intentional act like so many threads. But then language gets convoluted quickly. So when we say an object is given in an act, or that the object is woven into the fabric of consciousness, this is what we mean (i.e., that the object of consciousness is the content of an act of consciousness like thread is the content of a fabric, where in this analogy, the fabric is an intentional act.)

2.4 Fixing our language: concrete and ideal, real, signify and transcendent

Sometimes we feel we understand something better when we translate into a language that is more familiar to us; this is not the goal of the present section. This section is merely to fix our language without any claim as to explanatory power. Note in particular that the terminology we are going to introduce is meant to be neutral in respect to the natural attitude and whether things really exist or not (whatever that means).

If we look at a tree, then the tree will be given to our consciousness by what Husserl calls *sensible intuition*. If we assume a metric space, then the space will be given to our consciousness by our conceptual grasp of its definition and the concepts involved. Husserl has a fancy word for conceptual grasp (or for comprehending the meaning of words): categorical or *eidetic intuition*. Following Husserl, we call objects given to sensible intuition *concrete objects*, while objects given to eidetic intuition we call *ideal objects*. Thus, given to consciousness, a tree is a concrete object, while the number π or the meaning of the word "chair" is an ideal object. (The distinction concrete/ideal resembles the distinction we make in plain English when we speak of physical and abstract objects, and often indeed Husserl differentiates the two according to the fact that the former is spatio-temporal.)

An intentional object is given to a consciousness as woven into the fabric of its act; this alone does not allow us to go beyond its presence to a consciousness. (We say "present to" and not "present in" to avoid the misleading associations of the container theory of consciousness.) While we have a natural impulse to jump to conclusions about the reality of certain objects (whether they exist in the real world or are mere hallucinations, say, caused by high fever), all we have to start is that they are present to our consciousness. In the words of Husserl:

> It makes no essential difference to an object presented and given to consciousness whether it exists, or is fictitious, or is perhaps completely absurd. (Husserl 1970, vol. II, 99; cf. Hua XIX, V, § 11)

Note that, insofar both ideal objects and concrete objects are present to a consciousness, they share the same ontological status of "being present to a consciousness." Thus, the tree I see in the back yard and the number π I use in an equation share the same ontological status. (And that therefore, if concrete objects are real, so are *ideal objects*.) This is one point where we can identify a platonic bent to Husserl's phenomenology, but as we argue, misguided, for Husserl is saying that ideal objects are as real as concrete ones, whereas Plato would never say such a thing.

According to Husserl, intentional objects *signify*. To signify in consciousness is what to denote is for language: to point at something beyond (i.e., consciousness or language). But, while we are engaged in *epoché*, there is no a priori ground for assuming that what an intentional object signifies—say, a real tree in the real world, or the true metric space in some Platonic realm—exists somehow independently of consciousness. We call what is signified (e.g., the alleged real tree) a *transcendent object*. And we call it so, since its putative existence goes beyond, or transcends what is given to a consciousness. We thus see that a concrete object in consciousness serves as a sign for a transcendent object. We say that the transcendent object is what is *meant* by the concrete object, which is to say that the intentional object (i.e., a presentation) is a sign that directs us towards the existence of some object *not* of consciousness (i.e., a thing). We can conceive of ideal objects as signs, too, but what they signify may not be beyond consciousness.

Note the radical claim we sneaked in above when we said that comprehending word meaning is an intentional act (eidetic intuition) best described in analogy to a perceptual act (sensible intuition). This runs counter to the traditional divide between, on one hand, intuition, which passively receives but does not comprehend, and, on the other hand, understanding, which is active and does comprehend. Husserl disagrees; perception is also an activity, one of the many possible forms of consciousness. According to a naive epistemology, we are given something in intuition (patch of green) and then our understanding forms a judgement (it is a tree). According to Kant, what appears as a raw given in intuition is already constituted by a rational mind (greenish substance causally connected in space-time); this then allows understanding and judgement to find traction (Kant's schematism). According to Husserl, who tries to correct and extend Kant's analysis, everything, whether concrete or ideal, is first intuited in an intentional act whose fabric is much more complex than Kant assumed. The more accurate the phenomenological analysis, the more details we can bring out of what is given in the intentional act "tree in the back yard" or "let *x* be a set." Note that the analysis of eidetic intuition may go beyond a mere conceptual analysis, the familiar analysis of meaning as the hallmark of philosophic methodology since Plato's *dihairesis*. The phenomenological analysis of the full conscious experience will discern many different layers, among them layers of body awareness and situatedness in the world. For, according to Husserl, consciousness is embodied.[23] This richness of intentional acts will prove critical for Husserl's transcendental constitution of intersubjectivity. Recall, however, that we do not claim any special first-order privileges: the richness of the given may not be immediate, and it may take time and training to bring it out.

2.5 Aspects and full/partial presentations

Suppose we look at a tree from different sides. You see things I do not see, and vice versa. Is it a different tree, because we ascribe it different properties? No, we assume it to be the exact same, identical tree and our differently perceived properties to be complementary. Suppose we both say "number π," but you think of the ratio of a circle's circumference and diameter: $\pi = C/d$, while I think of the arc length of the top half of the unit circle:

$$\int_{-1}^{1} \frac{dx}{\sqrt{1-x^2}},$$

or some infinite series:

$$\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{4}{2k-1} \right) = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \ldots .$$

Is this a different number? No, we assume it to be the exact same, identical number and our different descriptions to be complementary. (Frege's distinction according to sense and meaning comes to mind here.)

Generalizing the observation above, namely, that we may miss something based on our perspective in space, we say that an intentional object may have many different aspects, not all of which must be fulfilled in a single presentation to consciousness. If all its aspects are fulfilled, we call it a *full* presentation; otherwise, a *partial* (i.e., a partially fulfilled) presentation.

For instance, the die in front of me has six sides, but I see only three. Thus, the intentional object "die" given in sensible intuition has unfulfilled aspects, namely, the three sides I cannot see, and is therefore partially presented. Likewise, the number π, given in eidetic intuition, will have many unfulfilled aspects, namely, all those representations of it as an infinite series that I am not aware of (but Euler was). It makes sense, therefore, to speak of fulfilled and unfulfilled, or partially fulfilled, presentations.

The same observations we just made apply to the situation where we speak not of two presentations of the same intentional object but speak of a concrete object and the transcendent object it signifies. They are the same, but they may be presented in different aspects to my consciousness. Because you and I are faced with different presentations does not mean that there are two distinct objects, one present to each of us as conscious subjects. The presentation of an object does not duplicate it, even if the object itself maintains unfulfilled aspects. As Husserl put it:

> It need only be said that the intentional object of a presentation is *the same* as its actual object, and on occasion as its external object, and that it is absurd to distinguish between them. (Husserl 1970, vol. II, 127)[24]

It is a fallacy of the natural attitude to reduplicate the intentional object and split into what is given and into what it signifies; the impulse to do so is strong but, in light of a proper *epoché*, still misguided.

We can now say how concrete and ideal objects differ. If the signified object is a concrete object, a partially fulfilled presentation of its aspects is inevitable. Concrete objects are always presented in space, and this is a simple reason for their necessarily partial presentation; for spatial objects will always have sides I cannot see. If, however, the signified object is an ideal object, then the degree of fulfillment of its presentation will depend on my grasp of its conceptual complexity. We might never arrive at a full presentation of what are usually called empirical concepts (such as tree or chair), but full presentation seems like a viable option for mathematical concepts (e.g., by definition up to isomorphism).

The proper application of the *epoché* does, however, justify our distinguishing between two types of objects. The two modes of presentation indicate two types of objects, be they only objects of consciousness. We argue that this does constitute an ontological difference.

2.6 An ontological difference

We may want to use the difference in fulfillment to distinguish the ontological status of what concrete and ideal objects signify. (And we can use it instead of or in addition to spatial presentation.) On one hand, there is the concrete object. Its inevitably partially fulfilled presentation to consciousness makes our natural attitude assume that the transcendent object, which it signifies, is external to consciousness. On the other hand, there is the ideal object. Its, at least in principle, possibly full presentation to consciousness makes our natural attitude assume that it is identical to what it signifies and hence, internal to consciousness (that ideal objects are subjective, the original problem Husserl and Gödel were facing).

We said earlier (in **2.4**) that ideal objects and concrete objects have a comparable ontological status, namely, the status of being real, *provided* they are both considered as intentional objects of a consciousness which is itself real. We further said that all intentional objects are signs. What they are signs *of* does not necessarily enjoy comparable ontological status. To put it differently, all objects of consciousness, considered as such, are real. What they signify, however, can be either outside or inside consciousness. (Note. If that were our topic, we could go from here and recover features of the natural attitude we bracketed earlier.)

To think of something is therefore not to make its transcendent object external to consciousness, i.e., we cannot conclude that because we can think an ideal object, that it therefore exists outside of consciousness, perhaps in and amongst the spatio-temporal objects (see **3.2** below). For example, while we say that consciousness is consciousness *of*, it could be conscious of itself. In that case, consciousness is conscious of something ideal. Another example is unity, which, for Husserl, is a form of consciousness but also the basis of our numerical system. It is real, because consciousness is real. But that does not mean it is external to consciousness. The same applies to Dedekind's proof that an infinite sets exist (see Dedekind 1888, 357: Theorem 66); what he meant, in light of Husserl's analysis, is that such an infinite set is real as an ideal object. Husserl mentions Anselm's ontological argument as an example of how philosophy has, at several points, been mistaken in equating an ideal object with an external object.[25]

With that being said—that the real existence of ideal objects that transcend the intentional acts that signify them does not indicate a consciousness-independent existence—we can answer half of our questions about Platonism. To answer them all, we need to take two more steps.

2.7. Intersubjectivity

Husserl spilled much ink on the topic of intersubjectivity, esp. after his so-called transcendental turn, but little did he include to his published writings.[26] Kant could adduce the validity of Aristotelean logic, which was considered a given, to argue for the intersubjectivity and objectivity of knowledge. Husserl of the earlier, realist phase could have appealed to a shared human condition and, consequently, a shared form of consciousness that would guarantee intersubjectivity and objectivity. But once we commit to an e*poché*, pretty little is left to build on. It is no surprise, then, that Husserl clearly saw the specter of a "transcendental solipsism" (Hua I, V, §42, 91) looming as a threat over his entire phenomenological project.[27] Husserl's argument is long; it spans the entire Fifth of his *Cartesian Meditations* (Hua I, 121–177). We report here only on the main steps.

Assume we practice radical *epoché*, bracket everything and especially what we came to call the "intentional stance." (see Dennett 1987, ch. 2) We are left with a primordial world that, although purged of all traces of alien subjectivity, still forms a coherent layer of the *Weltphänomen* present to my consciousness (i.e., a "phenomenon of a world" without pretense to its existence) (Hua I, §44, 127)[28]. In this remaining layer of "mere nature" I find "my own body as the unique one that is not just a body but the only object to which I empirically ascribe fields of sensations." (Hua I, §44, 128; our translation)[29] From here, the primordial world, Husserl proceeds in three steps: First, the constitution of other conscious agents; second, the ontological promotion of my primordial world to that of a world I share with other conscious agents; third, intersubjectivity, i.e., the constitution of a world all conscious agents share.[30]

Husserl distinguishes between perception and apperception (but different than Leibniz or Kant did). When I see a hammer, its front side is what I perceive. But more is present to my consciousness than what I see; e.g., I know that it forms a solid in three-dimensional space and therefore has a backside. This extra is what I apperceive. In phenomenology, it is "schematized". While this much may sound familiar, Husserl goes on step further and explains how natural kind terms come about. There was a first time I saw someone using a hammer, say, for driving a nail into a wall. Rooted in this first experience—which added meaning to what was initially presented to consciousness—every time I now see a hammer, I anticipate its use for driving nails into a wall. Husserl calls this kind of analogical apperception, which is based on forming matching pairs between a past and a present experience, an appresentation (because additional meaning gets co-presented to consciousness; see Hua I, §§ 50ff.). He then argues that in the primordial world, where my body is the only seat of subjectivity, when I encounter another human body that looks and moves like mine, both form a matching pair—the role of empathy!—and I immediately appresent the subjectivity in the other body that I know to inhabit my own. This completes Husserl's first step: appresentation populates my primordial word with other conscious human agents.[31]

Once these other consciousnesses are given, they change the ontological status of transcendent objects. The way appresentation was defined, I can appresent any other human consciousness only in likeness to my own. This entails that when a concrete object is present to my consciousness, I appresent that it is potentially present to other human consciousnesses as well. In other words, any concrete object is now apperceived as being a potential concrete object for any other human agent. (We say "potential" since it will depend, among others, on the relative position of the other body to my own.) As Husserl puts it:

> "The ontological meaning of world, and of objective nature in particular, includes this '*thereness-for-everyone*' that we always appresent." (Hua I, §43, 124; our translation)[32]

Suppose, by some freak accident of nature, someone living a fairly normal life on a remote island without having ever encountered an animal body. When they see a tree, it would miss the appresentive quality of "it's-there-for-everyone-to-see" which is so absolutely fundamental to our human experience. Thus, the appresentation of the other human body as the seat of a consciousness turns my subjective world into an intersubjective one; here, "intersubjective" has still the limited meaning of "my subjective world is shared with other human consciousnesses."

In a third step, I realize that the "thereness-for-everyone" that I learned to apperceive with every concrete object is likewise presented to any other human consciousness.

For I conceive of those only as likenesses of my own. Thus, the world I first appresented as shared with other conscious agents, I now appresent as the world they appresent as shared with me and everyone else. We thus arrive at the constitution of an intersubjective world in its full meaning: my subjective world is the one I share with others, and they share theirs with me and everyone else. Moreover, the claim that it is the same world we share and hence can agree on—rather than everyone living in their own bubble—follows from the basic fact of appresentation: I can conceive of any other human consciousness only in likeness to my own.[33]

While full intersubjectivity is a notion we can put to good use in Section **3.3** below, it is not yet objectivity as defined in **2.1** above. To this task we turn in the next section, where we recover a notion of objectivity as independence from any individual subjectivity; but do so only in respect to mathematics.

2.8 Objectivity: The Fuller Account

In the *Cartesian Meditations*, Husserl makes already a number of remarks about cultural artifacts (e.g., books, tools, works of art, etc.)—"objects with spiritual predicates" he calls them, since their presence points to alien subjectivity as their origin—and states that they, too, have the objectivity of "thereness-for-everyone" relative to a shared cultural background (e.g., European or French).[34] As humans, we are denizens of both the natural world and a cultural world; moreover, we experience both as objective: the objectivity of the former is unconditional (since the natural world stays the same across cultures) while the latter is conditional (i.e., relative to being a member of a certain culture) (see Hua I, §58, 159–163, *passim*). These ideas are further developed in the manuscript *On the Origins of Geometry*.

Some cultural artifacts such as paintings or architecture are created in a medium that automatically grants them a certain amount of permanence; but others are of a more fleeting nature: a tune, a poem, a prayer. They are conceived in a single human consciousness where they can be retained in memory and, if all goes well, recreated; but they will perish unless they get transferred to a more permanent medium (see Husserl 1976a, 370). Language, according to Husserl, is such a medium; language provides the fleeting creations of a human consciousness with a lasting physical basis: they are grafted onto the body of a language and thus inherit the latter's permanence.[35] They last as long as the language is spoken or its records are understood. While these fleeting creations are, in Husserl's terminology, ideal objects, they thus receive a concrete avatar: be it the artist's brain or a book that preserves them for the next generation. Tradition forms when successive generations pass down what they have preserved.

Fleeting cultural artifacts acquire objectivity through tradition for two reasons. First,[36] a written record allows other members of the community to recreate the original creative idea or insight; repetition reinforced retention in the first consciousness, and frequent repetition by others consolidates its permanent place within and keeps it a living tradition (see Husserl 1976a, 371.18–378.7 in particular). And this opportunity to recreate constitutes, according to Husserl (see ibid., 367.44), thereness-for-everyone, his favorite hallmark of objectivity. Furthermore, since we are dealing with ideal objects, when we recreate, we create tokens of the same type—we sing the same national anthem every time we do—and do not add copies as we do when we recreate concrete objects (see ibid., 368). The second reason is a constant unspoken companion but made explicit in appendices to the *Crisis*; in short, it is the idea that tradition generates a historical a priori. Coming out of history, it is contingent; but for those born into it, it is experienced as a priori.

> "What about the objectivity of these ideal entities? What about this a priori of history and its objectivity? We are thus lead again to the precondition of a non-interrupted tradition." (1976b, 362)[37]

> "The whole cultural present 'implies' the whole cultural past. To be more precise, it implies a gapless sequence of cultural pasts that imply one another: the universal a priori of history." (1976a, 379)[38]

Cultural objects, appropriated by generations of speakers, are experienced like objects of nature: something that is there-for-everyone and whose existence is independent of one's individual making. We all can read Shakespeare, and as individuals

Beethoven's Ninth is as much beyond our control as the planet Venus is. Husserl is clear, though, that culture must not be monolithic[39]: a currency is for all, Kant probably less so.

Husserl conceives of mathematical objects (definitions, theorems, proofs, techniques for calculation) as cultural artifacts. They originate in a single human consciousness, get shared, and eventually become part of the oral or written mathematical subculture (or, to a certain extent, part of the general culture). As such, we encounter them as objective: Euclid is there for everyone, and his theorems are facts beyond my control. New mathematics originates in an individual consciousness; but once is has become part of the folklore or textbook knowledge, it is experienced as independent of my individual subjectivity: it is part of the mathematical tradition that is in place whether I know or think about it or not.

Is objectivity, naturalized by a historical a priori, incompatible with Husserl's earlier, more boldly Platonic language? If we look the program underlying the *Crisis*, namely,

> "We can obtain a seriously scientific foundation of our a priori sciences only by an appeal to this a priori [of the life world] we have to develop," (Hua VI, §36, 144; our translation)[40]

then it becomes clear that Husserl is not going back on what he said, but that he rather admits to his own former oversight and tries to remedy it:

> "The supposedly totally independent logic qua universal basic science a priori is nothing but a naïveté. Its evidence lacks the scientific justification from the universal a priori of the life world." (ibid., 144; our translation)[41]

Consequently, Husserl's language in the *Crisis* is not changed much; to wit, he freely invokes Bolzano's terminology of "as-such" (ibid. §34.e, 132) or writes:

> Our apodictic [mathematical] thinking—progressing in stages according to concepts, theorems, reasoning, and proofs towards infinity—'discovers' only what in truth is already there. (ibid., §8, 19)

If we limit ourselves here to the case of mathematics (and not try address the bigger issue of phenomenology and history) then what we see Husserl emphasizing in *Crisis* is that mathematical concepts are obtained by a process of idealization. But once we have them, it is business as usual: we explore the logical edifice they form as an extended Leibnizian *mathesis universalis* (ibid., 44).

3. Platonism

As we said above, Platonism has existed much longer than it has been a problem for mathematics. And the definition of "Platonism" Bernays had in mind when he applied the term to mathematics does not necessarily accord with any Platonism Plato might have held. In order to figure out how Gödel might have thought Husserl could support his Platonic conception of mathematics, we first distinguish, in a very coarse manner, between types of Platonism.

3.1 Platonic Platonism

We (the philosophical community) generally think that "Platonism" indicates one or more beliefs that Plato held, subject to interpretation of course. For example, "Platonism" entails a theory of forms, the participation of sensible objects in those forms, and the assumed inferiority of everything sensible since, on Plato's account, every sensible object, i.e., the entirety of nature, is a shoddy replica of its ideal form. On Plato's account, mathematical objects are granted a reality above the sensible but below the forms and are accessed through a distinct kind of thought (*dianoia*). Plato explains in the *Republic* Book VI, as part of his exposition of the divided line, that mathematical objects are a subset of intelligible objects, but subordinate to the intelligible objects of which we become aware through the process of dialectic (i.e., the intelligible objects accessible to *nous*). Mathematical objects are not sensible objects, but are assumptions illustrated through sensible objects, as when a geometer draws a triangle, all the while understanding that the proper object of analysis is not the drawn triangle but the ideal triangle that the drawn triangle exemplifies. The intelligible triangle is *assumed* but not clarified. (Thus, it appears to *dianoia* rather than *nous*.) The understanding of mathematical objects is mediated by sensible experience, unlike the intelligible objects appearing through rationality alone.[42] None of this is part of what we find when we talk about Husserl's or Gödel's views. Gödel's view is actually contrary to Plato's Platonism, on the point that Gödel's takes sensible reality as the standard against which the reality of ideal objects is to be measured. Likewise for Husserl, whose phenomenology has no room for forms and their shoddy replicas. So if we should find cause to call Husserl, or Gödel, or maybe both, a Platonist, then they are not Platonic Platonists.

3.2 Lost Island Platonism

A bold version of mathematical Platonism is what we may want to call "Lost Island Platonism." Go back in time before Google Earth and before airplanes. Think of an island that no one has ever set foot on because it was never spotted from a sailing boat. It exists, although no one has discovered it; and if no one does, it remains forever a lost island. Once you happen to discover it, however, you can take full possession of its treasures. Are mathematical objects like a lost island waiting to be discovered; and if no one does, do they still exist?

Husserl, for one, is not a "Lost Island Platonist." He may seem to be one, however, especially in Book 1 of the *Logical Investigations*; for instance, when he writes in his critique of the anthropologism he found, among others, in Christoph Sigwart:

> "The validity of these [logical] laws does not depend on whether we, or anyone else, is able actually to perform acts of conceptual grasp."
>
> "What is true is absolute, is true 'as such;' truth is identically one whether humans or non-humans, angles or gods grasp it in their judgements."
>
> "But every truth as such remains what it is, it keeps its ideal being. It belongs to the realm of what is absolutely valid." (Hua XVIII, I, §29, 109; our translation)[43]

We first note that Husserl does not speak about the existence, be it of concrete or ideal objects, but about validity (*Geltung*). He explicitly rejects the proposal to translate the "ideal being" of validity as a being in some Platonic realm ("to hang somewhere in the void" (*irgendwo im Leeren*, tr. J.H. Findlay),

p. 136) but insists that "we experience [the idea of truth] like any other idea in act of ideation based on an intuition;" i.e., ideas are woven into the fabric of intentional acts. We note, second, that this denial of a Platonic realm is consistent with his three-step program for a pure logic, or *mathesis universalis*, that he sketches at the end of Book 1 of the *Logical Investigations*:

> (1) The identification and systematization of all primitive concepts which, to be clear, "can only originate in respect to the diverse functions of thinking, can only have their foundation in possible acts of thinking" (Hua XVIII, I, §67, 245)[44]
>
> (2) Theoretical laws that hold for these primitive concepts and form a unified theory. These laws have "objective validity" and are "directly rooted in the primitive concepts." (ibid., §68, 247)[45]
>
> (3) A theory of manifolds "whose deduction rest entirely in those theories [formed in the second step]." (ibid., §69, 249)[46]

Here, we see Husserl making the same point: the entire project of a *mathesis universalis* rests ultimately in the acts of thinking agents. If, then, there is no room for a Platonic realm, where does validity live? It lives in the potential of what thinking agents can do:

> "The being, or the validity, of what holds generally is the same as ideal possibility. The statements 'the truth is valid' and 'thinking beings are conceivable that comprehend the relevant meaning' are equivalent." (Hua XVIII, I, §39, 135)[47]

Thus, validity does not translate into a claim about the existence of objects but about the possibility of actions. If you give me two Lego bricks, I can snap them together top to bottom; this is a concrete possibility. If you give me two premises (e.g., $a<b$ and $b<c$), I can infer what they entail (i.e., $a<c$); this is an ideal possibility. We therefore find that even Husserl's bold Platonic language in the first book of the *Logical Investigations* does not, on a closer reading, support a Platonist ontology.

Earlier in his career, Husserl took it for granted that concepts have nice properties suitable for general laws and theories a priori (that, so to speak, Lego bricks have studs). Properties, by the way, that are given in intentional acts and are as such objective because not under our control. Where do these properties, where does the a priori come from? This is what we see him addressing in his last writings when he includes the life-world and the historical a priori (see **2.7** above). As Johanna Tito (1990, xlvi) put it so nicely:

> "Contrary to Plato, for whom ideas are known prior to life, for Husserl ideas are lived before they are known."

3.3 Husserlian Responses to Gödelian Worries

In a first response to Gödel, we connect his requirements for a satisfactory philosophy of mathematics (collected earlier, see **1.A+B** above) and pair them up with various observations from Husserl.

Gödel wanted to argue that the *mathematical realm consists* in the "well-determined" fabric of "relations between concepts" that makes all axioms and theorems "either true or false." He wanted to argue that mathematical knowledge is "purely conceptual" and "true owing to the meaning of the terms." The project of a *mathesis universalis* (see **3.2** above) so closely resembles Gödel's views on this that it is easy to find matching quotations in Husserl for each of Gödel's claims.

Gödel also wanted to argue that *mathematical concepts* and the relations among them exist *objectively*. There are three senses in which mathematics can be said to be objective according to Husserl. First (see **2.8**), the basic concepts are originally given in the intentional acts of the first mathematician. As such, as something given, they are experienced as objective. Any property they have cannot be changed, but described (ideation) and genetically explained (life-world). Second (see **2.8**), once they have become part of mathematics, we can either recreate the original experience of the first mathematician (keeping the tradition alive) or encounter them as dead cultural artifacts; either way, we experience their ontological status as something objective. Third (see **3.2**), it is a brute fact that certain relations among concepts just hold. Any sufficiently prepared being can find this out. This, their validity, is objective, too. And it does not go away, even if the human race falls out of existence. For in Husserl, validity translates into possibility and not into an object (that then would have to reside in some location, be it in a consciousness or "somewhere in the void").

Gödel aimed to demonstrate that the mathematical realm "confronts our thinking as nature" does. If we understand this as the requirement that physical and abstract objects share the same ontological status, then they do so since concrete and ideal objects are both real (in the sense of **2.4** and **2.6**). If we understand this as the requirement of a "second plane of reality," then it still comes out as true. For we encounter nature as intersubjective (thereness-for-everyone) and objective (no single consciousness can change it at will). But this is how we encounter the cultural word: as "harsh realities" (*harte Wirklichkeiten*) (Hua IV, §152, 354). Therefore, since mathematics is an integral part of our cultural word, we encounter it the same (see **2.7–8** above). The mathematical realm loses, however, any metaphysical nimbus; it becomes part of our mundane culture. It is, if you will, a phenomenological version of semantic externalism (see Putnam 1975).

Gödel believed that mathematical truths could be "directly perceived" by an "additional sense of mathematical intuition" which is "strikingly similar to the "physical sense." According to Husserl (see above) everything, whether concrete or ideal, is first intuited, in case of mathematics in eidetic intuition (see **mo**). We may intuit things blurry at first and need closer inspection to intuit them sufficiently sharply. Eidetic intuition is not really an additional sense, though. It is a faculty we employ all the time; false epistemology just made us overlook this.

Some may object that what we just proposed is cheating; we foisted a position on Gödel that is a Platonism by the letter but not in spirit. Maybe so; maybe Gödel was hoping for a more traditional account, where Platonists exercise divine thoughts in heavenly abodes. But we believe that he kept studying Husserl for a reason, and we believe the reason was that he was willing to reconsider a reinterpretation of his initial Platonic hunches and to get educated on how to spell them out in a more defensible way.[48] If this is correct, then his education would have included another twist that Husserl brings to Gödel's worries, an almost complete reversal of assumptions a Cartesian common sense makes.

Gödel argues that mathematical objects "confront us as objectively and independently of our thinking." Gödel thus places mathematical objects among the objective as opposed to the subjective, which is to say that he believes in independently existing mathematical objects that would not fall out of existence along with the conscious entity that is conscious of it. The subjectivist, however, would place mathematical entities in the consciousness of the human subject. If we apply the subjective-objective dichotomy to theories of philosophical idealism then Husserl's phenomenology clearly falls on the side of objective idealism: there are ideal objects, and they are objective—but objective in the senses above, as opposed to any colloquial sense.

One of the fundamentals principles of phenomenology is, however (think *epoché*), not to assume too much. In particular, do not to assume we know what the subject is, and then to define the object relatively to consciousness. The familiar tenet "consciousness is always consciousness of" defines the object insofar as the object is that of which consciousness is conscious of—whatever that may be. If consciousness is conscious of something, then that something is *objectivated*, and now it is an object. The distinction according to subject and object is, then, according to Husserl, not a pre-given primitive fact we are forced to accept as our starting point. Rather, the primitive fact is consciousness and the subjective-objective distinction is derived from the relative role both take in a particular intentional act. The terms "subject" and "object" are defined secondarily only and relatively to the act of consciousness. To say, therefore, that ideal objects exist independently of consciousness is already to assume the subject–object distinction that phenomenology has already denied. Along with the subject–object distinction, or, more technically speaking, with the specification that subjects and objects are defined relatively according to what role they take in a particular intentional act, the ideas of something's being "internal" and "external" to consciousness disappear as well.

A similar argument can be made to refute dualism. The dualist conception of reality led us to believe in two distinct substances (the mind and body) and then to try to define their interaction, as if they were two things completely distinct from one another, with a possibly unbridgeable gap in between. But the fact that *consciousness is always consciousness of* means that *consciousness* is not an independent existent, nor probably even a complete existent (in the sense of something whole); it depends on an object (a world of objects), without which, there would be no such thing as consciousness. That is to say, the world is assumed in the definition of consciousness, and consciousness is nothing separate or independent from the world ("world" here meant to indicate the whole bevy of both sensible and ideal objects or, in fact, any object whatsoever), and in fact depends on it for its existence. More properly speaking, it is not accurate to say that anything exists "in" a subject, as "subject" is just a word we use to indicate whatever it is that is conscious *of* the world (as a fabric is a fabric of threads). Rather than ideal objects existing *in* consciousness, we should say that consciousness exists *in* the world, insofar as it is conscious *of the world*, and *only* insofar as it is conscious of the world. This fundamental tenet of phenomenology is the reason for the equivalence of objects of consciousness and objects of the world—there are not, in fact, two objects, but one. (It is an object by definition, if it is something of which we are conscious—that does not mean it is "in" a subject, for that is what *makes it* an object.) Upon closer phenomenological scrutiny, some of Gödel's most basic worries just disappear.

4. Concluding Remark

We went quite some distance. Actually, we have rushed through a difficult terrain with no time to stop and rest while still trying to point out some great vistas. The goal was to discover what Gödel might have found in Husserl with specific regard to mathematical realism. We have explored, in particular, the potential the language of phenomenology has for describing the experience that mathematicians deal with objects that are ideal, objective, and real. We saw, however, that Husserl is not a Platonist even though he may have sometimes used their language. This is what we see as one of the main trajectories running through his life's work: learning how to naturalize Platonic hunches and how to translate their language into a language that can be accounted for in a sober scientific spirit. What Husserl therefore had to offer was not the rehabilitation of a naïve Platonism. Rather, what Husserl had to offer to Gödel was a reconciliation of subjective and objective idealisms. Husserl's contribution to establishing the objectivity of ideal objects is his insistence on the numerical identity between the object and the object of consciousness. Whether this amounts to a Platonism depends on what definition of "platonism" we adopt. Clearly, Husserl's reinterpretation is not a Platonic Platonism and a weak version only of Bernays' Platonism. For the criterion championed by Bernays, the "consciousness-independent existence," refers in Husserl either to validity, but not to basic concepts which owe their properties to the life-world, or to the independence a culture enjoys from its particular members, but then without any distinguished metaphysical aura.

Our final observation. In the beginning, we sketched the development within modern mathematics—we singled out the unrestricted use of the law of excluded middle and impredicative definitions—that made Bernays inject a new term to the philosophy of mathematics: Platonism. We know that Husserl himself was deeply influenced by these developments (see, e.g., Schmit 1981, Willard 1984, Lohmar 1989, or Centrone 2010 and the literature cited). Ironically enough, Husserl's general take on Platonism does not permit us to take a stance on the two issues that caused the whole debate. This would require a detailed phenomenological explication of what we find in certain mathematical acts. It was Gödel's hope that such a concentrated effort would lead to new axioms of set theory that would then settle open problems. He saw his hopes dashed, though.

NOTES

[1] "Euclide *postule : on peut relier deux points par une droite ; tandis que M.* Hilbert *énonce l'axiome : deux points quelconques étant donnés, il existe une droite sur laquelle ils sont tous les deux situés. « Existe » vise ici le système des*

droites. / Cet exemple montre déjà que la tendance dont nous parlons consiste à envisager les objets comme détachés de tout lien avec le sujet refléchissant. / Cette tendance s'étant faite valoir surtout dans la philosophie de Platon*, qu'il me soit permis de la qualifier du nom de « platonisme ».*" The symbol "/" indicates, here and in other quotes, a paragraph break in the original. Likewise, we use "$_{n}\,|\,^{m}$" to indicate a page break (from page *n* to page *m*) in the text we quote. We render emphasis in the original (spaced lettering, caps) as underlining.

[2] See d'Alembert (1765) and Brown (1991) for context; we owe Craig Fraser for the pointer to Brown.

[3] See, e.g., Gray (2008, chs 2.1, 3.1, and 4.1) on geometry and Volkert (1986, esp. ch. I.6) on non-differentiable functions.

[4] See, e.g., Corry (1996) on algebra and Ferreirós (1999) on set theory.

[5] See Biermann (1969, *passim*); the German *Nostrifikation* meant (and in Austria still means) the approval of out-of-state transfer credits or diplomas.

[6] Parsons (1995) gathers and discusses the evidence available on Gödel's realism; Tieszen (1992) collects what is known about Gödel and Husserl in general terms and sketches, in his (1998), Gödel's way to Husserl, a topic that was subsequently treated by van Atten & Kennedy (2003) in much more detail.

[7] See, e.g., Gödel (2003d, 244): "The fact is that I have completed several versions [of the text], but none of them satisfies me […] it may do more harm than good to publish half done work" (and so he did not). We owe this quote to Goldfarb (1995, 324).

[8] See Gödel (1995c), (1995d), and (1995e) resp. Here and throughout we adopt the notation used in the edition of Gödel's collected works. Likewise, when there is no risk of confusion, we drop the name Gödel or Husserl from references.

[9] See Bernays (1946) and Weyl (1946) for reviews of Gödel (1990b), and Kleene (1948), Jónsson (1948), and Buchdahl (1965) for reviews of Gödel (1990c) and Gödel (1990d), resp.

[10] First in Wang (1974), later followed by Wang (1987), (1996); see Parsons (1998).

[11] See Wang (1987, 20, 112), and Wang (1974, 8–11), resp. These letters are now reproduced, in full, in Gödel (2003b, 396–399, 404f.). Russell's statement was known, however, to readers of his *Autobiography*.

[12] Gödel owned Husserl's *Logical Investigations* (1900-01), *Ideas*, vol. 1 (1913), *Cartesian Meditations* (1931), and *Crisis* (1936), copies he heavily annotated, but also shorter essays such as "*Philosophie als strenge Wissenschaft*" (1910) or the entry "Phenomenology" (1929) written for the *Encyclopedia Britannica*; see Føllesdal (1995, 367, note b), for details. He did not have a copy of *Formal and Transcendental Logic* (1929). Sometimes, we refer to these books by their acronyms LI, Id, FTL, CM, and C, and cite according to chapter and section. Usually, we offer both the German text and a translation, often our own. And since Husserl wrote in the Teutonic style of convoluted sentences, we often shrink an English quotation to the main assertation but indicate any omissions in the German original.

[13] "[…] weil man nie den Mut hatte […] der peinlichen Frage ins Angesicht zu sehen, wie die Subjektivität […] Gebilde schaffen kann, die als ideale Objekte einer idealen 'Welt' gelten können."
[14] For our purposes we do not need to decide on the question whether or not there are conscious experiences that are non-intentional (e.g., moods or hollow urges). Husserl discusses the question in (Hua XIX, V, §15).
[15] This is what links Husserl to Kant and makes him adopt the term "transcendental." Kant argued that the objects of experience are not given in intuition but that their objecthood (e.g., unity) is mostly the result of a rational mind applying Aristotelian logic (in form of pure concepts) to what is given in intuition; objectivity then results from the validity of the logic involved. What Kant did for objects of experience, Husserl wants to do for abstract objects; including logic, which Kant took for granted.
[16] Taxonomy is big, though; take botany, for example. (Thanks to Ben Datillo for educating us on this.) Naked-eye observation of plant morphology can be used for the purpose of taxonomy, but is not the most direct evidence, and can be misleading (similarities from convergence, differences from adaptations); a phylogenetic reconstruction based on the plant's genetic make-up is more defensible. A rough, "naked-eye" taxonomy of conscious phenomena was a by-product of Husserl's investigations. But when it comes to the classification of all conscious phenomena we miss a scientifically sound approach comparable to what phylogeny is for botany. It is not even clear what the most appropriate definition consciousness is. We think humility is the appropriate response, not to stop trying.
[17] Husserl describes the natural attitude: "I am conscious of a world endlessly spread out in space, endlessly becoming and having endlessly become in time. I am conscious of it: that signifies, above all, that intuitively I find it immediately, that I experience it. By my seeing, touching, hearing, and so forth, and in the different modes of sensuous perception, corporeal physical things with some spatial distribution or other are *simply there for me, 'on hand'* in the literal or the figurative sense, whether or not I am particularly heedful of them and busied with them in my considering, thinking, feeling, or willing." (Husserl 1982, §27, 51).
[18] See Schwitzgebel (2014) for an overview of the trouble with introspection.
[19] « *Il est sensible, en effet, que, par une nécessité invincible, l'esprit humain peut observer directement tous les phénomènes, excepté les siens propres. Car, par qui serait faite l'observation?* […30 | 31…] *L'individu pensant ne saurait se par tager en deux, dont l'un raisonnerait, tandis que l'autre regarderait raisonner. L'organe observé et l'organe observateur étant, dans ce cas, identiques, comment t'observation pourrait-elle avoir lieu?* »
[20] For an overview, see Gallagher & Zahavi (2014).
[21] See Henrich et al. (2010) for the notion of WEIRD people and Klein et al. (2018) for the latest batch of research efforts in this direction.
[22] See, e.g., Kim (2011), ch. 2, for a discussion of such Cartesian privileges.
[23] See, e.g., the second volume of Husserl's *Ideas,* composed 1912–28 but published in 1952
[24] See Hua XIX, V, Appendix to § 11 and § 20, 439.12: "Man braucht es nur auszusprechen […] daß der intentionale Gegenstand der Vorstellung *derselbe*

ist wie ihr wirklicher und gegebenenfalls ihr äußerer Gegenstand und daß es widersinnig ist, zwischen beiden zu unterscheiden.". Most emphasis suppressed in translation; translation by Findlay.

[25] "Such errors have dragged on through the centuries – one has only to think of Anselm's ontological argument – they have their source in factual difficulties, but their support lies in equivocal talk concerning 'immanence' and the like." (Husserl 1970, 127 ; Hua XIX, V, Appendix to 11 and 20, 595)

[26] The two main sources from his published writings is CM, Med V, and his essay "On the origin of geometry," published as an appendix to *Crisis*. His manuscripts on intersubjectivity 1905–1935 were collected and published as *Husserliana*, vol. XIII–XV.

[27] To be clear, it is not the failure to prove that the world and other people in it exist; this much is taken for granted. Rather, the menace is that phenomenology might turn out not to be the all-encompassing first philosophy it claims to be in case its methods should fail to account for the constitution of an objective world and people with a shared intersubjectivity in it.

[28] "In der Abstraktion verbleibt uns eine einheitlich zusammenhängende Schicht des Phänomens Welt."

[29] "So gehört zu meiner Eigenheit als von allem Sinn fremder Subjektivität gereinigte, ein Sinn bloße Natur [… unter deren Körpern] finde ich dann in einziger Auszeichnung meinen Leib, nämlich als den einzigen, der nicht bloßer Körper ist, sondern eben Leib, das einzige Objekt innerhalb meiner abstraktiven Weltschichte, dem ich erfahrungsgemäß Empfindungsfelder zurechne."

[30] Hua I, §49, 137.1ff.:"Der Seinssinn objektive Welt konstituiert sich auf dem Untergrunde meiner primordinalen Welt in mehreren Stufen. Als erste ist abzuheben die Konstitutionsstufe des Anderen […] Damit in eins und zwar dadurch motiviert vollzieht sich eine allgemeine Sinnesaufstufung auf meiner primordinalen Welt, wodurch sie zur Erscheinung von einer bestimmten objektiven Welt wird [… und] letztlich eine Monadengemeinschaft [… / deren] transzendentale Intersubjektivität […] die objektive Welt intersubjektiv konstituiert."

[31] Since appresentation is a special kind of apperception, we could keep terminology simple and use apperception throughout. We use, however, appresentation and its derivatives wherever we feel it is helpful to recall the act of co-presenting a consciousness with another human body.

[32] "Zum Seinssinn der Welt und insbesondere der Natur als objektiver gehört ja […] das *Für-jedermann-da*, als von uns stets mitgemeint."

[33] What we called "full intersubjectivity" is Husserl's notion of a "transcendental intersubjectivity:" an open community of exchange among peer human consciousnesses, or monads; see *Cartesian Meditations* (Hua I §49, esp. pp. 137–138, and §§56–58, *passim*).

[34] Hua I, §43, 124.11ff.: "Zudem gehören zur Erfahrungswelt Objekte mit geistigen Prädikaten, die […] auf fremde Subjekte […] verweisen: so alle Kulturobjekte (Bücher, Werkzeuge und Werke irgendwelcher Art usw.), die dabei aber zugleich den Erfahrungssinn des Für-Jedermann-da mit sich führen (scilicet für Jedermann der entsprechenden Kulturgemeinschaft, wie

der europäischen, eventuell enger: der französischen etc.).” See also his remarks on cultural predicates (*Kulturprädikate*) and alien spirituality (*Fremdgeistiges*), ibid. §44, 126f.

[35] Husserl calls it a *Sprachleib* and *Verleiblichung* (ibid., 369.5f.),which is a powerful metaphor in German. For the German word *Leib*—unlike *Körper*, which is the translation of (Platonic) “solid” or “field” (in algebra)—can only denote the body of a higher organism. Husserl thus suggests that the linguistic body of a fleeting product of consciousness is somehow comparable to the human body that grants consciousness persistence through time. Derrida (1962, 69) emphasizes the same point when he renders *Sprachleib* as *la chair linguistique* (linguistic flesh).

[36] Husserl is clearly biased towards written records; see (Husserl 1976a, 371.26). He was inattentive to the feats of memorization that are common in oral cultures; see, e.g., Kelly (2015).

[37] “Wie steht es mit der Objektivität dieser idealen Gebilde, dieses Apriori [der Geschichte], wie mit seiner Objektivität? Da kommen wir wieder auf die Voraussetzung [… einer] nicht abbrechenden Tradition.”

[38] “[D]ie gesamte Kulturgegenwart […] ‘impliziert’ die gesamte Kulturvergangenheit […] Genauer gesprochen, sie impliziert eine Kontinuität einander implizie379 | 380render Vergangenheiten […/…:] das universale Apriori der Geschichte.”

[39] This is how we understand ibid., 366.20: “In einer Unzahl von Traditionen bewegt sich unser menschliches Dasein. Die gesamte Kulturwelt ist nach allen ihren Gestalten aus Tradition da.”

[40] “Nur durch Rekurs auf dieses […] zu entfaltende Apriori können unsere apriorischen Wissenschaften […] eine ernstlich wissenschaftliche Begründung gewinnen.”

[41] “[D]ie vermeintlich völlig eigenständige Logik […] als universale apriorische Fundamentalwissenschaft […] ist nicht anderes als eine Naivität. Ihre Evidenz entbehrt der wissenschaftlichen Begründung aus dem universalen lebensweltlichen Apriori.”

[42] Plato’s exposition of mathematical objects in the hierarchy of objects appears at 510c–511b in the *Republic*. At 511a, Socrates says (*Works*, tr. Paul Shorey): “This then is the class that I described as intelligible, it is true, but with the reservation first that the soul is compelled to employ assumptions in the investigation of it, not proceeding to a first principle because of its inability to extricate itself from and rise above its assumptions, and second, that it uses as images or likenesses the very objects that are themselves copied and adumbrated by the class below them, and that in comparison with these latter are esteemed as clear and held in honor.” The Greek reads, Plato, *Opera*:
“τοῦτο τοίνυν νοητὸν μὲν τὸ εἶδος ἔλεγον, ὑποθέσεσι δ᾽ἀναγκαζομένην ψυχὴν χρ
ῆσθαι περὶ τὴν ζήτησιν αὐτοῦ, οὐκ ἐπ᾽ἀρχὴν ἰοῦσαν, ὡς οὐ δυναμένην τῶν ὑποθέσ
εων ἀνωτέρωἐκβαίνειν, εἰκόσι δὲ χρωμένην αὐτοῖς τοῖς ὑπὸ τῶν κάτωἀπεικασθεῖσ
ιν καὶ ἐκείνοις πρὸς ἐκεῖνα ὡς ἐναργέσι δεδοξασμένοιςτε καὶ τετιμημένοι.”

[43] “Die Geltung dieser Gesetze […] hängt nicht davon ab, ob wir und wer immer begriffliche Vorstellungen faktisch zu vollziehen […] vermag.” Ibid.

§36, 125.9ff.: "Was wahr ist, ist absolut, ist "an sich" wahr; die Wahrheit ist identisch eine, ob sie Menschen oder Unmenschen, Engel oder Götter urteilend erfassen." Ibid. §39, 136.6–9: "Aber jede Wahrheit an sich bleibt, was sie ist, sie behält ihr ideales Sein […] Sie gehört zum Bereich des absolut Geltenden."

[44] "Beiderseits handelt es sich um Begriffe, die, […] nur im Hinblick auf die verschiendenen 'Denkfunktionen' 245 | 246 entspringen, d.h. in möglichen Denkakten […] ihre konkrete Grundlage haben können."

[45] "Gesetze […] die unmittelbar in den kategorialen Begriffen wurzeln."

[46] "[A]ndererseits is es von vornheren klar, daß ihre Deduktion […] ausschließlich in jenen Theorien fussen muß."

[47] "[W]ie das Sein oder Gelten von Allgemeinheiten auch sonst den Wert von idealen Möglichkeiten besitzt [… so] auch hier: Die Ausagen 'die Wahrheit gilt' und "es sind denkende Wesen möglich, welche Urteile des bezüglichen Bedeutungsgehaltes einsehen', sind von gleichem Werte."

[48] Here we wish to point out that additions he made post-1960 to an earlier paper seem directly informed by Husserl's philosophy; see Gödel (1990d, 268).

REFERENCES

d'Alembert, Jean Le Rond. 1765. "Mathématique." In *Encyclopédie, ou, Dictionnaire raisonné des sciences, des arts et des métiers,* edited by Denis Diderot, *vol. 10: Mam–My*, 188–189. Neufchastel: Faulche.

Benacerraf, Paul & Hilary Putnam (eds.). 1964. *Philosophy of Mathematics: Selected Readings*. Englewood Cliffs: Prentice-Hall.

Bernays, Paul. 1935. "Sur le platonisme dans les mathématiques." *L'enseignement mathématique* 24 (1935): 52–69.

________. 1946. "Review of 'Gödel (1944)'." *Journal of Symbolic Logic* 11 (3): 75–79.

Biermann, Karl-Reinhard. 1969. "Did Husserl take his doctor's degree under Weierstrass's supervision?" *Organon* [Warsaw] 6: 261–264.

Brown, Gary I. 1991. "The Evolution of the Term 'Mixed Mathematics'." *Journal of the History of Ideas* 52 (1): 81–102.

Buchdahl, Gerd. 1965. "Review of 'Benacerraf & Putnam (1964)'." *Mathematical Reviews* # MR0023780.

Centrone, Stefania. 2010. *Logic and Philosophy of Mathematic in the Early Husserl*. Vol. 345 of *Synthese Library*. Dordrecht: Springer.

Centrone, Stefania (ed.). 2017. *Essays on Husserl's Logic and Philosophy of Mathematics*. Vol. 284 of *Synthese Library*. Dordrecht: Springer.

Comte, Auguste. 1869. *Cours de philosophie positive*. Tome 1. Paris: Baillière. [First Edition 1830].

Corry, Leo. 1996. *Modern Algebra and the Rise of Mathematical Structures*. Vol. 17 of Science Networks – Historical Studies. Basel: Birkhäuser.

Crosilla, Laura. 2017. "Predicativity and Feferman." In *Feferman on Foundations. Logic, Mathematics, Philosophy*, edited by Gerhard Jaeger and Wilfried Sieg, 423–447. Cham: Springer.

Da Silva, Jairo José. 2017. "Husserl and Weyl." In Centrone 2017, 317–352.

Dawson, John W. Jr. 1997. *Logical Dilemmas. The Life and Work of Kurt Gödel*. Wellesley, MA: Peters.

Dedekind, Richard. 1888. *Was sind und was sollen die Zahlen?*. In *Gesammelte mathematsche Werke*, edited by Robert Fricke, Emmy Noether, and Øystein Ore, Bd. 3, 335–390. Braunschweig: Vieweg. First published 1932.

Dennett, Daniel C. 1987. *The Intentional Stance*. Cambridge, MA: MIT Press.

Derrida, Jacques. 1962. "Introduction." In *Edmund Husserl. L'Origine de la Géométrie,* translated by Jacques Derrida, 3–171. Paris: Presses Universitaires de France.

Feferman, Solomon. 1984. "Kurt Gödel: conviction and caution." *Philosophia Naturalis* 21: 546–562.

Ferreirós, Jóse. 1999. *Labyrith of Thought. A History of Set Theory and Its Role in Modern Mathematics*. Vol. 23 of Science Networks – Historical Studies. Basel: Birkhäuser.

Fraser, Craig. 2019. "Edmund Husserl's Contributions to the Theory of the Second Variation in the Calculus of Variations."

To appear in *Festschrift for Umberto Bottazzini*.

Føllesdal, Dagfinn. 1995. "Introductory note to **1961/?*" In Gödel 1995, 364–373.

Gallagher, Shaun & Dan Zahavi. 2014. "Phenomenological approaches to self-consciousness." In *The Stanford Encyclopedia of Philosophy*. Edited by Edward N. Zalta. https://plato.stanford.edu/archives/win2016/entries/self-consciousness-phenomenological/

Gödel, Kurt. 1986. *Collected Works. Vol. I: Publications 1929–1936*, ed. by Solomon Feferman et al. New York: Oxford UP.

________. 1990a. *Collected Works. Vol. II: Publications 1938–1974*, ed. by Solomon Feferman et al. New York: Oxford UP.

________. 1990b. "Russell's mathematical logic (1944)." In Gödel 1990a, 119–141.

________. 1990c. "What is Cantor's continuum problem? (1947)." In Gödel 1990a, 176–187.

________. 1990d. "What is Cantor's continuum problem? (1964)." In Gödel 1990a, 254–270. [Revised version of Gödel 1990c (1947)].

________. 1995a. *Collected Works. Vol. III: Unpublished Essays and Lectures*, ed. by Solomon Feferman et al. New York: Oxford UP.

________. 1995b. "The present situation in the foundations of mathematics (*1933o)." In Gödel 1995a, 45–53.

________. 1995c. "Some basic theorems on the foundation of mathematics and their implications (*1951)." In Gödel 1995a, 304–323.

________. 1995d. "Is mathematics syntax of language? (*1953/9-III and *1953/9-V)." In Gödel 1995a, 324–362.

________. 1995e. "The modern development of the foundations of mathematics in the light of philosophy (*1961/?)." [German transcript and English translation]. In Gödel 1995a, 374–387.

________. 2003a. *Collected Works. Vol. IV: Correspondence A–G*, ed. by Solomon Feferman et al. Oxford: Clarendon Press.

________. 2003b. *Collected Works. Vol. V: Correspondence H–Z*, ed. by Solomon Feferman et al. Oxford: Clarendon Press.

________. 2003c. "Letter to Gotthard Günther, dated June 30, 1954" (German-English). In Gödel 2003*a*, 500–505.

Goldfarb, Warren. 1995. "Introductory note to **1953/9*." In Gödel 1995a, 324–334.

Gray, Jeremy. 2008. *Plato's Ghost. The Modernist Transformation of Mathematics*. Princeton: Princeton UP.

Hartimo, Mirja. 2017a. "Husserl and Hilbert." In Centrone 2017, 245–263.

________. 2017b. "Husserl and Gödel's incompleteness theorem." *The Review of Symbolic Logic* 10 (4): 638–650.

Henrich, Joseph, Stephen J. Heine & Ara Norenzayan. 2010. "The weirdest people in the world?" *The Behavioral and Brain Sciences* 33 (2–3): 61–135.

Husserl, Edmund. 1950–2012. *Husserliana: Edmund Husserl – Gesammelte Werke*. Bde. 1–41. Haag: Martinus Nijhoff (Bde. 1–26, 1950–1987); Haag: Kluwer Academic Publishers (Bde. 27–37, 1988–2004); New York: Springer (Bde. 38–41, 2005–2012). [Abbreviated *Hua*].

________. 1950. [*Hua* I] *Cartesianische Meditationen und Pariser Vorträge*. In *Husserliana*, Bd. I, edited and introduced by Stephan Strasser. Haag: Martinus Nijhoff. [Also mentioned in the text as CM].

________. 1952. [*Hua* IV]. *Ideen zur einer reinen Phänomenologie und phänomenologischen Philosophie. Zweites Buch: Phänomenologische Untersuchungen zur Konstitution*. In *Husserliana,* Bd. IV, edited by Marly Biemel. The Hague: Martinus Nijhoff. [Also mentioned in the text as *Ideen II*].

________. 1970. *Logical Investigations*. Volumes I and II. Translated by J. N. Findlay. Edited by Dermot Moran. New York: Routledge & Kegan Paul.

________. 1974. [*Hua* XVII]. *Formale und transzendentale Logik. Versuch einer Kritik der logischen Vernunft*. In *Husserliana*, Bd. XVII, edited by Paul Janssen. Haag: Martinus Nijhoff.

________. 1975. [*Hua* XVIII]. *Logische Untersuchungen. Erster Teil. Prolegomena zur reinen Logik*. In *Husserliana,* Bd. XVIII, edited by Elmar Holenstein. Haag: Martinus Nijhoff. [Text of the 1st and 2nd edition, Halle, 1900, 1913].

________. 1976. [*Hua* VI]. *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie. Eine Einleitung in die phänomenologische Philosophie*. Edited by Walter Biemel. Den Haag: Martinus Nijhoff.

________. 1976a. "Die Frage nach dem Ursprung der Geometrie als intentional-historisches Problem (1939)." In Husserl 1976, 365–386.

________. 1976b. "Beilage II zu §9a (*1936)." In Husserl 1976, 357–364.

________. 1982. *Ideas pertaining to a pure phenomenology and to a phenomenological philosophy: First Book. General Introduction to a Pure Phenomenology*. Translated by Fred Kersten. Dordrecht, Boston & London: Kluwer. [Also mentioned in the text as *Ideen I*].

________. 1984. [*Hua* XIX]. *Logische Untersuchungen. Zweiter Teil. Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*. In *Husserliana,* Bd. XIX, edited by Ursula Panzer. The Hague: Martinus Nijhoff. [Text of the 1st and 2nd edition, Halle, 1901, 1922]. [Also mentioned in the text as LI].

Ingarden, Roman. 1975. *On the Motives which led Husserl to Transcendental Idealism*. Translated by Arnór Hannibalsson. Volume 64 of *Phenomenologica*. The Hague: Martinus Nijhoff.

Jónsson, Bjarni. 1948. "Review of 'Gödel (1947)'." *Mathematical Reviews* # MR0023780.

Kelly, Lynne. 2015. *Knowledge and Power in Prehistoric Societies: Orality, Memory and the Transmission of Culture*. Cambridge: Cambridge UP.

Kim, Jaegwon. 2011. *Philosophy of Mind*. Boulder: Westview Press.

Kleene, Stephen Cole. 1948. "Review of 'Gödel (1947)'." *Journal of Symbolic Logic* 13 (2): 116–117.

Klein, Richard A. et al. 2018. "Many Labs 2: Investigating

Variation in Replicability Across Sample and Setting." *PsyArXiv*. November 21. doi:10.31234/osf.io/9654g

Kleiner, Israel. 2007. *A History of Abstract Algebra*. Boston: Birkhäuser.

Lohmar, Dieter. 1989. *Phänomenologie der Mathematik. Elemente einer phänomenologischen Aufklärung der mathematischen Erkenntnis nach Husserl*. Vol. 114 of *Phaenomenologica*. Dordrecht: Kluwer.

Mohanty, Jitendra N. 1995. "The development of Husserls' thought." In *The Cambridge Campanion to Husserl*, edited by Barry Smith and David W. Smith, 45–77. Cambridge: Cambridge University Press.

Moore, Gregory. 1990. "Introductory note to *1947* and *1964*," In Gödel 1990, 154–175.

Parsons, Charles. 1995. "Platonism and mathematical intuition in Kurt Gödel's thought." In Parsons 2014, 243–266.

________. 1998. "Hao Wang as philosopher and interpreter of Gödel." In Parsons 2014, 243–266.

________. 2002. "Realism and the debate on impredicativity, 1917–1944." In Parsons 2014, 40–66.

________. 2014. *Philosophy of Mathematics in the Twentieth Century: Selected Essays*, Harvard: Harvard University Press.

Plato. 1903. *Platonis Opera*. Edited by John Burnet. Oxford: Oxford University Press.

________ .1969. [Works]. *Plato in Twelve Volumes*. Vols. 5 & 6. Translated by Paul Shorey. Cambridge, MA: Harvard UP.

Putnam, Hilary. 1975. "The meaning of meaning." In *Mind, Language and Reality*. *Philosophical Papers, vol 2*, 215–271. Cambridge: Cambridge University Press.

Schmit, Roger. 1981. *Husserls Philosophie der Mathematik. Platonistische und konstruktivistische Momente in Husserls Mathematikbegriff*. Vol. 10 of *Conscientia. Studien zur Bewußtseinsphilosophie*. Bonn: Bouvier.

Schuhmann, Karl. 1977. *Husserl – Chronik. Denk- und Lebensweg Edmund Husserls*. In *Husserliana Dokumente*, Bd. I, edited by Karl Schumann. Den Haag: Nijhoff.

Schwitzgebel, Eric. 2014. "Introspection." In *The Stanford Encyclopedia of Philosophy,* edited by Edward N. Zalta. https://plato.stanford.edu/archives/win2016/entries/introspection/

Tieszen, Richard. 1992. "Kurt Gödel and phenomenology." In Tieszen 2005, 93–111.

________. 1998. "Gödel's path from the incompleteness theorems (1931) to phenomenology (1961)." In Tieszen 2005, 125–148.

________. 2005. *Phenomenology, Logic, and the Philosophy of Mathematics*. New York: Cambridge UP.

Tito, Johanna Maria. 1990. *Logic in the Husserlian Context*. In *Northwestern University Studies in Phenomenology and Existential Philosophy*. Evanston, IL: Northwestern UP.

Troelstra, Anne Sjerp & van Dalen, Dirk. 1988. *Constructivism in Mathematics. An Introduction, vol. I.* Vol. 121 of *Studies in Logic and the Foundations of Mathematics*. Amsterdam: North-Holland.

van Atten, Mark & Juliette Kennedy. 2003. "On the philosophical development of Kurt Gödel." *The Bulletin of Symbolic Logic* 9(4): 425–476.

Volkert, Klaus Thomas. 1986. *Die Krise der Anschauung: Eine Studie zu formalen und heuristischen Verfahren in der Mathematik seit 1850*. Vol. 3 of *Studien zur Wissenschafts-, Sozial- und Bildungsgeschichte der Mathematik*. Göttingen: Vandenhoeck & Ruprecht.

Vongehr, Thomas. 2013. "Husserl und Hilbert – Familienbeziehungen." *Mitteilungsblatt für die Freunde des Husserl-Archivs* 36: 15–20.

Wang, Hao. 1974. *From Mathematics to Philosophy*. London: Routledge & Kegan Paul.

________. 1987. *Reflections on Kurt Gödel*. Cambridge: MIT Press.

________. 1996. *From Gödel to Philosophy*. Cambridge: MIT Press.

Weyl,Hermann. 1946. "Review of 'The Philosophy of Bertrand Russell, ed. by P. A. Schilpp'." *American Mathematical Monthly* 53(4): 208–214.

Willard, Dallas. 1984. *Logic and the Objectivity of Knowledge. A Study in Husserl's Early Philosophy*. Series in Continental Thought. Athens, OH: Ohio UP.

Zahavi, Dan. 2005. *Subjectivity and Selfhood: Investigating the First-Person Perspective*. Cambridge: MIT Press.

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Husserl's Early Genealogy of the Number System

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Abstract

This article accomplishes two goals. First, the paper clarifies Edmund Husserl's investigation of the historical inception of the number system from his early works, *Philosophy of Arithmetic* and, "On the Logic of Signs (Semiotic)". The article explores Husserl's analysis of five historical developmental stages, which culminated in our ancestor's ability to employ and enumerate with number signs. Second, the article reveals how Husserl's conclusions about the history of the number system from his early works opens up a fusion point with his investigations from his mature texts, *The Crisis of the European Sciences* and "The Origin of Geometry". On the one hand, the essay shows that Husserl's methodology was similar, as he sought in both his early and late writings to uncover the essence of the history of the formal sciences and was not executing mere intellectual history. On the other hand, the article discloses that Husserl's insights from both time periods are strikingly analogous. Already in his early texts, Husserl saw that the sciences emerged from pre-theoretical experiences of the world and that the sciences are the result of a historical process, which involves the psychic activities of past individuals and the maintaining of discoveries over time by intersubjective communities. I conclude by showing how, in light of the analysis of this paper, we can rethink the evolution of Husserl's philosophy.

Keywords: Husserl; philosophy of arithmetic; semiotics; history of science; genealogy.

1. Introduction

The overarching goal of Edmund Husserl's 1891 Philosophy of Arithmetic (Hua XII; Husserl 2003. Hereafter PA) is to clarify the contemporary execution of arithmetic

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calculation by tracing it back to its origin in our everyday experience of number. Husserl demonstrates how the practice of arithmetic develops naturally and logically from those simple encounters with numbers. To begin this investigation, Husserl clarifies that I first experience numbers when I am "authentically presented" with them (Cf. Hua XII, 10–21; Husserl 2003, 15–22). During such authentic presentations, I have an in-person and immediate awareness of the number. I experience, what Husserl would later call, an "eidetic intuition" of the number species and I see that the number species applies to the counted elements of the multiplicity as a whole (Tillman 2012, 145).

These authentic presentations are not the only way I experience numbers. In fact, I am authentically presented with numbers in very few cases. Husserl claims that, as a result of the limitations of our psychic capacities, humans are capable of authentically presenting only those number species that are less than or equal to five, ten, or twelve, depending upon which quote one pulls from the text.[1] As one cannot authentically present these higher numbers, a tool was created by means of which one can be presented with, count, and solve arithmetical equations that concern or contain them; namely, number signs. I somehow become conscious of the numbers 38, 349, or 8,784 when I read the corresponding signs on the page. Husserl states that this manner of becoming aware of numbers via signs is a case of "inauthentic presentation". The number signs inauthentically present their higher numbers (Cf. Hua XII, 193–195; Husserl 2003, 205–207). The number sign does not provide me with a direct awareness of the number species. Rather, I am conscious of the number species via mediation of the concept.[2] When I employ the number sign, "13", the sign signifies the number species by way of the concept. The concept circumscribes the number species for me. It is these kinds of experiences, where I use the number signs to signify the number species via the concept, which Husserl calls the conceptual employment of signs.

This conceptual method is also not employed in all cases. When arithmetical calculation becomes very complex, I have to do without the concepts and use the number signs alone.

Husserl wrote that for mathematicians, "It is a fact that *in praxi* all numbering and calculating could dispense with recourse to the underlying concepts" (Hua XII, 242; Husserl 2003, 256). Here, the manipulation of the signs themselves stands in the place of not only the authentic presentation of the number species, but also the conceptual employment of the number sign. I use the signs themselves to come to the correct answer to the equation, rather than having to work with the concepts or the species.

Because of the complexity of the development of arithmetical calculation, Husserl spends nearly the entirety of PA tracing back the execution of contemporary arithmetic to our authentic presentation of numbers. Yet, Husserl was not content with performing that analysis alone. In a most curious and little studied passage from the end of chapter seven, Husserl employs his insights about the grounding of arithmetic calculation in our daily lives to develop a radical and new element of his philosophy. In the section entitled, "The Natural Origination of the Number System" (Hua XII, 244–252; Husserl 2003, 258–267), Husserl executes what can only be called a genetic-historical[3] examination of the inception of number signs and the number system. He studies the different historical stages of sign development, which culminated in the first instance of the number system for primitive mankind. He searches, in that section, to make "intelligible how, in general, a sign system, that is artificial in its type and constitution, and whose consciously intended invention and theoretical justification would require abstract reflections of the most complicated sort, can come about through the course of natural psychological evolution" (Hua XII, 253; Husserl 2003, 259).

With regards to, "The Natural Origination of the Number System", the paper has two goals. First, the essay simply elucidates Husserl's early and extra-ordinary conclusions about the historical psychological generation of number signs and the number system. This task is; however, more complicated than it initially appears. In PA, Husserl does not provide the reader with enough information to fully grasp his historically-minded insights. He justifiably conducts his historical examination as an aside, because it is tangential to the main objectives of his book. Husserl was fortunately not unaware of the limitations of that genetic investigation. Accordingly, in an often-overlooked[4] 1891[5] manuscript, entitled, "On the Logic of Signs (Semiotic)" (Hua XII, 340–373; Husserl 1994, 20–51 [Hereafter LZ]), Husserl supplements that historical investigation from PA. In LZ, he clarifies certain conscious mechanisms, which he did not properly discuss in PA, that allowed for the evolution of sign usage for primitive mankind. Accordingly, I will synthesize Husserl's insights from both PA and LZ to present a comprehensive and accurate picture of his early historical insights.

The second objective of the paper is to demonstrate how Husserl's 1891 philosophy of the historical origination of the number system prefigures his mature philosophical writings about the history of the sciences. Indeed, those familiar with Husserl would have already noted that his historical investigations from PA and LZ appear anachronistic, as they bear a striking similarity to Husserl's examinations from his final works, *The Crisis of the European Sciences* (Hua VI; Husserl 1970a [Hereafter KW]) and, "The Origin of Geometry" (Hua VI; Husserl 1970b [Hereafter UG]). Indeed, in all four texts – PA, LZ, KW, and UG – Husserl traces formal scientific projects back to historical psychic accomplishments in the world of pre-scientific experience. The paper will show that such similarities are by no means superficial. Naturally, in his first writings, Husserl could not yet have formulated his conclusions about history in the same terms or with the same nuance that he did in KW and UG. In the 1890s, he had not discovered concepts such as meaning-sedimentation, life-world, horizon, or passivity, amongst others. Despite this fact, the essay demonstrates that many of the methods and key ideas, which would play an important part in Husserl's last writings on history and science, were already well developed in his early texts. In other words, the second goal of the article is to reveal a fusion point between Husserl's writings from the 1890s and the 1930s.

To accomplish these objectives, the paper is divided into two further sections. In section two, I address Husserl's examination of the inception of number signs and the number system from LZ and PA. In section three, I then demonstrate how his method and insights from the early texts prefigure his historical analysis from KW and UG. I then conclude by briefly demonstrating how this connecting point, between Husserl's immature and mature writings, reveals an inadequacy with and frustrates contemporary interpretations of the evolution of his philosophy.

2. Husserl's Early Historical Genetic Analysis of the Origin of the Number System

In the following sections, I investigate Husserl's analysis of the historical generation of the number system, as he described it in LZ and PA. Before broaching Husserl's observations; however, it is necessary to more clearly outline the goals of this historical-genetic analysis and to examine the methodology employed during its undertaking. To be certain, Husserl's investigation is not an attempt to provide an accurate historical account of the development of arithmetic in, for example, ancient Greece, Egypt, or China. He writes that, "The periods within which the origination of number systems and number sign systems falls are unknown to any historical tradition. Therefore there can be no thought of a reproduction of the historical development." (Hua XII, 245; Husserl 2003, 259) Husserl sees that his theory must rather account for how arithmetic arose during its independent "discovery by different peoples, which is deducible with certainty from the existing differences (e.g. the choice of the base number of the system), found alongside all commonalities" (Hua XII, 245; Husserl 2003, 259). Husserl is also not seeking to demonstrate that the evolution of the number system *depends* or is *contingent* either upon the psychological composition of the human mind or on the particular factical developments of human history. He does not here – nor anywhere in PA – psychologize or historicize the number system or arithmetic.[6] Instead, while still accounting for the "general traits of human nature", Husserl's true objective is to disclose the "psychological evolution of such formations … in all its essential points" (Hua XII, 245; Husserl

2003, 259). He hopes to show what is *essential or necessary* in the historical inception of the number system.[7]

Methodologically considered, Husserl executes his analysis by identifying and exploring five stages of the psychological evolution of the number system. The grounding assumption of Husserl's theory is that the ancients were able to enumerate with inauthentic number signs and thereby develop the number system, because they had mastered the use of other kinds of signs first. There is a chain of increasingly more complex kinds of signs, where one must first be able to utilize the simplest kind of sign before one is capable of learning the next and more elaborate kind of sign in the series (Hua XII, 250–251; Husserl 2003, 263–265). Keeping in mind the goals of Husserl's overall historical-genetic analysis, in what follows, I examine how Husserl describes each of the developmental historical stages, which were necessary for the inception of the number system.

2.1 Natural Mediating Signs

The necessity of bringing in LZ to augment Husserl's historical analysis from PA is clear from the start, because his examination from the latter begins too late in humanity's development. In PA, Husserl initiates his investigation by looking at the second stage of this historical development – not the first. Husserl, in the 1891 book, examines how humans came to formulate the simplest kinds of number signs. Yet, in LZ, Husserl asserts that certain other experiences are necessary even before the creation of number signs is possible. In that manuscript, Husserl examines this first stage of development, which opens up the possibility of sign creation; namely, the experience of "natural signs" (Cf. Hua XII, 345–346; Husserl 1994, 24–25).

Even though Husserl, in LZ, does not thoroughly describe natural signs, the meaning behind this term can be made clear on the basis of his analysis. An example will help. As is well known, when cooking meat over a fire, it goes from a bright red to a brown color. Eating meat when it is red can be dangerous, as parasites and diseases may remain in the meat.

Yet, when brown, the meat is safe to eat (albeit, less savory). After years of cooking meat on an open fire, the ancients would have had many experiences of themselves or other's contracting certain illnesses after eating red meat and they would have experienced a certain rejuvenation after eating meat, which was cooked until it was brown. As a result, there could become established an association (an associative link) between the brown color of the meat and its healthiness. For the early Husserl, this association is the mechanism behind the signitive operation of natural (and all) signs. Once that associative link is installed, when the villager, who is cooking the meat, sees that the meat has a brown color, the associative link, which has been sedimented, is reactivated. The authentic presentation of the brown-ness of the meat would associatively awaken an awareness of the healthiness of the meat. The brown would function as a sign, as it – by means of association – points beyond itself, that is, signifies the meat as healthy. Husserl calls this sign and others like it, "natural", because they are not the result of human invention, but rather arose organically through man's pre-theoretical experiences of the world.

Of importance is that a natural sign – if it is univocal and sufficient to pick out the signified object – is a "temporary" inauthentically presenting sign or a "mediating" sign.[8] In LZ, Husserl describes these (simplest) kind of signs as, "mere intermediaries for the production of authentic representations corresponding to them" (Hua XII, 351; Husserl 1994, 31). When mediating signs signify by means of association, they *lead or prompt me* to authentically present their signified objects. They function by means of mediation and not via replacement. For the above example, when villagers would be authentically presented with the meat as brown, the brownness would function as a natural mediating sign, which would have associatively prompted the villagers to authentically present the meat as healthy.

2.2 Conventional Mediating Signs

The second phase of historical sign development occurs on the basis of the first. In LZ, Husserl concludes that once cavemen had "the capacity for understanding signs", which would arise organically or naturally, then and only then would they have the capacity to create signs, which he calls – for obvious reasons – "conventional" signs (Cf. Hua XII, 349–350; Husserl 1994, 28–29). The ancients needed to see that the one object can mediate our awareness of another, that is, signify the other, before they could themselves fashion conventional signs. Husserl writes that, "the natural modes of procedure must precede that of the conventional" (Hua XII, 366; Husserl 1994, 44). At the same time, Husserl emphasizes that that realization is not enough. After recognizing the mediating and signifying power of signs, our ancestors would then have also had to employ their will to fashion a sign, which was meant to communicate something to others or to themselves (Hua XII, 345; Husserl 1994, 24–25).

To return to the above example, if, after realizing that the brownness of meat awakens an awareness of the healthiness of meat (that is, functions as a sign, which associatively signifies the meat's healthiness), one caveman could use certain berries and the bark of trees to create a brown paste, where he could then "paint" certain warriors, who were most healthy and skilled, with a brown color. By doing so, that ancient person would be employing his will to conventionally use what was initially a natural sign. He would be willfully marking the warriors with signs, which could signify to others the fact that these warriors are most healthy and robust. When the warriors would be authentically presented to the villagers with the brown paste covering their bodies, the previously established associative link between brownness and healthiness could be reawakened. The brownness would function as a sign, which would signify the healthiness of the warriors for the villagers, as the brownness-sign on the warriors would prompt the villagers to authentically present the warriors as healthy. Importantly, Husserl claims that these first conventional signs are also, like the simplest natural signs, mediatory signs. Conventional signs were first created to mediate others' authentic awareness of the signified. The brown paste on the warriors would not replace, but rather mediate the villagers' authentic presentation of the warriors as healthy.

Husserl asserts that this first creation of conventional signs is possible on the basis of the experience of natural signs, because both operate by means of the same mechanism; namely the association that prompts the authentic presentation of the signified object. The difference between them is that conventional signs were created by humans by using their will. Indeed, because they both function via mediating association, this jump from natural to conventional signs is easy to make, such that, "we should not be amazed when animals make themselves understood through signs, to a certain extent" (Hua XII, 345; Husserl 1994, 25). At the same time, even though they both function by means of the same associative mechanism, Husserl asserts that conventional signs open many possibilities for communication and knowledge, which were otherwise closed off. He writes that, "For the conventional techniques do not merely do the same thing better than the natural ones. Rather, they do incomparably more" (Hua XII, 366; Husserl 1994, 44).

On the basis of the above remarks, it is possible to explore Husserl's genetic-historical analysis of the number system from PA, which begins at this second stage. Without mentioning that our forbearers must have first understood the power of natural signs before they could have fashioned conventional signs, Husserl just starts his discussion with an examination of the creation of the conventional signs, which can mediate the authentic awareness of number species. He states that, among the ancestors, there must have been an interest in sensible groups of the same kind and that there would be a "drive to communicate concerning the events of practical life, in which determinate groups of such objects played a great role" (Hua XII, 245–246; Husserl 2003, 260). There was, for example, a practical need for accurately determining how many sheep were in a herd and whether one of the sheep had been eaten by a wolf the previous night. This need could be met, Husserl claims, via, "an imitation[9] by sensible means of the things represented". There needed to be discovered some sensible objects, which could clearly "imitate" the objects, whose amount needed to be determined. These imitating objects should be easy to access and clearly differentiated from each other. For Husserl, the objects, which could perform this function, are self-evidently the fingers on the hand. He writes that fingers would "have come immediately to mind for the imitation and symbolization of corresponding groups of arbitrary other objects" (Hua XII, 246; Husserl 2003, 260).

It was not enough; however, for the ancients to have recognized that the fingers could imitate groups of objects, for those fingers to function as signs, which mediate our authentic awareness of the number species and number concepts. Number concepts and number species also have a generality, which needs to be realized. Number species are not instantiated in just one group of objects alone, but are rather instantiated in any concrete multiplicity, which contains that number of objects. Moreover, the concept, to which the finger sign refers (and can mediate our awareness of the species) is also applicable to any number of groups, which have that same number of members. In order to realize this generality, Husserl states that cavemen had to look back and forth between, on the one hand, different groups of the same number (for example, three arrows, three sheep, three warriors) and, on the other, the fingers (the three fingers that are held up). By doing so, they would see that the three fingers serve as a sign that can signify all of the different groups of three objects, where this would allow for a recognition of the generality of the species or concept. Husserl writes that, it was "only through constant back-reference from groups of the most various types to the finger groups, sharply distinct in sensible appearance [that] finger numbers rise to the level of representatives of general concepts" (Hua XII, 246; Husserl 2003, 260).

Finally, with the willful creation of the conventional mediating finger signs, the ancients could begin enumeration. According to Husserl, enumeration with finger signs must have been initially very difficult and would have required a great deal of psychic energy.[10] In order to enumerate in a secure manner, our ancestors would have to work through every single number to reach higher numbers, where they would raise one finger to represent each of the members of the group. For the first member, the pointer finger could have been raised, for the next member, the middle finger, then the ring finger, and so on (Hua XII, 246–247; Husserl 2003, 261). This sequencing of numbers by our forebears represented the dawn of the number system for mankind.

2.3 Surrogates

While enumeration represented an important step forward in the historical development of the number system, in its first form, it was still very limited. During the period when humans only had access to mediating signs, which are the first kind of signs that were discovered, they would have only ever been able to enumerate up to the number 12. To understand why this is the case, we remember first, that only numbers up to 12 can be authentically presented, and second, that mediating number signs prompt the authentic presentation of numbers. With these two ideas in mind, we can say that if I saw the number sign 13, and if that sign functioned as a mediating sign, it would prompt me to authentically present the number 13. Yet, this is impossible, because – as was discussed in the introduction – the number 13 cannot be authentically presented. As such, it seems that the number sign 13 and any greater number signs could not signify its number species nor signify at all.

Yet, Husserl claims that enumeration, "could be continued beyond [the narrow domain of authentic representations]" (Hua XII, 246; Husserl 2003, 261). This was only possible for ancient peoples when they were equipped with a new kind of sign, which could signify non-authentically-presentable numbers. The discovery of these novel signs, which Husserl terms, "surrogates", "replacements", or "permanently inauthentically presenting signs", is the third stage of historical sign development. A surrogative sign does not prompt one to authentically present the signified. Instead, when the number sign 13 functions as a surrogate, it can signify the number concept, even though that number is never authentically presented. The number surrogate simply deflects consciousness to the non-apparent or not-authentically-presented signified object.[11] By means of this replacing or deflecting function, surrogates allowed for ancients to count beyond those numbers, which could be authentically presented.

Concerning surrogates, it should also be noted that while they are different from mediating signs, the former also operate by means of the same general mechanism as the latter; association. The object, which will be the surrogate, also must first be associatively tied to a signified object, such that when I again see the surrogate, that associative link is reawakened, where the surrogate deflects me to or replaces the other signified and not authentically presented object.

The specific historical details of the evolution of mediating signs into surrogates are only briefly discussed by Husserl in LZ. He writes, "Only in consequence of constant usage, with the associations which develop, and occasionally also through experimentation – or through a mixture of the two – do conventional signs (provided they are actually suited for it) assume the character of surrogates" (Hua XII, 366; Husserl 1994, 44–45). In this quote, Husserl is affirming that the forbearers did not create conventional signs to function as surrogates. Conventional signs were first created as mediating signs. Yet, some conventional signs naturally evolved to become surrogates. Indeed, the fashioning of signs as surrogates could only occur at much later stages of history. Husserl writes, "We already have a high developmental level of spiritual culture when we invent conventional surrogates with full consciousness of their function" (Hua XII, 367; Husserl 1994, 45). It can; therefore, be concluded that fingers originally served as mediating conventional signs and that, only after their continual usage, did they then assume the character of surrogates. It can also be claimed that each of the three now outlined stages of development follow each other chronologically. By first experiencing natural mediating signs, ancients were then able to use their will to craft conventional mediating signs. Finally, these conventional mediating signs organically became conventional surrogative signs.[12]

By employing surrogates, our ancestors could then enumerate beyond the limits of authentic presentation. When discussing this development of enumeration, Husserl provides a more explicitly historical analysis and even employs numerous anthropological examples to support his case. He again begins at the simplest level, stating that ancient men would count up to five using the fingers on one hand, before using their other hand to count up to ten. Here, the ancients ran into a dilemma – a fortuitous dilemma – that could only be solved by creating a more advanced number system. Simply stated, the dilemma is; how should they continue to count beyond ten? There were no more fingers for them to count on and it seemed that this was thus the upper limit of enumeration. Yet, Husserl claims that the continuation of counting was possible, by making a note, which would allow for the cavemen to remember that they had already counted through all ten fingers. He writes that, "there obviously remained nothing left but to make a note – on the side, by a sensible sign – of the fact that the fingers had been numbered through once, and then to count off the objects yet remaining by means of the finger again" (Hua XII, 247; Husserl 2003, 261–262). In this case, the mark on the side operates as a surrogate. When cavemen saw this mark, they were not motivated to authentically present the ten objects; rather, the mark served to replace and signify the counting through of the ten fingers, which had already occurred. This solution would be reapplied when ten marks on the side became noted, that is, when all ten of the fingers were counted through ten times. A new sign would be created, which would signify the ten counting through of the ten fingers. To clarify exactly what he means with this idea, Husserl draws in an example from the anthropologists, E.B. Taylor. According to Taylor, the villagers in south east Asia enumerate, "by using in counting a small stone for the ones. When ten of these are together, they are replaced with a small piece of coconut shell. When ten of these latter are together, then a larger piece of coconut shell is used" (Hua XII, 247; Husserl 2003, 262).

Importantly, by utilizing their fingers and marks or coconut shells in this way for counting, cavemen had established a system of counting that was recursive. Once the cavemen had counted through all of the fingers once, after making a note on the side, they would begin the process over, by starting to count with the first finger again. They would go back to the first finger every time all of the fingers had been counted. This method of "restarting" the counting is what makes the system recursive. Moreover, this recursive system was also a decimal system, because the number at which the counting restarted was ten. When all ten fingers were counted, the counting would start back with the first finger after the mark had been made. Ten thus became the "base number" of this recursive system – hence the term decimal. Husserl summarizes these important points, by writing that, "In this way one was led to a general procedure for the enumeration of groups, through which each larger number is already constructed in the form of a polynominal function of powers of ten" (Hua XII, 247; Husserl 2003, 262).

2.4 Language Signs

Soon after the invention of sign language and the manipulation of small tokens, Husserl concluded that conventional language signs were created and used to signify numbers. The word signs themselves and the method of enumeration with word signs; however, did not somehow stand apart from enumeration with fingers and tokens. Instead, the latter serves as the foundation for the former. Husserl writes that the way in which enumeration was developed with the word signs, "was not merely a "fortunate move", but rather was a necessary consequence of the further development of counting with fingers" (Hua XII, 250; Husserl 2003, 264). How is this the case? On the one hand, the first several word number signs were, "a mere translation of finger numbers into word numbers" (Hua XII, 246; Husserl 2003, 261). The first finger was translated into 1, one finger and one finger was translated into 2, one and one and one finger was translated into 3.

On the other hand, the *method* of enumeration was a direct working out of the *method* of enumeration with fingers. We know that when the ancients finished counting with both hands, they would set aside a mark and begin again. This starting over at ten established that number as the base number of the system. That is, it established the recursive decimal system. Because that base number was already established, when ancients began to enumerate using word signs, they counted in a recursive manner and often employed ten as the base number for their system. When they reached ten number words and wanted to add one more item, they also would begin the process over. This time; however, they did not add a mark to the side to signify that the first ten digits had been run through, but rather placed a "1" in the tens column to show that the first ten numbers had been counted. That is, they took the "names for the numbers up through ten" and formed the higher numbers "through the mere combination of these" (Hua XII, 248; Husserl 2003, 262). In the same way, when ten tens were counted out, the ancient mathematicians, following the established decimal system, began the sequence again after placing a "1" in the hundreds column.

2.5 Mechanization

Husserl concludes that the transformation of the decimal system from the use of fingers and tokens, to that of language did "facilitate and simplify counting itself" (Hua XII, 248; Husserl 2003, 262). He states that, "Through these modifications, enumerations would become more cohesive and systematic and simultaneously independent of sense perceptible instruments other than words" (Hua XII, 248; Husserl 2003, 262–263). As a result of this simplification, Husserl claims that the mechanization of enumeration became possible. Because the decimal recursive system made it so effortless to generate the number language signs, "as soon as the systematic was mastered through practice, the mental process of concept formation automatically had to vacate the field to the external reproduction mechanism of name formation" (Hua XII, 250; Husserl 2003, 265). Husserl outlines two ways in which the linguistic decimal system allowed for this mechanization – which is the fifth stage of development – to take place.

First, this linguistic decimal number system, which is an extension of counting with fingers, allowed for language number signs to be easily brought to consciousness. Concerning generation of these signs, I do not have to memorize ten thousands distinct signs to be capable of counting to that large sum. Instead, I only must remember 10 number signs (0–9) and continually implement them in the recursive manner.[13] Second, the decimal system, which employs words, allows for the number signs to signify in a univocal manner, such that enumeration can be trusted – and does not have to be checked (or double checked) by some other means. The ancients were assured that the smaller numbers signs signify in an unambiguous manner, because they set them up in such a way that they have a one-to-one correspondence to their authentically presented numbers. By continuing the formulation of number signs according to the established recursive method beyond those that have authentically presentable numbers, the univocal link between the sign and its number is maintained, where each higher number sign continues to have a one-to-one correspondence to its number.

With these ideas in mind, it is possible to understand Husserl's brief historical outline of the mechanization of enumeration. He first writes, "Originally one counted by a mental action, picking out of the group one member after another: one, one and one is two, two and one is three, and so on" (Hua XII, 250; Husserl 2003, 265). During this enumeration, the ancients experienced those signs as surrogates for their number concepts. After one had learned the numbers 1-9 – by following the recursive decimal pattern, it became easier and easier to generate the number signs. Ultimately, our forbearers could mechanically or 'instinctually' count through the numbers without experiencing them as signifying their concepts, that is, without experiencing their conceptual content. Husserl writes that after long practice, "one counted mindlessly, so to speak, or mechanically, by following out the sequence of names … without any reflection on their conceptual signification" (Hua XII, 251; Husserl 2003, 265). This led to a further simplification, where the ancestors did not have to sequence every number, by picking out one member after another (1, 1 + 1 = 2, 2 + 1 = 3), as this sequence was rather "abbreviated into the sequence of terms 1, 2, 3 …" (Hua XII, 251; Husserl 2003, 266).

To conclude this section of the essay, I note that, during his genetic-historical analysis, Husserl does not discuss any further steps of the development of arithmetic, which would have occurred after the ancients had formulated the language decimal number system. Most noticeably, he does not investigate the mechanization of arithmetical calculation, whereby one follows the rules of arithmetic to calculate without any reference to the concepts. He simply mentions that with the development of the "Indic system, [the number signs] first assume the character of a logically perfect instrument of arithmetic, but also of an instrument which originated through scientific reflection" (Hua XII, 252; Husserl 2003, 266). The fact that Husserl does not discuss this development is in line with the goals of this passage from PA, because he asserts that he only seeks to clarify the historical evolution of the number system and is not accounting for the genesis of contemporary arithmetic calculation here.

3. The Fusion Points with Husserl's Mature Investigations

As stated in the introduction, I conclude this essay by revealing the fusion points between Husserl's early examination of the historical development of the number system and his historical analysis of the generation of physics and geometry from KW and UG. I discuss how Husserl's analyses from the early 1890s already provided him with many of the tools and insights necessary to conduct his final genetic-historical examinations. In other words, I will demonstrate that Husserl's methodology is similar during both time periods and I show that the results of his studies of historical genesis are analogous. Finally, I briefly discuss how these conclusions challenge the standard reading of Husserl's philosophy and thus require a rethinking of the development of his phenomenology.

To reveal the methodological similarities between the works arising from the distinct periods of Husserl's life, we first remember that, in LZ and PA, Husserl was not concerned with discussing the factical historical developments of the number system. Instead, he disclosed the essence of the historical emergence of the number system. Husserl adopts a very similar methodology in KW and UG. In these final writings, he is not interested in conducting intellectual history. If Husserl were doing so, as David Carr writes, "he would seem to share the ontological commitment of the 'natural attitude' involved in all normal historical inquiry by his own accounts, i.e. the concern with men who actually existed" (Carr 1970, xxxii). Rather, Husserl is – as he was in his early works – seeking to uncover the *essence* of the historical development of the sciences, here physics and geometry. Even when he investigates the insights of, for example, Galileo and Thales, he is not examining the particular historical details of their discoveries, but is rather analyzing the insights of those thinkers as *examples* of the historical development. In UG, Husserl writes, "For, as will become evident here in connection with one example, our investigations are historical in an unusual sense, namely, in virtue of a thematic direction which opens up depth problems quite unknown to ordinary history … Our problems and expositions concerning Galilean geometry take on an exemplary significance" (Hua VI, 365; Husserl 1970b, 353). By studying those thinkers as examples, Husserl's late analysis remains focused on determining the essence of history and does not engage in a study of factical events.

More importantly, in his final writings, Husserl executes his investigation by taking up and revising the idea, which guided his analysis of history in LZ and PA; namely, that the number system was generated by means of different psychic discoveries, which compound on each other over the course of time. In those early texts, he concluded that after the first and simplest kind of sign had been learned and often employed, the next and more complex kind of sign in the series could be discovered. Even for the early Husserl, the number system did not present itself to the ancients (and does not present itself to us) in a pre-formed manner, but rather manifests itself as the production of the psychic activities of distinct individuals, whose discoveries became maintained by future generations. In alignment with this, Husserl writes in UG that, "These sciences are not handed down ready-made in the form of documented sentences; they involve a lively, productively advancing formation of meaning, which always has the documented, as a sediment of earlier production, at its disposal that it deals with logically" (Hua VI, 375; Husserl 1970b, 365). Here, Husserl is telling the reader, as Mohanty states, that geometry "is thus a moving process. It is related to an entire generation of workers in the field sharing a common horizon" (2017, 421). With this guiding idea in mind, in KW and UG, Husserl sought – as he did in PA and LZ – to dig back up the meaning-sedimentations, which were required or essential for the development of the sciences – now, the sciences of geometry and physics. He looks to the distinct essential stages of the historical development of the sciences and reveals how the discoveries of each stage became sedimented and how individuals and communities worked from the previous stages to develop their own insights, which would in turn become sedimented. In sum, in his later writings, Husserl unearthed geometry's "first acquisition, out of first creative activities", and traced "one set of acquisitions to another", thereby discovering, "a continuous synthesis in which all acquisitions maintain their validity" (Hua VI, 367; Husserl 1970b, 355).

Not only Husserl's methodology, but also many of the conclusions of his studies from his first and last philosophical writings are strikingly analogous. I here mention two similarities. First, we remember that Husserl concluded, in PA, that the invention of finger number signs did not arise from a theoretical interest, but rather from within a pre-theoretical attitude. Finger signs were developed, because there was a need for communicating with others about, "the events of practical life, in which determinate groups of such objects played a great role" (Hua XII, 246; Husserl 2003, 260). The number of sheep and the number of arrows needed to be determined not for a science or theory of nature, but rather simply for the needs of survival. The number system was generated out of the practical needs of the pre-theoretical and everyday world of the ancients. In line with this, in his mature works, Husserl claims that the theoretical attitude of, for example, the Greeks, was preceded by and arose out of "original natural life" (Hua VI, 327; Husserl 1970a, 281). Husserl writes that natural life, "can be characterized as a life of naively, straightforwardly directed at the world" (Hua VI, 327; Husserl 1970a, 281). This world of natural life, that is, the world experienced prior to theoretical interests, is what Husserl famously called the "life-world" in KW (cf. Carrr 1970, xl). The life-world, the late Husserl concludes, serves as the context within which the sciences were and continue to be developed and is that which the sciences study and relate back to in distinct ways.[14] On the basis of these similarities between Husserl's insights, it can thus be concluded that Husserl's seminal idea – that the life world is the ground for all theoretical activities – can be traced back to his writings about the number system from the early 1890s.

A second important commonality between Husserl's conclusions from his immature and final writings can be discovered by looking at his investigation of the consciousness of idealities, such as that of numbers, squares, circles, or formulae. As we know, in PA, Husserl observed that the ancients were able to become aware of the number concepts and species only after they had invented signs for them. By "constant back-reference" from the finger signs to groups of objects, the forebearers discovered the generality of the concepts and species. Moreover, the invention and employment of linguistic number signs simplified and standardized their meanings. Similarly, in UG, Husserl seeks to determine, "how does geometrical ideality proceed from its primary intrapersonal origin … to its ideal objectivity" (Hua VI, 369; Husserl 1970b, 358)? Just as he decided in PA, Husserl now states that the invention of certain signs is the condition of possibility for the consciousness of ideal objectivities. When the geometrical ideality takes on a linguistic garb, it has the possibility of becoming an ideal object. He writes, "In advance we see that [this realization of ideal objectivity] occurs by means of language, through which it receives, so to speak, its linguistic living body" (Hua VI, 369; Husserl 1970b, 358). From these insights, we see why Husserl executes his studies of the historical development of the number system and of geometry, in part, as investigations of the development of signs. As he concludes in the works from both time periods that signs are the pre-condition of our thinking of ideal numbers and shapes, he also concludes in all four texts that it is by accounting for the evolution of sign-manipulation that he can clarify the (historical) development of the sciences.[15]

On the basis of the above conclusions,[16] it can be made clear why the revelation of these connecting points between the methods and conclusions of Husserl's early and final historical analyses challenges the current understanding of the development of his philosophy. Simply stated, while other scholars frequently claim that Husserl revolutionized his philosophy when he executed a historical investigation of the origin of physics and geometry in his very late works (e.g. Bernet et. al. 1993, Hopkins 2011, Mohanty 1995, Zahavi 2002), the paper has shown that this is not the case. This essay demonstrated that Husserl's concern with understanding the historical inception of the sciences was there from the start. He had, already in 1890, executed a robust historical analysis of the essential meaning-sedimentations necessary for the development of the number system and – by extension – the formal science of arithmetic. He saw that the sciences emerged from the pre-theoretical experiences of the world (the life world) and from practical demands that that world placed on mankind. He had, at the very first stages of his career, realized that the sciences are the result of a historical process, which involves the psychic activities of individuals and the maintaining of discoveries over time by intersubjective communities. These insights further problematize interpretations of Husserl's works, because they seem, in some ways, inconsistent with his critiques of Dilthey, which he famously put forward in his 1911, "Philosophy as a Rigorous Science" (Hua XXV/2002). However, this is not the place to engage in an analysis of Husserl's argument against Dilthey, as it has instead been the goals of this essay to clarify Husserl's early prodigious conclusions about the historical inception of the number system and to reveal the important links those insights have to his later works.

NOTES

[1] For five being the maximum number of things one can authentically present, see Hua XII, 114; Husserl 2003, 120. For ten being the greatest, see, in the respective texts, 224/ 236, and for twelve, see 192/202.

[2] What exactly a concept is for the early Husserl, is difficult to determine. However, I think Willard comes closest to properly elucidating Husserl's

understanding of concepts when he claims that the notions of presentation (in his terminology representation (Vorstellung)) and concept are equivalent, "without interesting exception (1984, 26) in PA. According to Willard, "a concept or representation is treated by Husserl as a repeatable and shareable thought (1984, 27).

[3] Understandably, the reader may be hesitant to accept my use of Husserl's later terminology during my discussion of his early works. Yet, on my reading, these terms are the only ones that would correctly convey the meaning of Husserl's historical analysis of the genesis of the number system from the 1890s. Indeed, it is a goal of this essay to show that there is little difference between Husserl's conclusions about the origin of geometry and physics from his final writings and his ideas concerning the generation of the number system from these first texts.

[4] To the best of my knowledge, there are nine articles that discuss the tenets of LZ in some detail. These are: Byrne 2017a, 2017b, 2017c; D'Angelo 2013; Ierna 2003; Majolino 2010, 2012; Zuh 2008, 2012.

[5] If one assumes Carlo Ierna's dating (2005, 36–40), Husserl wrote LZ immediately after composing his letter to Carl Stumpf, within which he admitted that the project of his forthcoming Philosophy of Arithmetic, to ground mathematics in the concept of number, was fundamentally misguided. In contrast, if one follows Willard (1986, 111–116) or Hopkins' (2002, 60–71) interpretations, he composed LZ even prior to the correspondence with Stumpf!

[6] While there are indeed many problems with Husserl's descriptions in PA, advocating for psychologism is not one of them. Husserl clearly did not believe that numbers were mental entities. Rather, he states that the collection or numbers of objects are objective. He writes that, "the domain of numbers takes in an unrestricted manifold of species", which are "the numbers in themselves, that is, the numbers that are in general inaccessible to us." (Hua XII, 260; Husserl 2003, 275). Hopkins explains, "The collection is not an objective (sachliche) unity grounded in the contents of the collected things. This is not to say, however, that Husserl thought that the unity of the collection is not objective. The objectivity of its unity is never in question for him" (Hopkins 2006, 92). In line with this, Husserl concludes that the concept of number applies to the number of objects and not to the collecting act: He asserts that a concept applies to the object if the object possesses certain determinations or relations that the concept connotes. Finally, Husserl believed that the truth or falsity of numerical calculations were not relative to human psychological composition. He asserted that the truths of arithmetic were necessary truths, which could be demonstrated by the "analysis of concepts" (Hua XII, 268; Husserl 2003, 284).

[7] Important to note is that Husserl does not claim that this is an analysis of the historical a priori. Instead, he asserts that he is describing this evolution "in an aposteriori fashion" (Hua XII, 245; Husserl 2003, 259).

[8] Husserl calls only those signs that univocally signify and thus can lead us to assured truth and knowledge of the world in a scientific sense, "inauthentic presentations" (Hua XII, 351; Husserl 1994, 30–31).

[9] Husserl later recognized that his use of the term “imitation” was misguided and required revision. He wrote in a footnote, “This mode of expression, although incorrect is nevertheless appropriate here, because it is suited to the mental level concerned. The psychical activities brought to bear upon sensible groups supply concepts which the more naïve consciousness regards as abstract positive moments of the respective intuitions themselves” (Hua XII, 246 n. 1; Husserl 2003, 260 n. 2).

[10] Husserl even uses here an anthropological example to prove this point, stating that, “reports about counting among savage peoples confirms this” (Hua XII, 246; Husserl 2003, 261).

[11] This interpretation of Husserl’s theory of surrogates is fundamentally different from my reading of those signs from my previous publications. In those previous articles, I claimed that the surrogate replaces the signified object by being confused for it. As a result, I asserted that Husserl had an “intuitive theory of meaning” in his writings from the early 1890s (In particular, see Byrne 2017b, 223–226).

[12] Husserl does mention that, in some cases, it is possible to skip the second stage of sign development: He observes that a natural sign could, without further ado, become a natural surrogate (Hua XII, 367; Husserl 1994, 45).

[13] A further development of linguistic number signs, which Husserl discusses in PA, is that it allows for one to straightforwardly compare and contrast number signs, because the recursivity is structured via columns (ones, tens, hundreds, etc.). I place a number sign in a distinct column depending upon how many amounts of tens that sign is supposed to signify. When I am then presented with two number signs, I can immediately ascertain which quantity is greater by first examining the left most column (which concerns the greatest multiples of ten) and contrast the number signs found there (778 > 341 and 778 > (0)78). If this does not settle the matter, I continue comparing the number signs in the columns from left to right until I find a disparity (665 > 663) or ultimately see them as equal (1,356 = 1,356). Concerning all of these points, see Hua XII, 256 –257 and 238; Husserl 2003, 281–292 and 252.

[14] In these mature works, Husserl also goes into more detail about how intersubjectivity plays a critical role in the historical development of the sciences. Because we operate within the context of the sciences, which were discovered and maintained by past thinkers, Husserl even concludes that consciousness essentially possesses an intersubjective and historical component. The thinkers of the past serve as the background of our individual and collective consciousness. Carr states that, for the Husserl of the Crisis, “The background of the past now becomes that of the social or intersubjective past, which now belongs to the individual subject by virtue of membership in a community” (2016, 161).

[15] Indeed, Husserl already emphasized this point in LZ, writing that the developments of sign usage, “do not merely accompany psychic development, but rather they essentially condition it, making it possible to begin with. Without the possibility of external, enduring marks of reference as supports for our memory, without the possibility of symbolic representations … there would simply be no higher mental life – much less, then, science” (Hua XII, 349; Husserl 1994, 29).

[16] I must highlight that one should by no means read these conclusions as entailing that there are no important differences between Husserl's first and final works. Indeed, there are. To merely mention one significant example, in KW, Husserl considers his genetic-historical analysis to be an essential introduction to phenomenology, where this conclusion holds its own set of problems. In contrast, Husserl's study of the history of the number system in PA is executed as an aside; it is tangential to the overarching objectives of that book.

REFERENCES

Bernet, R., E. Marbach, and I. Kern. 1993. *An introduction to husserlian phenomenology*. Evanston: Northwestern.

Byrne, T. 2017a. Husserl's early semiotics and number signs: Philosophy of Arithmetic through the lens of "On the Logic of Signs (Semiotic)". *Journal of the British society for phenomenology* 48(4): 287–303.

________. 2017b. Surrogates and empty intentions: Husserl's On the Logic of Signs as the blueprint for the First Logical Investigation. *Husserl studies* 33(3): 211–227.

________. 2017c. "The dawn of pure logical grammar: Husserl's study of inauthentic judgments from 'On The Logic of Signs' as the germ of the Fourth Logical Investigation." *Studia Phaenomenologica* 17, 285–308.

Carr, D. 1970. "Introduction." In *The Crisis of the European Science*, translated by D. Carr, XV-XLIII. Evanston: Northwestern University Press.

________. 2016. *Experience and History: Phenomenological Perspectives and the Historical World*. Oxford: Oxford University Press.

D'Angelo, D. 2013. "The foundations of alterity. Husserl on referencing and indicating." *Investigaciones Fenomenológicas* 10: 53-71.

Ierna, C. 2003. "Husserl and the infinite." *Studia Phaenomenologica* 1-2: 179-194.

________. 2005. "The beginnings of Husserl's philosophy, Part 1: From "Über den Begriff der Zahl" to *Philosophie der*

Arithmetic." *The New Yearbook for Phenomenology and Phenomenological Philosophy* 5: 1-56.

________. 2006. "The beginnings of Husserl's Philosophy, Part 2: Philosophical and mathematical background." *The New Yearbook for Phenomenology and Phenomenological Philosophy* 6: 33-81.

Hopkins, B. 2002. "Authentic and symbolic numbers in Husserl's *Philosophy of Arithmetic*." *The New Yearbook for Phenomenology and Phenomenological Philosophy* 2: 39-71.

________. 2006. "Husserl's Psychologism and Critique of Psychologism, Revisited." *Husserl Studies* 22: 91-119.

________. 2011. *The Origin of the Logic of Symbolic Mathematics: Edmund Husserl and Jacob Klein*. Bloomington: Indiana UP.

Husserl, Edmund. 1950-2012. *Husserliana: Edmund Husserl – Gesammelte Werke*. Bde. 1-41. Haag: Martinus Nijhoff (Bd. 1-26, 1950-1987); Haag: Kluwer Academic Publishers (Bd. 27-37, 1988-2004); New York: Springer (Bd. 38-41, 2005-2012). [Abbreviated *Hua*].

________. 1976. [Hua VI]. *Die Krisis der europäischen Phänomenologie. Eine Einleitung in die phänomenologische Philosophie*. In *Husserliana,* Bd. VI, hrsg. von Walter Biemel. Den Haag: Martinus Nijhoff. [Also mentioned in the text as KW].

________. 1970. [Hua XII]. *Philosophie der Arithmetik. Mit ergänzenden Texten*. In *Husserliana,* Bd. XII, hrsg. von Lothar Eley. Den Haag: Martinus Nijhoff. [Also mentioned in the text as PA.]

________. 2003. *Philosophy of arithmetic*. *Psychological and logical investigations with supplementary texts from 1887-1901*. Translated by D. Willard. New York: Springer.

________. 1983. [Hua XXI]. *Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass*. In *Husserliana,* Bd. XXI, hrsg. von I. Strohmeyer. Den Haag: Martinus Nijhoff.

________. 1986. (Hua XXV) *Aufsätze und Vorträge. 1911-1921. Mit ergänzenden Texten.* In *Husserliana,* Bd. XXV, hrsg. von T. Nenon and H. Sepp. Den Haag: Martinus Nijhoff.

________. 1970a. *The Crisis of the European Sciences and Transcendental Phenomenology. An introduction to Phenomenological Philosophy.* Translated by D. Carr. Evanston: Northwestern. UG

________. 1970b. "The Origins of Geometry (1936)." In Husserl 1970a, 353-378. [Also mentioned in the text as UG].

_________. 1994. "On the Logic of Signs (Semiotic) (1891)." In *Early writings in the philosophy of logic and mathematics.* Translated by D. Willard, 20–51. New York: Springer. [Also mentioned in the text as LZ].

________. 2002. "Philosophy as a Rigorous Science." *New Yearbook for Phenomenology and Phenomenological Philosophy* 2: 249-295.

________. 2003. *Philosophy of Arithmetic. Psychological and Logical Investigations with Supplementary Texts from 1887-1901.* Translated by D. Willard. Dordrecht: Springer.

Majolino, C. 2010. "Structure de l'indice et équivocité du signe. A l'origine du partage. Anzeige/Ausdruck dans les 'Recherches logiques.'" *Histoire Épistémologie Langage* 32(2): 1-56.

________. 2012. "La fabrique du renvoi. Sur quelques thèmes sémiotiques chez le premier Husserl." In *Sémiotique et phénoméologie*, edited by C. Majolino, 93-114. Paraddigmi: Franco Angeli.

Mohanty, J.N. 1995. "The Development of Husserl's Thought." In *The Cambridge Companion to Husserl*, edited by B. Smith and D.W. Smith, 45-77. Cambridge: Cambridge University Press.

________. 2017. *Edmund Husserl's Freiburg Years: 1916-1938.* New Haven: Yale University Press.

Tillman, M. 2012. "Husserl's genetic philosophy of arithmetic: An alternative reading." *American Dialectic* 2(2): 141-190.

Willard, D. 1980. "Husserl on a logic that failed." *The Philosophical Review* 89(1): 46-64.

________. 1984. "Logic and the objectivity of knowledge: A study of Husserl's early philosophy." *Philosophical Review* 95 (4): 611-614.

________. 2003. "Introduction." *Philosophy of Arithmetic. Psychological and logical investigations with supplementary texts from 1887-1901, translated by* D. Willard, XIII-LXIV. New York: Springer.

Zuh, D. 2008. "How do categorial representations influence everyday intuition? On Husserl's early attempt to grasp the horizontal structure of consciousness." *Studia Universitatis Babes Bolyai Philosophia* 12: 49-62.

________. 2012. „Wogegen wandte sich Husserl 1891? Ein Beitrag zur neueren Rezeption des Verhältnisses von Husserl und Frege." *Husserl Studies* 28: 95-120.

Zahavi, D. 2002. *Husserl's Phenomenology*. Stanford: Stanford University Press.

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On Husserl's Thin Combination View: Structuralism, constructivism, and what not

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Abstract

After a brief outline of his method, the paper discusses Husserl's view of mathematics by means of two theses, namely the Incompleteness Claim and the Dependence Claim, with which Øystein Linnebo (2008) has characterized non-eliminative structuralism as opposed to the more traditional Platonist view of mathematics. According to the Incompleteness Claim, mathematical objects are incomplete in the sense that they have no non-structural properties. The Dependence Claim holds that the mathematical objects are dependent on each other and/or structure to which they belong. Husserl's view is shown to be a combination view: It is generally a species of non-eliminative structuralism, of which the two claims hold. However, in addition the Incompleteness Claim motivates constructivist approach to the mathematical objects. Moreover, due to the "thinness" of his "mathematics-first" approach, he is also open to the more traditionally Platonist approaches to mathematical objects.

Keywords: Husserl, structuralism, mathematical Platonism, constructivism, mathematical naturalism

1. Introduction, Husserl's method: radikale Besinnung

Husserl's philosophy of mathematics is primarily a method with which to approach mathematics. Hence, any attempt to explain his views about mathematics has to be preceded by an account of the used method. He explained it in the most mature way in the introduction to the *Formal and Transcendental Logic* (1929), where Husserl claims that the work is a result of *Besinnung* (for more detail, see Hartimo 2018a). He defines *Besinnung* as follows:

> "*Besinnung* signifies nothing but the attempt actually to produce the sense 'itself', …, it is the attempt to convert the 'intentive sense' … the sense 'vaguely floating before us' in our unclear aiming, into the fulfilled, the clear, sense, and thus to procure for it the evidence of its clear possibility" (Husserl 1969, 9).[1]

Assuming that rational activities are goal directed, *Besinnung* means clarifying the sense of the activity by explicating the typically implicit goals that guide the activity. Husserl assessed these goals by looking at the history of formal sciences, from ancients onwards, trying to capture the "point" of these sciences and how the mathematicians' goals are situated within the tradition of formal sciences. He sorted these activities into two kinds: to formal mathematics that has *non-contradictoriness* as its primary goal. The search for this is manifested in the search for definite manifolds. In logic, that is theory of science, the primary goal is *truth*.

According to Husserl, finding out what people, here the scientists, are aiming at requires entering in "a community of empathy with the scientists" [Mit den Wissenschaftlern in Einfühlungsgemeinschaft stehend oder tretend] (Husserl 1969, 9; Hua 17, 8). Husserl thus claims that *Formal and Transcendental Logic* (1929) is based on his empathetic engagement with the goals of the mathematicians and logicians around him (see Hartimo 2018b for the list of books he had read). Indeed, *Formal and Transcendental Logic* can be read as a commentary on foundations of mathematics and logic in the 1920s, and especially of David Hilbert's aims (Hartimo 2017).

The role of transcendental phenomenology is crucial for Husserl's method. Transcendental logic is ultimately about examining the presuppositions assumed in formal sciences and clarifying the evidence striven at in them. In other words, it is examination of mathematicians' aims, i.e., reflection of what exactly mathematicians are after when they seek non-contradictoriness or truth. Husserl's transcendental reflections showed that pure mathematics is guided by what he calls evidence of distinctness, that is, *Deutlichkeit*. In contrast, logic, striving at critically verified judgments, aims at having the objects themselves in the evidence of clarity, *Klarheit*. Husserl claims that the difference between the kinds of evidence made him realize that pure mathematics (what Husserl calls formal mathematics) has to be separated from logic that has a (different) notion of truth as its goal. In a way then, transcendental logic served to Husserl as a heuristic device for foundational research. However, its primary aim is to sort out conceptual confusions and making sure that the activities have a "point" that its practitioners have a clear awareness of. With the help of transcendental logic, the norms guiding formal mathematics and logic could be clarified, and if needed, revised.

In starting from examining mathematicians' activities and in the attempt of making sense of these activities, Husserl's approach is "mathematics first", and reminiscent of Penelope Maddy's naturalistic method, summarized to be to: "identify the goals and evaluate the methods by their relations to those goals" (Maddy 1997, 194). To be sure, Husserl incorporates into it transcendental phenomenological reflection, which is not of interest to Maddy. Nevertheless, his method is similarly "mathematics-first" and in it activities are criticized in so far as they do not serve the purposes they were supposed to, or their goals are unclear, conflated, or confused. The clarification of these goals leads to amelioration of the used concepts so that the renewed norms will be adopted habitually into the practices. Thus Husserl's "mathematics-first" approach is also revisionist: transcendental and historical study aims at finding out what the used concepts, norms, and values should be. However, this is not philosophy-first revisionism, in which, in words of Shapiro, "[t]he criticism does come from outside, from pre-conceived first principles" (Shapiro 2012, 13). It is criticism that arises from the consideration of the goals and values within the activities themselves. This may lead to embracing a plurality of normative goals, as I believe Husserl was led to. Despite of this, Husserl's picture is not relativist either: it aims at one unified picture within which all genuine practices have their proper roles. In it the confused goals and aims of the mathematicians are clarified and sorted out to form one sensible whole.

In what follows I will try to draw a picture of this whole as it seems to have looked to Husserl, when approached with the method characterized above. I will argue that Husserl sees mathematics mainly as a structuralist enterprise. I will argue that his structuralism differs from the more traditional Platonism and is Platonist in a "Lotzean" sense. Husserl also finds a need for more "material determination," which shows in his occasional constructivism. Finally, Husserl is open to a possibility of there being Platonistic, independent abstract objects, if mathematics develops in the way that commits mathematicians' to their existence. Thus, his view can be characterized to be a combination view, a combination of structuralism, constructivism, and even Platonism – all considered "thinly," as views to which mathematicians are committed, rather than as philosophical postulations about what there is.

2. Husserl's non-eliminative structuralism vs. Platonism: Dependence and Incompleteness

The role of *Besinnung* in Husserl's methodology makes his views contextual and "mathematics first". It led Husserl to a belief that mathematics is ultimately about striving for "definite manifolds", domains of categorical theories that should also be syntactically complete – something to which he still refers to in FTL (Hua 17, §31). Husserl's structuralism is particularly clear in the following passage from the *Prolegomena* (1900):

> "The *objective correlate* of the concept of a possible theory, definite only in respect of form, is *the concept of a possible field of knowledge over which a theory of this form will preside.* Such a field is, however, known in mathematical circles as a *manifold*. It is accordingly a field which is uniquely and solely determined by falling under a theory of such a form, whose objects are such as to permit of *certain* associations which fall under certain basic laws of this or that *determinate* form (here the only determining feature). The objects remain quite indefinite as regards their matter, to indicate which the mathematician prefers to speak of them as 'thought-objects'. They are not determined directly as individual or specific singulars, nor indirectly by way of their material species or genera, but solely by the form of the connections attributed to them. These laws then, as they determine a *field* and its *form*, likewise determine the theory to be

> constructed, or more correctly, the theory's form. In the theory of manifolds, e.g. '+' is not the sign for numerical addition, but for any connection for which laws of the form $a + b = b + a$ etc., hold. The manifold is determined by the fact that its thought-objects permit of these 'operations' (and of others whose compatibility with these can be shown *a priori*)." (Husserl 1970, 156; Hua 18, §70)

A formally definite manifold has a form. In terms defined by Stewart Shapiro this form is a *structure*: "A structure is the abstract form of a system, highlighting the interrelationships among the objects, and ignoring any features of them that do not affect how they relate to other objects in the system" (1997, 74). The objects, the pure positions in the structure, are abstract "thought-objects." They are determined "solely by the form of the connections attributed to them". They comprise what Husserl calls a '*manifold'*, and formal mathematics is about such manifolds and their relationships to each other. Husserl seems to suggest that the formally definite manifolds are domains of categorical theories, i.e., theories for which any two realizations are isomorphic with each other. For Husserl, the "thought-objects" are bona fide objects, even though they are only "formally determined." (To anticipate what is to come later, Husserl seems to have two notions of definiteness in mind: one merely formal, and the other more "material.") Husserl's discussion of mathematical objects by means of structures does not aim at eliminating them, but at demarcating a legitimate domain of formal objects. Husserl's structuralism is thus a species of non-eliminative structuralism (cf. Parsons 2008, 52).

On Husserl's view, the mathematicians are thus committed to the existence of abstract objects in so far as they are guided by the notion of definite manifold (HUA 17 §31). To determine how his view relates to Platonism about mathematics, it is very useful to consider two claims, termed Incompleteness Claim and Dependence Claim, with which Øystein Linnebo (2008) has characterized mathematical (non-eliminative) structuralism. The Incompleteness Claim holds that mathematical objects are incomplete in the sense that they have no non-structural properties. According to the Dependence Claim the mathematical objects are dependent on each other and/or structure to which they belong. The more traditional

Platonists differ from structuralists in ascribing richer and more independent nature to the mathematical objects.

On Husserl's *Prolegomena* formulation given above, both of these claims hold. The mathematical objects are "quite indefinite as regards to their matter" and "They are not determined directly as individual or specific singulars, nor indirectly by way of their material species or genera, but solely by the form of the connections attributed to them" (Incompleteness), and they are determined "solely by the form of the connections attributed to them" (Dependence). Husserl further explains that this approach banishes "all metaphysical fog and all mysticism from the mathematical investigations in question" (Huserl 1970, 157 [§70]). Husserl wrote this remark when he was in Halle with Georg Cantor as his colleague, so the suspicion is that the remark is directed at Cantor. In his *Grundlagen*, Cantor explicitly suggested that his definition of a manifold or a set captures something akin to the Platonic idea (Cantor 1996, 916). Whether Husserl thinks of Cantor or not, consistently with his remark about "metaphysical fog" (*Prolegomen*a, §70), in the [*Logical*] *Investigations* (II, § 7), Husserl is critical of Platonic realism, i.e., "the metaphysical hypostatization of the universal, the assumption that the Species really exists externally to thought" (Husserl 1970, 248). On this formulation universals are given richer nature and independence, which is ruled out with the two structuralist claims.

However, to Brentano Husserl has conceded that already the *Prolegomena* had been influenced by Lotze's interpretation of Plato (BW 1, 39). In his attempt to rewrite the introduction to the 1913 edition of the *Logical Investigations* Husserl credits Lotze's discussion of Plato for his development towards anti-psychological idea of logic, calling Lotze's interpretation genial:

> "The fully conscious and radical turn and the related „Platonism" I owe to the study of Lotze's Logik. As little as Lotze himself could overcome contradictions and psychologism, as much his genial interpretation of Platonic ideas helped me and my further studies. Lotze's discussion of truths in themselves suggested to me the thought to place all mathematics and a good part of traditional logic into the realm of ideality." (Hua 20/1, 297)

For Lotze's Plato, the ideas do not exist as things do, but they possess validity in virtue of the relations between them.[2] On Lotze's view the Platonic ideas are thus dependent on other ideas and the structure they are a part of. Husserl's view could thus be said to be Platonist in Lotze's sense, that is, within the limits of the two structuralist theses.

In sum, for Husserl the structures, i.e., the unique formal domains, form a clearly circumscribed idea of the "essential content of logic" (Hua 18, §3). In so doing, they banish "the metaphysical fog" out of his Platonism in substituting modern mathematics in place of the doctrine of ideas in Lotze's Plato. In mathematics, the structures, but nothing else, exist "in themselves". Stewart Shapiro calls this kind of structuralism *ante rem* structuralism in accordance to the traditional distinction between *ante rem* and *in re* theories of universals.[3]

3. Structuralism and its thinness in Husserl's later works

Husserl's *ante rem* structuralism can be found more or less unchanged in Husserl's *Ideen I* (1913). In it, the notion of definite manifolds is presented as an ideal norm for scientific rationality (Hua I, §72). Husserl writes, for example, that

> "the closer an experiential science comes to the 'rational' level, the level of 'exact,' of nomological science - ….- the greater will become the scope and power of its cognitive-practical performance." (Husserl 1982, 19; Hua 3/1, §9)

The definite manifolds provide the empty forms of any region whatever. Husserl defines this as formal ontology that thus contains the forms of all ontologies "and prescribes for material ontologies *a formal structure common to them all*" (Hua 3/1, §10). Husserl establishes the term "essence" to refer to mathematical objects "themselves":

> "One occasionally reads in a treatise that the series of cardinal numbers is a series of concepts and then, a little further on, that concepts are products of thinking. At first cardinal numbers themselves, the essences, were thus designated as concepts. But are not cardinal numbers, we ask, what they are regardless of whether we 'form' or do not form them?" (Husserl 1982, 42)

These essences still conform to categorical axiomatic theories, so that, considered purely formally, there are mere essence-forms (i.e., the thought-objects of the *Prolegomena*) that fit all possible essences. Husserl also points out that these exact essences should be regarded as Kantian ideas (Husserl 1982, 97; Hua 3/1, §83). As such they have a normative, guiding role for our perception and also for concept formation.

In the *Formal and Transcendental Logic* (1929) Husserl explains that the concept of the definite manifold "has continually guided mathematics from within" (Hua 17, §31; Husserl 1969, 95); it is thus a typical example of goal-senses he thinks guides mathematics and is revealed to him by *Besinnung*. Husserl cites his *Prolegomena* discussion of the definite manifolds and terms pure positions, i.e., thought-objects (*Prolegomena*), essence-forms (*Ideas I*), as 'pure modes of anything-whatever' (Hua 17, §24; Husserl 1969, 78). In his transcendental examination of the givenness of the objects (whether abstract or real), Husserl describes them as somethings-themselves that are transcendent (§61). In other words, even though the mathematical objects are dependent, they nevertheless are given as *bona fide* objects, transcendent even though ideal. He also refers to the axiomatic ideal as a regulative ideal norm "beneath actually experienced Nature" (Husserl 1969, 292; Hua 17, 257).

But then in the *Crisis*, written in the 1930s Husserl suddenly renounces such realism as a misleading view:

> "Mathematics and mathematical science, as a garb of ideas, or the garb of symbols of the symbolic mathematical theories, encompasses everything which, for scientists and the educated generally, represents the life-world, dresses it up as "objectively actual and true" nature. It is through the garb of ideas that we take for true being what is actually a method—a method which is designed for the purpose of progressively improving, in infinitum, through "scientific" predictions, those rough predictions which are the only ones originally possible within the sphere of what is actually experienced and experienceable in the lifeworld. It is because of the disguise of ideas that the true meaning of the method, the formulae, the "theories," remained unintelligible and, in the naive formation of the method, was never understood." (Husserl 1976, §9h).

Until the *Crisis*, Husserl's view of mathematics is a species of *non-eliminative structuralism*, in particular *ante rem* structuralism, which however explicitly turns into a normative ideal, or an ideal which reason places into nature. It is the "garb of ideas that we take for true being" as he puts it in the above quote. But this is an illusion, in fact, the "substructed" structure is only a method. Husserl is now instrumentalist about the structures.

This turn of the events can be explained in many ways, for example, psychologically as a result of a general crisis Husserl went through in the early 1930s. A philosophically more satisfactory explanation highlights the importance of Husserl's newly acquired awareness of the Löwenheim-Skolem Theorem. Husserl's primary source to developments in the foundations of mathematics in the 1930s, around the time he was writing the *Crisis* was Friedrich Waismann's (1896-1959) *Einführung in das mathematische Denken: die Begriffsbildung der modernen Mathematik* (1936).[4] In this work Waismann states about unique structures of natural (and later similarly also of real numbers) that:

> "It is now extremely significant that Skolem has thwarted every hope of this kind. That is, he proved a general proposition which says that it is impossible to characterize the number series by finitely many axioms. For, every statement which is valid in the arithmetic of natural numbers is also valid for structures of another kind, so that it is impossible to distinguish the number series by any inner properties from sequences of another kind." (Waismann 1936, 84; 1966, 105)

On Waismann's understanding Löwenheim-Skolem Theorems show that there are no unique structures such as the structure of natural numbers or the structure of reals. It makes Husserl's belief in formal structures underlying his view of formal ontology a wild goose chase. Either Husserl should have shown the well-known results false, or else he had to admit that his own earlier beliefs were illusions. Husserl chose the latter alternative. This development of Husserl's views from *ante re*m structuralism to instrumentalism about the structures show that his metaphysical commitments cannot be discussed independently of his *Besinnung*, that is, his understanding of the mathematicians' view about the nature of mathematics. Instead of taking it as a philosophy-first defense of a certain metaphysical position, Husserl's view is about mathematicians' view of the mathematical reality.

4. The incompleteness of mathematical objects and Husserl's constructivist leanings

In the above analyses, I have mainly assessed Husserl's view of mathematical objects in terms of the Dependence Claim, i.e., the claim that mathematical objects are dependent on each other or the structure to which they belong to. I will now move on to consider the Incompleteness of mathematical objects. According to this claim, the structuralist objects are pure positions of structures, and thus they have no identity or features outside of a structure. Husserl puts this kind of claim forward in the *Prolegomena* in the passage cited already once*:*

> "The objects remain quite indefinite as regards their matter, to indicate which the mathematician prefers to speak of them as 'thought-objects'. They are not determined directly as individual or specific singulars, nor indirectly by way of their material species or genera, but solely by the *form* of the connections attributed to them" (Husserl 1970, 156; Hua 18, §70).

The incompleteness of structuralist objects has been criticized in the literature. Probably the best-known criticism is due to Paul Benacerraf (Benacerraf 1964, 291). The argument is that it must be possible to individuate the abstract objects independently of the role they play in the structure. The structuralist objects are "incomplete," because they can only be ascribed certain properties defined by the structure. This indeterminateness poses problems for example for the applications of mathematics (Parsons 2008, 106, 151).

Husserl's view with regard to the Incompleteness Claim is extremely interesting. He acknowledges that the thought-objects are incomplete, and have no more nature than what the structure ascribes to them. But to him, this motivates using constructive means to further determine these otherwise incomplete objects. Linnebo would call this a "compromise view". He interestingly takes Kant to represent such a view with his [Kant's] distinction between *totum analyticum* (totality prior to its constituents) and *totum syntheticum* (totality synthesized from its parts) (Linnebo 2008).

Indeed, the incompleteness or inauthenticity of the structuralist objects occupied mathematicians already in the 19^{th} century. In the 19^{th} century the problem was typically construed in terms of existence; it seemed that the structuralist "thought-objects" could not be thought as properly existing as such but their existence had to be established by some other means. Dedekind, too, established the existence of the simply infinite systems by correlating them with the realm of things that can be objects of his own thought (Dedekind 1996, 806-807).

Sometimes the worry about the actual existence of the structural objects was related to the worry about their applicability. Citing Frege's remark that "It is applicability alone that raises arithmetic from the rank of a game to that of a science. Applicability therefore belongs to it of necessity" (*Grundgesetze* II §91, cited by Parsons 2008, 73), Parsons (2008) points out that Frege and Russell seemed to have regarded this as an objection towards structuralists.

Husserl shared such concerns. Accordingly, in his Double Lecture he first discusses manifolds as structures, holding that such a domain is "a determinate, but formally defined, manifold" [in German "eine bestimmte, aber formal definierte Mannigfaltigkeit"]. The expression invites the thought that there could be also materially determined manifolds; and as we go further, it becomes clear that this indeed seems to have been Husserl's idea, and that the sought for "material determination" is related to computable constructibility. This is because the purely formal mathematics is difficult to apply. In the Double Lecture, Husserl writes for example that: "But the difficulties lie precisely in the relationship between formal mathematics and its employment in substantive mathematics or in the particular domains of knowledge" (Husserl 2003, 411 {92}). He then starts developing a more constructive method to determine the objects, with a view to determine the objects more individually (see Hartimo 2018c):

> "The essential point is the following: In the axiom system I define not only sentences which hold true for all members of the manifold in general. I therefore operate not only with general, indeterminate concepts of objects, but rather I also introduce individually designating concepts of objects – as it were proper names for objects (or species of objects) – and I axiomatically establish their existence" (Husserl 2003, 445 {116}).

He seems to have thought that for the sake of application the formal objects should have more "material" definiteness. To enable the use of the formal objects he correlated them, or named them, with determinate numbers, and then established the "term-re-writing" reductions to equalities. Thus he came to give the criteria of identity for different kinds of expressions of natural numbers (see Hartimo 2018c). Similarly motivated, in *Ideas I* Husserl adds to structuralist formal ontology material ontologies by means of eidetics, and in *Formal and Transcendental Logic* Husserl regarded the structuralist objects too abstract to have something to do with truth related to experience of objects 'themselves':

> "Each multiplicity defined by a system of axiom-forms presented them with the task of explicitly constructing the form of the corresponding deductive science itself; and the execution of the task involved precisely the same work of constructive deduction that is done in a concrete deductive science with concepts having material contents" (Husserl 1969, 98; Hua 17, §32).

In the end, for him, the proper objects of formal ontology have to be intuited, whether immediately or mediately. They must draw 'fullness' from the evidence of clarity (e.g., Husserl 1969, 203). For Husserl, this takes place in judgments about individuals. The evidence can be 'transferred' by the rules of the judgment theory to more complex judgments (I discuss this in more detail in Hartimo, forthcoming). These evident judgments determine the objects suitably for the needs of sciences and truth. Husserl thus thinks that formal mathematics should be thus constructively proven, and thus it should be ideally, not only formally, but also materially definite.

But in line of his "mathematics first" attitude, Husserl also writes that this kind of evidence is of no particular interest to mathematicians (Husserl 1969, 203), to whom *distinctness*, instead of *clarity* is of interest. (ibid., §52-53). The existence of abstract objects in Husserl's view can thus be either thin or thick, either as indeterminate thought-objects (in pure mathematics) or else as immediately or mediately evident objects (in logic). While the structuralist mathematical objects are distinctly given, Husserl clearly thinks that it would be desirable to bring them into evidence of clarity. For Husserl then, the Incompleteness Claim and the Dependence Claims do not hold of all mathematical objects; they do not hold of the constructed formal objects that are given in the evidence of clarity. Husserl often seems to imply that all structuralist formal objects could be constructively given. This is a belief that the development of mathematics showed false in the 1930s. Given his methodological, "mathematics-first" views, it should not be taken as a thesis about the nature of mathematics but rather as a belief about what he thought mathematicians of his time were thinking about mathematics, hence dependent on the stage of mathematical research.

5. Husserl and the iterative conception of sets

Linnebo (2008) discusses sets on the iterative conception as a counter-example to the structuralist Dependence Claim. Since sets are formed from their elements, the elements of the sets have to be "available" before they can be comprehended into a set. The elements of the sets are thus not dependent of the sets they are members of. The sets they will be members of may not even yet exist on iterative conception. In it sets are formed in stages so that on each stage the sets will be formed from the objects of the previous stage. This results in an open-ended hierarchy that can always be further extended (Linnebo 2008, 2013). The cumulative hierarchy motivates most of the axioms of set theory, rendering them in some sense evident (Boolos's phrase is that "there is a thought behind" it) (Linnebo 2017, 140), and I will argue next that this kind of evidence could have well be of interest to Husserl, too.

This brings us to a consideration about Husserl and the Dependence Claim, but this time possibly in favor of the more traditional Platonism. While in the case of constructive judgment theory the fullness is added to the background structure, set theory might suggest a Platonist departure from structuralism. Husserl may have known about iterative conception of sets developed by Zermelo.[5] Zermelo's iterative view of sets postulates a dynamic, open-ended sequence of bigger and bigger domains (models), uniquely characterized by the cardinality of their basis and a "boundary number" that is the least ordinal not in the model. In the hierarchy of models, the sets of one layer are 'grounded' [würzelnd] in the preceding layers, so that their elements are in the previous layers and serve as material for the following layers (Zermelo 1996, 1219).

There are some indications in Husserl's texts that might indicate awareness of Zermelo's cumulative hierarchy, or at least some similar phenomenon. In FTL, for example, in the context of discussing idealizations involved in analytics and, in particular, the fundamental form 'and so forth', Husserl discusses the reiterational 'infinity' that is presupposed in mathematics that "is the realm of infinite constructions, a realm of ideal existences, not only of 'infinite' senses but also of constructional infinities" (§74). And then he continues:

> "Obviously we have here a repetition of the problem concerning subjective constitutive origins: as the hidden method of constructions which is to be uncovered and reshaped as a norm, the method by which 'and so forth', in various senses, and infinities as categorial formations of a new sort become evident… Precisely this evidence, in all its particular formations, must now become a theme" (Husserl 1969, 189; Hua 17, §74)

Husserl refers to a new kind of evidence related to "constructional infinities." The new task of transcendental phenomenology would then be to examine the notion of evidence involved.

A possible reference to the cumulative hierarchy can be found in Husserl's correspondence with Dietrich Mahnke. In a letter to Mahnke in 1933 Husserl discusses his view of the infiniteness of transcendental subjectivity and infinity or endlessness of phenomenology, which studies the infinity of being within the totality [Alleinheit]. He calls the transcendental subjectivity as 'constitutive infinity' and then compares it to the infiniteness of the structural system of the world:

> "The infiniteness of the world, the infiniteness of teleology, that, as the world that prevails for the infinity of monads, recedes and becomes, in evermore new and changing ways, but yet remains as one identical world, - that is not a one-dimensional or multi-dimensional infinity, it is an infinite system of rays of infinity, I think with an infinity of levels, that each has its axiomatics." (BW3, 498)[6]

In fact, Husserl's use of the word "Strahlensystem von Unendlichkeiten" is curious. The reference to an infinity of levels, Stufen, could be informed by Zermelo's cumulative hierarchy introduced in (Zermelo 1996) or it could refer to

Gödel's incompleteness results and the ensuing infinity of axiomatizations where the Gödel sentence is decided by adopting a hierarchy of ever stronger theories in which the Gödel sentence of the previous level can be decided. The topological wording is curious and suggests something related to Riemann. In any case, it indicates Husserl's general attitude towards mathematics at the time to be one that endorses a kind of inexhaustibility or incompleteability: one cannot exhaust the reality with any finite set of axioms. In Zermelo's hierarchy individual domains may be uniquely characterized, which thus suggest structuralist existence for the objects defined by them and relative to them. But the characterization of the infinite sequence of them makes existence claims about, e.g., boundary numbers. So, the question arises: "does this make Husserl more traditionally Platonist, after all?" Indeed, for Husserl, this seems to be a question about the evidence related to "and so forth," to which he refers to already in the FTL. In any case, whether the cumulative hierarchy can be understood structurally or not, Husserl's methodological considerations imply that he should think that the task of transcendental logic is to examine any kind of evidence that surfaces in mathematics, hence certainly the one given by cumulative hierarchy.

6. Conclusion

In sum, while Husserl's (to be more specific, Husserl's view of mathematicians') view of mathematics is mostly *ante rem* structuralist, a closer examination of the Dependence and Incompleteness Claims shows that his *ante rem* structuralism does not hold of all mathematical objects. It holds of algebraic structures, hence much of what is studied in mathematics. Husserl would also like to bring as much of it as possible into clarity by means of computable constructions. Thus Husserl's structuralism has preferably "thick constructive patches," but it is also open to there being Platonist objects beyond it.

These different approaches ultimately differ in their normative goals, in particular, in the kinds of evidence they are after. Depending on the kind of evidence, different kinds of methods of proofs and definitions are adopted in different areas.

Husserl's transcendental logic demands consideration of these various evidences, purifying them [his terms], and then, if found worthy, adopting them as new norms guiding mathematical practices. In the *Formal and Transcendental Logic* these goals were divided into two main kinds of evidences: clarity and distinctness. In the early 20th century the distinction between these two kinds of evidences provided an important new insight to the developing modern structural mathematics as opposed to the more constructive or applied approaches. Nothing in Husserl's approach precludes new evidences and new goals to surface in mathematics. Consideration of all of them and how they relate to each other gives a unified picture of pluralistically given mathematics and should help in understanding its place in our conception of the world and our lives.

NOTES

1 "*Besinnung* besagt nichts anderes als Versuch der wirklichen Herstellung des Sinnes ‚selbst', der in der bloßen Meinung gemeinter, vorausgesetzter ist; oder den Versuch, den ‚intendierenden Sinn', ... den im unklaren Abzielen ‚vage vorschwebenden' in den erfüllten Sinn, den klare überzuführen, ihm also die Evidenz der klaren Möglichkeit zu verschaffen" (Hua 17, 8).

2 Lotze concludes his discussion of Plato's *Ideenlehre* as follows "Thus we readily understand the significance of Plato's endeavour to bind together the predicates which are found in the things of the external world in continual change, into a determinate and articulated whole, and how he saw in this world of Ideas the true beginnings of certain knowledge; for the eternal relations which subsist between different Ideas, and through which some are capable of association with each other and others exclude each other, form at all events the limits within which what is to be *possible* in experience falls; the further question what is real in it, and how things manage to have Ideas for *their* predicates, appeared to Plato not to be the primary, and was for the time reserved."(Lotze 1884, §315). After having established the unchangeable validity of the world of Ideas, the next task for Plato "was to investigate the universal laws which govern its structure, through which alone, in an Ideal world as elsewhere, the individual elements can be bound together into a whole" (Lotze 1884, §321). Thus the Dependence Claim is true of Lotze's Plato.

3 Shapiro's ante rem structuralism is a species of Parsons' non-eliminative structuralism, but not *vice versa*. Parsons' structures are not defined by Dedekind abstraction but by taking the language of mathematics as the background structure. On Parsons' view, the most elementary way of describing a mathematical structure is by introducing a one-place predicate

true of an object, with other predicates and functors true of this same object. The uniqueness is not central to him but the intended interpretations of the language of mathematics, which quantify over formal objects that are then, in a Quinean manner, thought to exist (Parsons 2004; Parsons 2008, esp. 111-115). In contrast to Shapiro, in the dilemma between first order logic and determinate ontology, Quine and Parsons opt for the first option on the expense of the latter.

[4] In the end of *Mathematische Existenz*, Oskar Becker discusses Löwenheim-Skolem theorem. However, Husserl probably did not read it until in Mars 1937. According to Husserl-Chronik, this is when "H. hat grössere Abschnitte gelesen (insbesondere zum ersten Mal auch [?] die zweite Hälfte) von Oskar Becker, Mathematische Existenz, 1927." (Schuhmann 1977, 484)

[5] The theory was developed by Zermelo in Freiburg, where Husserl, too, lived at the time (Zermelo 1996). It seems likely that Zermelo's theory is at least indirectly influenced by Husserl. Husserl's assistant of the time, Oskar Becker, apparently had lectured in Zermelo's seminar on problems in the theory of transfinite ordinals that same year (Mancosu 2010, 281, 539). Becker in turn, in his discussion of transfinite "Strukturkomplikationen" of the consciousness refers to Husserl's *Ideas I* (§100), where Husserl discusses hierarchical structures of intentionalities, such as remembering in remembering and so forth. Husserl thinks that they build up a hierarchy. According to him, "[a]ll the types of objectivation-modifications previously dealt with are always accessible for always newer hierarchical formations of such a kind that the intentionalities in the noesis and noema are hierarchically built up on one another or, rather, in a unique way, encased in one another". The intentional acts and the objectifications of them allow for a hierarchy of levels of them. Husserl even assigns indices for these levels. "To every noematic level there belongs a characteristic appropriate to that level as a kind of index with which each thing characterized manifests itself as belonging to its level… For indeed to every level belong possible reflections at that level, so that, e.g., with respect to remembered things at the second level of remembering, [there are] reflections on perceivings of just these things belonging to the same level (thus presentiated at the second level). Furthermore: each noematic level is an 'objectivation' 'of' the data of the following [level]." (Hua 3/1, §101). Acknowledging that Husserl is not motivated to iterate intentional acts infinitely many times, Becker suggests using such hierarchy to clarify Cantor's view of transfinite numbers. Using intuitionist terminology, he then characterizes Cantor's transfinite numbers as a "werdende Folge, deren 'zukunft' nicht voraussehbar ißt" (Becker 1973, 112); as a becoming succession that has a future that is not foreseeable. Becker is explicit about the potential character of the hierarchy. (Similar hierarchies were at the time also proposed by many, e.g., by Russell in his type theory and Weyl's construction in The Continuum (1917) to combat paradoxes of set theory).

[6] „Die Unendlichkeit der Welt, die Unendlichkeit der Teleologie, die in der Unendlichkeit von Monaden waltend Welt werden ließ und fortwerden, immerfort neu und anders werden lässt, und doch als identische Welt – das ist nicht eine einlinige oder mehrlinige Unendlichkeit, es ist ein unendliches

Strahlensystem von Unendlichkeiten, ich denken mit einer Unendlichkeit von Stufen, deren jede ihre Axiomatik hat“ (BW3, 498).

REFERENCES

The edition of Husserl's critical writings, the *Husserliana* vol. 1-42 (The Hague: Kluwer, 1950ff.), is cited in text as *Hua* 1-42. The letters published in *Husserliana Dokumente* vol. 3/1-10 (The Hague: Kluwer, 1994) are cited as *BW* 1-10.

Becker, Oskar. 1973. *Mathematische Existenz, Untersuchungen zur Logik und Ontologie mathematischer Phänomene*. 2., unveränderte Auflage. Tübingen, Max Niemeyer Verlag. [First published 1927].

Benacerraf, Paul. 1964. “What numbers could not be” In *Philosophy of mathematics, Selected readings*, by Paul Benacerraf and Putnam Hilary, 272-294. Cambridge, New York, Melbourne: Cambridge University Press.

Cantor, Georg. 1996. “Foundations of a general theory of manifolds: a mathematico-philosophical investigation into the theory of the infinite.” In *From Kant to Hilbert, A Source Book in the Foundations of Mathematics*, edited by William Ewald, 879-920. Oxford: Clarendon Press. First published 1883 by Teubner, Leipzig.

Dedekind, Richard. 1996. “Was sind und was sollen die Zahlen?“. In *From Kant to Hilbert, A Source Book in the Foundations of Mathematics*, edited by William Ewald, 879-920. Oxford: Clarendon. First published in 1888 by Vieweg, Braunschweig.

Hartimo, Mirja. 2017. “Husserl and Hilbert.” *Essays on Husserl’s Logic and Philosophy of Mathematics*, edited by Stefania Centrone, 245-263. Springer: Synthese Library.

________. 2018a. “Radical *Besinnung* in *Formale und transzendentale Logik* (1929)”, *Husserl Studies* 34: 247-266. DOI: 10.1007/s10743-018-9228-5

________. 2018b. “Husserl’s Scientific Context 1917-1938: A look into Husserl’s private library.” *The New Yearbook for*

Phenomenology and Phenomenological Philosophy XVI: 317-336.

________. 2018c. "Husserl on completeness, definitely." *Synthese* 195: 1509–1527. DOI: 10.1007/s11229-016-1278-7.

________. 2019. "Husserl on '*Besinnung*' and formal ontology." *Metametaphysics and the Sciences: Historical and Philosophical Perspectives*, edited by Frode Kjosavik and Camilla Serck-Hanssen, 200-215. London: Routledge.

Husserl, Edmund. 1950-2012. *Husserliana: Edmund Husserl – Gesammelte Werke*. Bde. 1-41. Haag: Martinus Nijhoff (Bd. 1-26, 1950-1987); Haag: Kluwer Academic Publishers (Bd. 27-37, 1988-2004); New York: Springer (Bd. 38-41, 2005-2012). [Abbreviated *Hua*].

________. 1969. *Formal and Transcendental logic*. Translated by Dorion Cairns. The Hague: Martinus Nijhoff. [Also mentioned in text as FTL].

________. 1970. *Logical Investigations*. Volume I. Translated by J. N. Findlay. Edited by Dermot Moran. New York: Routledge & Kegan Paul.

________. 1974. [Hua 17]. *Formale und transzendentale Logik. Versuch einer Kritik der logischen Vernunft*. In *Husserliana (1929)*, Bd. XVII, hrsg. von Paul Janssen. Haag: Martinus Nijhoff. [Also mentioned in text as FTL].

________. 1975. [Hua 18]. *Logische Untersuchungen. Erster Band. Prolegomena zur reinen Logik*. In *Husserliana*, Bd. XVIII, hrsg. von E. Holenstein. Haag: Martinus Nijhoff. [Also mentioned in the text as *Prolegomena*].

________. 1976. [*Hua 3/1*]. *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Erstes Buch: Allgemeine Einführungin die reine Phänomenologie.* 1. Halbband: Text der 1.-3. Auflage - Nachdruck. In *Husserliana,* Bd. III.1, hrsg. von Karl Schuhmann. Haag: Martinus Nijhoff. [Also mentioned in the text as *Ideen I*].

________. 1982. *Ideas pertaining to a pure phenomenology and to a phenomenological philosophy: First Book. General Introduction to a Pure Phenomenology*. Translated by Fred

Kersten. Dordrecht, Boston & London: Kluwer. [Also mentioned in the text as *Ideas I*].

________. 1994. *Briefwechsel* (I–X). In Husserliana: Edmund Husserl Dokumente, Bd. 3/1-10, hsrg. von K. Schuhmann & E. Schuhmann. Dordrecht: Kluwer Academic Publishers. [Abbreviated *BW*].

________. 2003. "Essay III, Double Lecture: On the transition through the impossible ("imaginary") and the completeness of an axiom system." In *Philosophy of Arithmetic, Psychological and Logical Investigations with Supplementary Texts from 1887-1901*, translated by Dallas Willard, 409-473. Dordrecht: Kluwer.

Linnebo, Øystein. 2008. "Structuralism and the Notion of Dependence," *Philosophical Quarterly* 58: 59-79.

________. 2017. *Philosophy of Mathematics. Princeton Foundations of Contemporary Philosophy*. Princeton and Oxford: Princeton University Press.

Lotze, Hermann. 1884. *Logic, in three books, of thought, of investigation, and of knowledge*. Translated by Bernard Bosanquet. Oxford: Clarendon Press.

Maddy, Penelope. 1997. *Naturalism in Mathematics*. Oxford: Oxford University Press.

________. 2011. *Defending the Axioms*. Oxford: Oxford University Press.

Mancosu, Paolo. 2010. *The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic 1900-1940.* Oxford: Oxford University Press.

Parsons, Charles. 2004. "Structuralism and Metaphysics." *The Philosophical Quarterly* 54 (214): 56-77.

Parsons, Charles. 2008. *Mathematical Thought and its Objects*. Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi: Cambridge University Press.

Shapiro, Stewart. 1996 [1985]. "Second-Order Languages and Mathematical Practice". In *The Limits of Logic: Higher-Order Logic and the Löwenheim-Skolem Theorem*, edited by Stewart Shapiro, 509-527. Ashgate Publishing.

________. 1997. *Philosophy of Mathematics: Structure and Ontology*. New York: Oxford University Press.

Schuhmann, Karl. 1977. *Husserl-Chronik, Denk- und Lebensweg Edmund Husserls*. The Hague: Martinus Nijhoff.

Zermelo, Ernst. 1996. "On Boundary Numbers and Domains of Sets: New Investigations in the Foundations of Set Theory." Translated by Michael Hallett. In *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, edited by William Ewald, 1219-1233. First published 1930 by *Mathematische Annalen* (65).

Waismann, Friedrich. 1936. *Einführung in das mathematische Denken: die Begriffsbildung der modernen Mathematik*. Mit einem Vorwort von Karl Menger. Wien: Gerold.

________. 1966. *Introduction to Mathematical Thinking: The Formation of Concepts in Modern Mathematics*. Translated by Theodore J. Benac. Harper & Row, New York.

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Husserl on Sets and the Causes of the Set-theoretical Paradoxes

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Abstract

Few realize that Edmund Husserl theorized about sets and the causes of the set-theoretical paradoxes. Interpreted here are his statements that: 1) the paradoxes show that his contemporaries did not yet have the real and genuine concept of set needed; 2) that if one is clear and distinct with respect to meaning, one readily sees the contradiction involved in the set-theoretical paradoxes; 3) that the solution to them would lie in demonstrating the shift in meaning that makes it that one is not immediately aware of the contradiction and that, once it is perceived, one cannot indicate wherein it lies. I study these convictions in connection with Frege's and Russell's ideas about sets and the conclusions that they came to regarding the causes of the paradoxes derivable within Frege's system.

Keywords: Husserl, Frege, Russell, Set Theory, Set-theoretical Paradoxes, Meaning

1. Introduction

The full story of set theory's role in shaping modern logic and in redrawing the boundaries between mathematics and philosophy in both the analytical and the phenomenological traditions is yet to be told and its full implications drawn. In particular, the fact that Edmund Husserl thought and taught about it well before the logic shaped by *Principia Mathematica* (Russell & Whitehead 1927) came to play a key role in laying the foundations for analytic philosophy has barely been investigated.

Husserl searched for clarity about the meaning of sets all throughout his career. For example, in his late work *Formal and Transcendental Logic* (Husserl 1978), he described his first book, *Philosophy of Arithmetic* (Husserl 2003a), as an initial attempt on his part "to obtain clarity regarding the original genuine meaning of the fundamental concepts of the theory of sets and cardinal numbers" (Husserl 1978, §27a). He was well-versed in the set theories being created by his contemporaries and lucid about their logical, epistemological and ontological implications. As a colleague and best friend of Georg Cantor, the creator of set theory, during the last 14 years of the 19th century, he had a front row seat at the creation of set theory and the discovery of the paradoxes of the transfinite. So he was sort of a victim *avant la lettre* of the crisis in foundations that broke out once Bertrand Russell publicized the famous contradiction about the set of all sets that are not members of themselves that he discovered while studying Cantor's theories (Hill & Rosado Haddock). Then, appointed to the University of Göttingen in 1901, Husserl engaged in exchanges with mathematicians who knew the set-theoretical paradoxes before Russell did. In November 1903, David Hilbert wrote to Gottlob Frege that Russell's antinomy was already known in Göttingen, that Ernst Zermelo had found it three or four years earlier after having learned of other, even more convincing, contradictions from Hilbert himself as many as four or five years before (Frege 1980a, 51; Peckhaus & Kahle 2000/2001). Concrete evidence corroborating Zermelo's finding of the paradox is found in a note he sent to Husserl on in April 1902. (Husserl 1994, 442; Rang and Thomas 1981)

Guillermo Rosado Haddock drew attention to Husserl's notes on set theory (Husserl, Ms A 1 35) in his 1973 doctoral thesis (Rosado Haddock 1973), but this did not arouse any excitement. Here I seek to provide the conceptual framework for interpreting Husserl's statements in those notes that: 1) the set-theoretical paradoxes show that his contemporaries did not yet have the real and genuine concept of set needed; 2) that if one is clear and distinct with respect to meaning, one readily sees the contradiction involved in the set-theoretical paradoxes; and 3) that the solution to them would lie in demonstrating the shift of meaning that makes it that one is not immediately aware of the contradiction and that once one perceives it one cannot indicate wherein it lies. To do this I present those of

Husserl's ideas that I think are necessary for interpreting those statements. To this end I juxtapose issues involved in Frege's use of the extensions that lead to the contradiction about the set of all sets that are not members of themselves within his system and the conclusions that he and Russell came to regarding the causes of that contradiction. Finally I interpret Husserl's statements in light of what I have said. My remarks are not about mathematics per se, but about set theory and the foundations of analytic philosophy and phenomenology.

2. Logical Laws and Laws of Meaning

By the time he wrote the *Logical Investigations* in the late 1890s, Husserl considered it very important to distinguish between logical laws and laws of meaning. According to him, logical laws serve to guard against formal or analytical contradiction, what he called *Widersinn*. What violates logical laws, what is contradictory (*widersinnig*), genuinely has a coherent meaning and can be determined to be true or false. However, though meaning is there, no existing object can correspond to the meaning. As examples of contradictions (*Widersinnigkeiten)*, he gave expressions like 'wooden iron'. 'round square', 'all squares have five corners' that have meaning, but no object. No thing or fact such as is described by such expressions exists or can exist (Husserl 1900/01, IV). In his notes on set theory, Husserl studied the sentences, 'The present emperor of France is blond' and 'The present emperor of France is not blond'. 'The present emperor is blond' implies that France presently has a blond emperor, while she has no emperor at all. He contended that the sentence is not valid, because it is objectless either in actual fact or owing to a contradiction.

Along these same lines, he maintained that to the objection that there is no set that contains itself as an element, one need merely respond that that is *widersinnig*. (Husserl, Ms A 1 35)

In contrast to logical laws, laws of meaning serve to distinguish meaningfulness from meaninglessness, sense from nonsense, by providing pure logic with possible coherent, meaningful meaning forms whose formal truth or falsehood and reference to objects, logical laws determine. Meanings, Husserl repeated over and over, are governed by *a priori* laws that regulate how they may be combined, fit together and constitute meaningful, coherent meanings instead of chaotic nonsense (*Unsinn*). The impossibility of combining meanings in certain ways is not subjective, but objective, ideal and grounded in the pure essence of meaning.

Husserl believed that the primitive, essential distinction between dependent and independent meanings formed the necessary basis for discovering the essential categories of meaning in which were grounded a number of essential laws of meaning. He, like Frege before him and Russell after him, stressed that fundamental differences between dependent meanings and independent meanings lying concealed behind inconspicuous grammatical distinctions are inviolable because they are "founded deep in the nature of things".

Husserl studied how one may be led astray by the fact that meanings of each category may figure in the subject position otherwise reserved for substantival meanings. The words are definitely in the subject position, but their meanings are not the same as they normally are. Not just any meaning can be substituted for *S* or for *p*. Once meaning categories are violated, the coherency of the meaning is lost. The underlining on Husserl's copies of Frege's "Concept and Object" and "Function and Concept" shows Husserl's fundamental agreement with Frege on this matter. (Hill & da Silva 2013)

3. A Natural Order in Formal Logic

In addition, Husserl found a natural order in formal logic and broadened its domain to include two levels above traditional Aristotelian logic, which he saw as being but a small area of pure logic that needed to be distinguished and segregated from the extended sphere of pure logic that includes the mathematical disciplines and is immense in range and wealth of content in comparison. He considered his understanding of the structure of the world of pure logic to be a radical clarification of the relationship between formal logic and formal mathematics and that it led to a definitive clarification of the sense of pure formal mathematics as a pure analytics of non-contradiction. (Husserl 2008, §§18-19; Husserl 1978, 11; Hill 2013; Hill 2015)

On the first level of Husserl's hierarchy, the traditional Aristotelian logic of subject and predicate propositions and states of affairs deals with what is stated about objects in general from a possible perspective. In the disciplines of the two higher levels, it is no longer a question of objects as such about which one might predicate something, but of investigating what is valid for higher-order objective constructions that are determined in purely formal terms and deal with objects in indeterminate, general ways. (Husserl 2008, 18c)

On the second level, Husserl located the basic concepts of mathematics, the theory of cardinal numbers, the theory of ordinals, set theory, mathematical physics, formal pure logic, pure geometry, geometry as *a priori* theory of space, the axioms of geometry as a theory of the essences of shapes, of spatial objects, but also the pure theory of meaning and being, *a priori* real ontology of any kind, ontology of nature, ontology of minds, natural scientific ontology, the sciences of value, pure ethics, the logic of morality, the ontology of ethical personalities, axiology or the pure logic of values, pure esthetics, ontology of values, the logic of the ideal state or the ideal world government as a system of cooperating ideal nation states, or the science of the ideal state, the ideal of a valuable existence, objective axioms (relating to *a priori* propositions as truth for objects, as something belonging in the objective science of these objects, or of objects in general in formal universality, essence-propositions about objects insofar as they are objective truths and as truths have their place in a truth-system in general. (Husserl 2008, §§18-19, 434-35; Husserl 1996, Chapter 11)

The third level is that of his theory of manifolds (Husserl 1900/01, *Prolegomena*, §§69-70; Husserl 1962, §§71-72; Husserl 1978, §33), which we shall not be concerned with here. The key thing to realize at this point is that, according to Husserl's theory, sets and numbers function in an entirely different way on the first level than in set theory and arithmetic, which Husserl put on the second level.

In the case of numbers, in expressions of the first level, for example, '2 men', '3 houses', numbers occur as form, but not as independent objects about which something is predicated. In that case, the sentence "Jupiter has four moons", to use Frege's example in the *Foundations of Arithmetic* (Frege 1884, §57), is a statement about Jupiter's moons in which the number characteristic four occurs as form and is thereby dependent. If one says w and x and y and z are φ, Husserl explained, then one has combined the objects $w \ldots z$ by 'and'. The 'and' is form and grounds the coherent form of the plural predication. Corresponding to this is a cardinal number, which is a new thought configuration. It is one thing, he stressed, to make statements about objects in which number properties occur as form, and are thereby dependent, and another thing to make statements about numbers as such in such a way that the numbers are the objects. We can make such forms independent, but then new higher-order objects, hypostatizations of forms, emerge that are not objects in their own right. This is why numbers function entirely differently in the propositional logic of the first level than they do in the arithmetic of the second level, where statements about numbers in which numbers are the objects are found, for example,

1. "Any number can be added to any number".
2. "If a is a number and b a number, then $a + b$ is as well".
3. "Any number can be decreased or increased by one".
4. "The numbers form a series continuing from 0 *in infinitum*". (Husserl 2008 18c)

4. Analytics

Instead of pure logic, Husserl taught, one might speak of analytics, or the science of what is analytically knowable in general, the science that establishes and systematically grounds analytic laws (Husserl 2003b, 244). He conceived of the second level of pure logic as an expanded, completely developed analytics in which one proceeds in a purely formal manner since every single concept used is analytic. One calculates, reasons deductively, with concepts and propositions. Signs and rules of calculation suffice because each procedure is purely logical. One manipulates signs that acquire their meaning in the game through the rules of the game. One may proceed mechanically in this way and the result will prove accurate and justified. (Husserl 2008, §§18-19, 434-35; Husserl 1996, Chapter 11)

In his logic courses, Husserl taught that the mathematical disciplines of the purely logical sphere proceed from given, purely logical concepts and axioms that are grounded in the essence of purely logical categories. It is a matter of a rigorously scientific, a priori theory that builds from the bottom up and derives the manifold of possible inferences from the axiomatic foundations a priori in a rigorously deductive way. (Husserl 2001b, 32-35, 39; Husserl 2008, §§13c, 19d, 25b)

From the late 1890s on, Husserl held that the "*world of the mathematical and purely logical is a world of ideal objects*, a world of 'concepts'…. *There all truth is nothing other than analysis of essences or concepts",* and pure logical, mathematical laws are laws of essence. (Husserl 2008, §13c)

In affirming this, he wanted to make it clear that he was not hypostatizing ideal entities or talking about the unwelcome, obscure "special and irreducible intermediary entities called meanings" that Quine called "illusory". (Quine 1961, 11-12, 22)

Husserl said that it was his failure "to obtain clarity regarding the original genuine meaning of the fundamental concepts of the theory of sets and cardinal numbers" in *Philosophy of Arithmetic* that had "compelled" him to recognize the purely logical ideal (Husserl 1978 §27a; §24 and note; Husserl 1975, 34-35). It is worthwhile pointing out in this regard that in Russell's article on the philosophical implications of mathematical logic that is translated in Husserl's notes on set theory, Russell affirmed that "all knowledge which is obtained by reasoning, needs logical principles which are *a priori* and universal" and that mathematics and logic force us "to admit a kind of realism in the scholastic sense… to admit that there is a world of universals and of truths which do not bear directly on such and such a particular existence". (Russell 1973, 292-93)

Husserl said that his concepts of ideal meanings and contents and the idea of transferring all of the mathematical and a major part of the traditionally logical to the realm of the ideal derived from Hermann Lotze, who had been Frege's teacher. Husserl repeatedly defended the view, which he attributed to Lotze, that pure arithmetic is a branch of logic that had undergone independent development. Husserl taught that the unending profusion of theories that arithmetic develops is already fixed, enfolded in the arithmetical axioms, and deduction effects the unfolding of them following systematic, simple procedures. Each genuine axiom is a proposition that unfolds the idea of cardinal number from some side or unfolds some of the ideas inseparably connected with the idea of cardinal number. (Husserl 2001a, 241-42, 271-72; Husserl 2001b, 19, 32-35, 39; Husserl 2008, §15, Hill & da Silva)

This is not necessary to my argument here, but because of the literature making Husserl into a sort of Brouwerian intuitionist (for example, Tieszen 1989; Van Atten 2007), it needs to be made clear that Husserl repeatedly, explicitly and emphatically stressed that, because they belong in the world of the purely logical, arithmetic and set theory are not phenomenology. He maintained that as long as we remain in pure theory of meaning and being, we need not concern ourselves at all with cognitive formations, with consciousness. He believed that everything 'purely' logical was an 'in itself', an 'ideal' that included in its proper essential content (*Wesengehalt*) nothing mental, nothing of acts, subjects, or empirically factual persons of actual reality. He believed that in the case of pure logic, of an 'analytics' in the broadest, radical sense of the word only certain of the most general cognitive-formations enter the picture for purposes of phenomenological elucidation. (Husserl 1975, 20, 31; Hill 2013)

5. Husserl on Sets and the Set-theoretical Paradoxes

So how do Husserl's ideas about sets and the set-theoretical paradoxes fit into the conceptual framework I have just described?

First, it is imperative to keep in mind that sets have an entirely different meaning in the subject-predicate propositions of the first level of Husserl's hierarchy than they do in the set theory of the second level. In the theory of proposition forms or forms of states of affairs of the first level, individual objects are the terms of the predication. Sets, however, do not occur as objects in the subject-predicate propositions, but function in them as dependent forms.

In contrast, in the set theory of the second level, truth is the analysis of essences or concepts, where "we make judgments universally about sets that in a certain way are higher order objects. We do not make judgments directly about elements, but about whole totalities of elements and arbitrary elements, and the whole totalities, the sets to be precise, are the objects-about-which.

He gave these examples of statements about sets on the second level,

1. "2 sets can each be joined into a new set".

2. "2 sets *a b* are each related to one another in such a way that either *a* is part of *b* or *b* is part of *a*, or that they intersect (a set having a part in common), or that it turns out that they are identical, coincide".

3. "The set formed of the elements *A B C* is part of the set formed of the elements *A B C D* containing 'more elements'". (Husserl 2008, 18-19)

On the second level, set theory is derived analytically from the concept of set, which if it is to be mathematical must have a "set essence" in view. This set essence is expressed in the relation between a set itself and its elements. An essence relation makes it impossible for the members of the relation to be identical. So it belongs essentially to the concept of set that no set can contain itself as an element without contradiction.

For Husserl, it is part of the idea of set to be a unit, a whole, comprising certain members as parts in such a way that it is something new that is first formed by them. It belongs essentially to the concept of whole that no whole can contain itself as a part. So, as a kind of whole, a set is subject to the formal rules governing wholes and parts that stipulate that a whole cannot, without contradiction, be its own part. So no set can contain itself as a member. Sets are *a priori* different from their members. (Husserl, Ms A 1 35)

Husserl's 1902 exchange with Zermelo turned upon remarks Husserl had made in 1891, in his review of Ernst Schröder's *Vorlesungen über die Algebra der Logik*, in which Schröder had tried to show that bringing all possible objects of thought into a class gives rise to contradictions. In his review, Husserl wrote that in "the sense of the calculus of sets as such, any set ceases to have the status of a set as soon as it is considered as an element of another set; and this latter in turn has the status of a set only in relation to its primary and authentic elements, but not in relation to whatever elements *of* those elements there may be". He warned that if "one does not keep this in mind, then actual errors in inference can arise". (Husserl 1994, 84-85, 442; Rang & Thomas 1981)

Third, Husserl repeatedly relegated the set theoretical paradoxes to the category of *Widersinnigkeiten*. For him, a set that contains itself as an element was *widersinnig*. By saying that the set of all sets that were not members of themselves is a *Widersinnigkeit*, Husserl was putting it into the same category as the round square, the golden mountain, and the present emperor of France. The formal logical construction "set of all sets which do not contain themselves as parts", he argued, may not be presupposed to be about something that already exists. Just as it is contradictory for a whole to be its own part at the same time, so it is contradictory for a set to be its own member. It proceeds from the paradox that a set that contains itself as an element or a set that does not must be a *Widersinn*. The classification is *widersinnig* as well.

Of what he referred to as "Zermelo's paradox", Husserl wrote that Zermelo argued that a set M that contains each of its partial sets as elements is an inconsistent set. 1) We consider those partial sets that do not contain themselves as elements. 2) In their entirety these form a set M′ that is contained in M. 3) M′ is thus an element of M. 4) M′ is not an element of M′. Proof: were M′ an element of M′, then it would contain a partial set of M (namely M′) that contains itself as element. However, M′ is to contain *ex definitione* partial sets of M that do not contain themselves as elements. 5) Thus M′, since it is not an element of M′, is a partial set of M, which does not contain itself as an element. But all such sets are *ex definitione* contained in the concept of M′, thus in opposition to 4. But M′ is an element of M′. We come to a direct contradiction. If it essentially belongs to the concept of set that (without contradiction) no set can contain itself as an element, then M′ and M are identically the same set, and we show that the whole reasoning was untenable. (Husserl, Ms A 1 35)

6. Frege's Recourse to Extensions

Husserl, Frege and Russell came to many of the same conclusions about the causes of the set-theoretical paradoxes, so we now need to look at the reasoning that led Frege to introduce sets and at Russell's struggles to avoid the contradiction derivable in Frege's system.

Frege thought that wherever we are concerned about truth, we must attach a reference to proper names and concept-words and that we are making a mistake that can easily vitiate our thinking if we do not to do this. So he considered the prime problem of arithmetic to be that of how one apprehends logical objects, in particular numbers. (Hill & da Silva 2013)

Operating only on the first level of Husserl's hierarchy, Frege argued that numbers were independent objects that must always be conceived substantivally and not as dependent attributes. He believed that the presence of the definite article 'the' in an expression like 'the number 4' served to class it as an object and that in arithmetic this independence comes out at every turn, as for example in an identity like 4 + 4 = 8. He thought that we should not be "deterred by the fact that, in the language of everyday life, number appears also in attributive constructions" for that "can always be got around". He proposed that:

> "Jupiter has four moons" can be converted into "the number of Jupiter's moons is four"… we can say: "the number of Jupiter's moons is the number four, or 4". Here "is" has the sense of "is identical with" or "is the same as". So that what we have is an identity, stating that the expression "the number of Jupiter's moons" signifies the same object as the word "four". (Frege 1884, §57)

He added that the independence that he was "claiming for number was not to be taken to mean that a number word signifies something when removed from the context of a proposition, but only to preclude the use of such words as predicates or attributes, which appreciably alters their meaning".

Seeing that many of the inferences that could be made by appealing to his formula for treating what is dependent as independent led to evidently false or nonsensical conclusions, or was sterile and unproductive, Frege settled for the definition: "The Number which belongs to the concept F is the extension of the concept 'concept equal to the concept F'" and for his axiom of extensionality, which he considered "an unprovable law" authorizing a transformation to "take place, in which concepts correspond to extensions of concepts…" (Frege 1979, 182)

Upon learning of the contradiction about the set of all sets that are not members of themselves that Russell derived in the system of *The Basic Laws of Arithmetic*, Frege tested the validity of the chain of inferences leading up to the contradiction and concluded that his law about extensions was false. He confessed that he had been reluctant to use classes, but had found no other answer to the question as to how to apprehend logical objects. (Frege 1980b)

He later described the shift of meaning that had made him not immediately aware of the contradiction. The paradoxes of set theory arise, he said, because a concept is connected with something that is called the set which appears to be determined by the concept and determined as an object. Such a transformation of a concept into an object is inadmissible, because the set formed only seems to be an object, while in truth there is no such object at all. He summed up the "essence of the procedure which leads to the thicket of contradictions":

> The objects that fall under *F* are regarded as a whole, as an object and designated by the name 'set of *F*s'. This is inadmissible because of the essential difference between concept and object, which is indeed quite covered up in our word languages…. Confusion is bound to arise if a concept word, as a result of its transformation into a proper name comes to be in a place for which it is unsuited. (Frege 1980a, 55)

In *Foundations of Arithmetic*, he had warned that it was a mere illusion to suppose that a concept can be made into an object without altering it. (Frege 1884, X)

7. Russell's Attempts to Evade the Paradoxes

As for Russell, he said that his struggle with the contradiction he derived in Frege's logic had taught him that if a word or a phrase that is devoid of meaning when separated from its context is wrongly assumed to have an independent meaning, false abstractions, pseudo-objects, and paradoxes and contradictions are apt to result (Russell 1973, 165). He had originally believed:

> When we say that a number of objects all have a certain property, we naturally suppose that the property is a definite object, which can be considered apart from any or all of the objects, which have, or may be supposed to have, the property in question. We also naturally suppose that the objects which have the property form a *class*, and that the class is in some sense a new single entity, distinct, in general, from each member of the class. (Russell 1973, 163-64)

However, the contradiction about the classes that are not members of themselves showed him that classes must be something radically different from individuals (Russell 1956, 81). He came to believe that if one assumes that the class is an entity, one cannot escape the contradiction (Russell 1973, 171). As he explained,

> if you think for a moment that classes are things in the same sense in which things are things, you will then have to say that the class consisting of all the things in the world is itself a thing in the world, and that therefore this class is a member of itself. (Russell 1956, 261)

Russell decided that he needed a way to make classes disappear from the reasoning in which they were present without really completely letting go of them (Russell 1919, 184), because he believed that "without a single object to represent an extension Mathematics crumbles". (Russell 1903, §489)

While wrestling with the problem of fake objects, he saw parallels existing between the problems arising when classes are treated as objects and those arising when descriptions, 'like the present king of France is bald', are treated as names. So, satisfied that classes and descriptions both fell into the same logical category of non-entities (Hill 1997), he reasoned that since:

> we cannot accept "class" as a primitive idea. We must seek a definition on the same lines as the definition of descriptions, i.e. a definition which will assign a meaning to propositions in whose verbal or symbolic expression words or symbols apparently representing classes occur, but which will assign a meaning that altogether eliminates all mention of classes from a right analysis of such propositions. We shall then be able to say that the symbols for classes are mere conveniences, not representing objects called

> "classes", and that classes are in fact, like descriptions, logical fictions…. (Russell 1919, 181-82)

Russell believed that his means of drawing objects out of descriptions provided a practical model of how to make non-entities function as entities without incurring contradictory results.

Early in his search for ways to evade (his choice of verb) the problem of the contradiction about the class of all classes that are not members of themselves, Russell thought that "the key to the whole mystery" was to be found by inventing (his choice of verb) a hierarchy of types (Russell 1903, §104). It had become clear to him that the contradiction about the classes that are not members of themselves could only be avoided by realizing that no class either is or is not a member of itself, that the entire question as to whether a class is or is not a member of itself is nonsense (Russell 1956, 261-62). So, he invented a hierarchy of classes according to which the first type of classes would be composed of classes made up entirely of particulars, the second type composed of classes whose members are classes of the first type, the third type composed of classes whose members are classes of the second type, and so on. The types obtained would be mutually exclusive, making the notion of a class being a member of itself meaningless (Russell 1973, 201; Russell 1903, §§104-105; Russell 1956, 264). His hierarchy of types was to perform "the single, though essential, service of justifying us in refraining from entering on trains of reasoning which lead to contradictory conclusions. The justification is that what seem to be propositions are really nonsense". (Russell 1927, 24)

Russell believed that no solution to the contradictions was technically possible without his theory of types, but he realized that it was not "the key to the whole mystery". After all, it was but an ad hoc effort to restore the hierarchical structure established by the fundamental differences between dependent and independent that ordinarily protects against invalid inference that was broken by Frege's Axiom of extensionality. He saw that deeper problems caused the old contradiction to break out afresh and he realized that "further subtleties would be needed to solve them". (Russell 1919, 135; Russell, 1956, 333; Hill & da Silva)

8. Interpretation of the Statements

In light of what I have said, how do I interpret the statements I said I was going to interpret?

The *first* statement concerned the set-theoretical paradoxes showing that Husserl's contemporaries did not yet have the real and genuine concept of set needed.

Those paradoxes were derived using a concept of set that allows one to form the expression "a set may be a member of itself", which Husserl judged to be *widersinnig*. In contrast, as we have seen, he would derive set theory analytically from the real and genuine a priori concept, or essence, of set, for which no set can be a member of itself and for which reasoning appealing to the notion of sets that do not contain themselves as members is entirely untenable. A set is a kind of whole and is subject to the formal rules governing wholes and parts that stipulate that a whole cannot be its own part.

The *second* statement says that if one is clear and distinct with respect to meaning, one readily sees the contradiction involved in the set-theoretical paradoxes.

It follows from the above that, if we are clear and distinct about the meaning of the real and genuine concepts of "set", "member", and more universally about the meaning of the real and genuine concepts of "wholes" and "parts", we readily see that all talk of sets being members of themselves is *widersinnig*.

As we have seen, for Husserl, being clear and distinct about meaning involved recognizing the primitive, essential, a priori, inviolable differences between the dependent and independent meanings that form the necessary basis for discovering the essential categories of meaning in which are grounded laws of meaning that provide logic with possible coherent, meaningful meaning forms whose formal truth or falsehood, reference to objects, *Widersinnigkeit* or lack thereof, is determined by logical laws.

For him, being clear and distinct about meaning also involved recognizing that sets have an entirely different meaning in the subject-predicate propositions of the first level of pure logic where they function as dependent forms, than in set theory of the second level where they function as higher order ideal objects and where truth is the analysis of essences or concepts.

In comparison, Frege reasoned on the first level, which obliged him to treat sets and numbers as objects. For example, he mixed the first level subject-predicate proposition "Jupiter has four moons" with what Husserl considered to be the second level arithmetical statement that 2+2=4. He considered numbers to be independent objects that must always be conceived substantivally and not as a dependent attributes (Frege 1884, §106 and note). He confused statements about objects in which number properties occur as form, and are thereby dependent, and statements about numbers in which numbers are the objects. This led him to introduce a law which he thought would permit him to treat what he recognized as dependent meanings as independent meanings. By making such forms independent, he generated new higher order objects, hypostatizations of forms that are not objects in their own right.

I interpret the *third* statement about the solution to the set-theoretical paradoxes lying in demonstrating the shift of meaning that makes it that one is not immediately aware of the contradiction and that once one perceives it one cannot indicate wherein it lies as having to do with Husserl's insistence upon the importance of the fundamental distinction between independent and dependent meanings that lies concealed behind inconspicuous grammatical distinctions.

Husserl and Frege were in fundamental agreement about what Frege called the "fatal tendency" of our "word languages" to cover up essential differences between concepts and objects and allow a concept word to be transformed into a proper name and so to come to be in a place for which it is unsuited. By unavoidable "awkwardness of language", by "a kind of necessity of language", one mentions an object, when one intends a concept.

Frege had thought that the presence of the definite article 'the' in an expression like 'the number 1' sufficed to class it as an object and that we should not be "deterred by the fact that in the language of everyday life number appears also in attributive constructions" for that "can always be got around".

He ultimately concluded that this propensity language to undermine the reliability of thinking by forming apparent proper names to which no objects correspond had allowed concept-words to be transformed into proper names and come to be in places unsuited to them and so had “dealt the death blow” to his set theory.

On his copy of Frege’s “On Concept and Object”, Husserl marked the sentence that reads, “Language has means of presenting now one, now another, part of the thought as the subject”. And he tellingly underlined the word ‘language’. According to his theory about the differences between logical laws and laws of meaning, something that violates logical laws can genuinely have a coherent meaning and can be determined to be true or false, but since it is *widersinnig*, no object can correspond to the existing meaning. So the formal logical construction “set of all sets which do not contain themselves as parts”, may not be presupposed to be about something that exists any more that the expression “the present emperor of France” denotes something that exists. (Hill & da Silva 2013)

Such shifts of meaning allow the pseudo-objects and type ambiguities to creep into reasoning unnoticed that Russell struggled to eliminate in his attempts to evade the paradoxes. As he once warned, when two words have two different types of meanings, the relations of those words to what they stand for are also of different types and the failure to realize this is “a very potent source of error and confusion in philosophy”. (Russell 1956, 133)

In addition, if, as Frege stressed, concept words and proper names must occupy essentially different places, and it is obvious that a proper name will not fit into the place intended for a concept word (Frege 1980a, 54-55), if, as he wrote, there is a radical difference between dependent and independent meanings concepts, which is such that an object can never stand for a concept or concept for an object (Frege 1980a, 92), then basic rules of inference like the principle of substitutivity of identicals and existential generalization will fail when one puts one in the place intended for the other.

Conclusion

In conclusion, I wish to emphasize that Husserl did not say that set theory itself was false. He considered it to be a legitimate mathematical discipline of the second level of the purely logical sphere. For him, set theory was a matter of a rigorously scientific, a priori theory that proceeds from the purely logical concepts and axioms that are grounded in purely logical categories such as those discovered by the essential distinction between dependent and independent meanings. He concluded that it was faulty reasoning about a faulty concept of set that had led to the set-theoretical paradoxes.

In particular, he found himself at odds with the concept of set underlying popular axioms of extensionality. While Russell's tactic was to invent ways to avoid the contradictions (Hill 1997), Husserl advocated making a fresh start and deriving set theory from a non-contradictory concept of set and element, or more universally of whole and part, without resorting to an axiom of extensionality. He was most disparaging when it came to the popular extensional definitions of sets *Principia Mathematica* and related systems and he was lucid enough to see that Mathematics would not crumble if it did not have "a single object to represent an extension". All the rigmarole that Russell went through to evade the contradictions derivable from Frege's system with its axiom of extensionality serves to illustrate what Husserl meant in *Formal and Transcendental Logic* when he said that extensions generate contradictions requiring every kind of artful device to make them safe for use in mathematical reasoning. (Husserl 1978, 74, 76, 83)

In comparison, Husserl's friend and colleague, David Hilbert, determined not to be thrown out of the set-theoretical paradise that Cantor had created (Hilbert 1967), seemed to think that the laws of inference were faulty. As he wrote,

> In their joy over the new and rich results, mathematicians apparently had not examined critically enough whether the modes of inference employed were admissible; for purely through the ways in which notions were formed and modes of inference used–ways that in time had become customary–contradictions appeared.... In particular a contradiction discovered by Zermelo and Russell had, when it became known, a downright catastrophic effect in the world of mathematics....

> The reaction was so violent that the commonest and most fruitful notions and the very simplest and most important modes of inference in mathematics were threatened and their use was to be prohibited…. Just think: in mathematics, this paragon of reliability and truth, the very notions and inferences, as everyone learns, teaches and uses them, lead to absurdities. (Hilbert 1967, 375)

In contrast to Hilbert's assessment of the problem, viewed from the angle of Husserl's theories about the inviolability of the laws governing the use dependent and independent meanings, Russell's contradiction is just faithfully telling us that: the set X of x's is not a member of what it is a set of (Hill 1997); what is predicated of an object is of a different logical type from the object itself; a concept is not an object; what is dependent is not independent…. In short, logic is doing what logic is supposed to do. Blurring distinctions between talk of sets on different levels by allowing the sets as dependent forms of the first level be transformed into proper names and come to figure on the wrong tier in the hierarchy of meaning breaks the logical structure. Flattening logical structure smooths the way for things to come into places not intended for them. Once logical structure is broken and meaning categories are violated trouble is ahead in the form of failures of inference. (Hill & da Silva, 2013)

Why should *Widersinnigkeiten*-producing theories about sets and the foundations of arithmetic have any lasting "downright catastrophic effect in the world of mathematics?" If those theories are producing contradictions, if they lead to the failure of the simplest and most important modes of inference, it is not logical to see that as posing any particular threat to the modes of inferences themselves and does not indicate that their use should be prohibited. It is more reasonable to conclude with Husserl that those logical laws are determining the truth or falsehood of conclusions just as they are supposed to do.

In my opinion, there is nothing particularly paradoxical or mysterious about the contradictions derivable in Frege's logical system. They are just cheap contradictions generated by an unclear theory of meaning. There is no reason at all why the paragon of reliability and truth that is mathematics should "crumble" as a consequence, as Russell once said it might or that basic rules of inference should be abandoned as Hilbert suggested.

REFERENCES

Frege, Gottlob. 1884. *The Foundations of Arithmetic*. Oxford: Blackwell.

________. 1979. *Posthumous Writings*. Oxford: Blackwell.

________. 1980a. *Philosophical and Mathematical Correspondence*. Oxford: Blackwell.

________. 1980b. *Translations from the Philosophical Writings*. Oxford: Blackwell, 3rd ed.

Hilbert, D. 1967. "On the Infinite". [1925]. In *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*, edited by J. van Heijenoort, 369-92. Cambridge (MA): Harvard University Press.

Hill, C. O. 1997. *Rethinking Identity and Metaphysics, On the Foundations of Analytic Philosophy*. New Haven (CT): Yale University Press.

________. 2013. "The Strange Worlds of Actual Consciousness and the Purely Logical". *New Yearbook for Phenomenology and Phenomenological Philosophy* (13): 62-83.

________. 2015. "Husserl's Way Out of Frege's Jungle". In *Objects and Pseudo-Objects Ontological Deserts and Jungles from Brentano to Carnap*, edited by B. Leclercq, S. Richard & D. Seron, 183-196. Berlin: de Gruyter.

Hill, C. O. & Rosado Haddock G. E. 2000. *Husserl or Frege? Meaning, Objectivity and Mathematics*. Chicago: Open Court.

Hill, C. O. & da Silva J. J. 2013. *The Road Not Taken, On Husserl's Philosophy of Logic and Mathematics.* London: College Publications.

Husserl, Edmund. 1900/01. *Logical Investigations*. London: Routledge and Kegan Paul.

________. 1962. *Ideas, General Introduction to Pure Phenomenology*. Translated by W. R. Boyce Gibson. New York: Collier-Macmillan.

________. 1975. *Introduction to the Logical Investigations: A Draft of a Preface to the Logical Investigations (1913).* Edited by Eugen Fink. Translated by Philip J. Bossert & Curtis H. Peters. The Hague: Martinus Nijhoff.

________. 1978. *Formal and Transcendental Logic*. Translated by Dorion Cairns. The Hague: Martinus Nijhoff.

________. 1994. *Early Writings in the Philosophy of Logic and Mathematics*. Vol. 5 of *Edmund Husserl Collected Works,* edited by Rudolf Bernet. Translated by Dallas Willard. Dordrecht: Kluwer.

________. 1996. *Logik und allgemeine Wissenschaftstheorie. Vorlesungen 1917/18, mit ergänzenden Texten aus der ersten Fassung 1910/11*. Hrsg. von Ursula Panzer. Dordrecht: Kluwer. [The English translation, *Logic and General Theory of Knowledge*, translated by Claire Ortiz Hill is forthcoming in 2019.]

________. 2001a. *Logik, Vorlesung 1896*. In *Husserliana: Edmund Husserl Materialien*, Bd. 1. Hrsg. von Elisabeth Schuhmann. Dordrecht: Kluwer.

________. 2001b. *Logik, Vorlesung 1902/03*. In *Husserliana: Edmund Husserl Materialien*, Bd. 2. Hrsg. von Elisabeth Schuhmann. Dordrecht: Kluwer.

________. 2003a. *Philosophy of Arithmetic, Psychological and Logical Investigations with Supplementary Texts from 1887-1901*. Vol. 10 of *Edmund Husserl Collected Works,* edited by Rudolf Bernet. Translated by Dallas Willard. Dordrecht: Kluwer.

________. 2003b. *Alte und neue Logik, Vorlesung 1908/09*. . In *Husserliana: Edmund Husserl Materialien*, Bd. 6. Hrsg. von Elisabeth Schuhmann. Dordrecht: Kluwer.

________. 2008. *Introduction to Logic and Theory of Knowledge, Lectures 1906/07*. Vol. 13 of *Edmund Husserl Collected Works,* edited by Ullrich Melle. Translated by Claire Ortiz Hill. Dordrecht: Springer.

________. *Ms A 1 35*. Untitled, undated manuscript on set theory available in the Husserl Archives in Leuven, Cologne and Paris.

Peckhaus, V. & Kahle R. 2000/2001. Hilbert's Paradox. *Report No. 38, 2000/2001*. Institut Mittag-Leffler, The Royal Swedish Academy of Sciences. https://www.mittag-leffler.se/preprints/files/IML-0001-38.pdf Accessed 17 March 2015.

Quine, W. 1961. *From a Logical Point of View*. 2nd rev. New York: Harper & Row.

Rang, B. & W. Thomas 1981. "Zermelo's Discovery of Russell's Paradox". *Historia Mathematica* (8): 16-22.

Rosado Haddock, G. E. 1973. *Edmund Husserl's Philosophie der Logik und Mathematik im Lichte der gegenwärtigen Logik und Grundlagenforschung*. Doctoral Thesis, Rheinischen Friedrich-Wilhelms-Universität zu Bonn.

Russell, Bertrand. 1903. *Principles of Mathematics*. London: Norton.

________. 1919. *Introduction to Mathematical Philosophy*. London: Allen & Unwin.

________. 1956. *Logic and Knowledge: Essays 1901-1950*. London: Allen & Unwin.

________. 1973. *Essays in Analysis*. London: Allen & Unwin.

Russell, B. & Whitehead A. N. 1927. *Principia Mathematica to *56*. Cambridge UK: Cambridge University Press.

Tieszen, R. 1989. *Mathematical Intuition, Phenomenology and Mathematical Knowledge*. Dordrecht: Kluwer.

van Atten, M. 2007. *Brouwer Meets Husserl. On the Phenomenology of Choice Sequences*. Dordrecht: Springer.

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From Grassmann, Riemann to Husserl: a brief history of concept of Manifold

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Abstract

Edmund Husserl's theory of manifold (*Mannigfaltigkeitslehre*) was formalized for the first time in his *Philosophie der Arithmetik*; in his *Logische Untersuchungen*, §§69–70; also discussed in *Ideen* I, §§72; in *Formale und Trascendentale Logik*, §§51–54; in *Logik und allgemeine Wissenschaftstheorie*, chapter two; and finally it appears in *Einleitung in die Logik und Erkenntnistheorie*, §§18–19. In each of these books, Husserl presents a concept of manifolds as an ontological form. Such form is necessarily axiomatic and appears as inspired by Bernhard Riemann's work. Indeed, Husserl, who studied and lectured extensively on Riemann's theories of space, presented his own conception of mathematics as a theory of manifolds as a generalization of Riemann's notion of manifold.

Keywords: Husserl, Riemann, Grassmann, Manifold, Phenomenology

1. Introduction

The goal of this paper is to take Husserl's first major work, *Philosophie der Arithmetik* (1891), as the starting point of our study on theory of manifold[1]. We articulate the claim that Husserl, using his classification of sciences, drove his mathematical work by a unitary philosophical program. Husserl's program includes several sections belonging to Ms. K I A, K I 15 y K I 4/9a-18a, most of them published in *Studien zur Arithmetik und Geometrie* (Hua XXI). In addition, we examine the ramifications of this concept in several areas of Phenomenology of Mathematics that have been the subject of recent commentaries and publications.[2] At the end, we conclude with a review of the second of two major themes in the aforementioned work of Husserl which have been the matter of a number of contemporary studies.

2. Husserlian manifolds or Riemannian manifolds?

The theoretical core of Husserl's theory of manifolds is dual. (*i*) It is a theory of theories, anchored in the German tradition of *Wissenschaftslehre*, guided by Fichte and Bolzano, and linked to the ancient tradition of *mathesis universalis*, explored by Descartes and Leibniz. (*ii*) The theory of manifolds is also a formal theory of everything (Milkov 2005). In fact, the theory of everything is intrinsically connected with the theory of theories.

Regarding this, Husserl refers to Riemann's conception of manifold and his generalization of geometrical theory, and takes his own notion of manifold as a kind of generalization of that of Riemann. But Riemann's influence on Husserl is not limited to the notion of mathematical manifold or to his views on pure mathematics. In fact, as attested by a posthumously published Husserl's book containing mostly material from the transition period of 1889–189 (edited as *Studien zur Arithmetik und Geometrie*, Hua XXI), Husserl not only extracts from Riemann his interest on the relationship between geometry and physical space, but also finds in Riemann the seed of his philosophy of mathematics as a whole (Rosado Haddock 2017).

It is indeed possible to show that Husserl's conception of mathematics as a theory of manifolds is a generalization and development of Riemann's views on mathematical knowledge and philosophy of mathematic. To clarify this statement, I will quote two passages from 1913 and 1901. The first one is from text No. 5 entitled "*Zwei Fragmente zum Entwurf einer Vorrede zur Zweiten Auflage der Logischen Untersuchungen*" (September 1913). In this "draft", Husserl presents a summary of his mathematical knowledge to the day and of his conversion from a philosophy of arithmetic (or philosophy of calculation), grounded on cardinal numbers to a *mathesis universalis*:

> Als ich aber daran ging, aufgrund der neuen Erkenntnis und unter Mithilfe Bolzanos meine logischen Vorlesungen völlig neu zu gestalten, erkannte ich das Unvollkommene des Bolzano'schen Entwurfs. Ihm fehlte die Idee einer rein formalen Mathematik bzw.

> "Mannigfaltigkeitlehre", die ich mir durch sachliche und historische Studien in einer Reinheit ausgebildet hatte, welche damals den Mathematikern noch keineswegs wie gegenwärtig vertraut war; und demgemäß (fehlte) auch jede Ahnung der inneren Einheit der formalen Logik mit der reinen Anzahlenlehre, der reinen Ordinalzahlenlehre, reinen Grössenlehre usw., schließlich der reinen Mannigfaltigkeits und Theorienlehre. (Hua. XX/1, 298)

A decade before, in September 7th 1901, Husserl sends a letter to Paul Natorp, in which confesses his acceptance of the Riemann's views on geometry and clearly states that accepts the existent of a diversity of geometric manifolds of *n*-dimensions, and, on the other hand, accepts that physical space, whose structure –curvature and dimensionality– should be empirical (Haddock 2017), i.e., the young Husserl not only did consider physical space as a particular case of the much more general concept of an *n*-fold extended magnitude, but also that the determination of the exact nature of physical space is not *a priori* available, but only empirically:

> In der Zeit von 1886/93 habe ich mich um die Theorie der Geometrie, der formalen Arithmetik u. Mannigfaltigkeits ehre sehr viel, periodenweise mit ausschließlicher Hingabe, bemüht. Davon giebt die Vorrede meiner Philosophie der Arithmetik 1891 entfernte Kunde (cf. den Hinweis auf Gauss' Anzeige zur 2. Abhandlung über biquadratische Reste, W.W. Bd. III), und zwar auch Kunde von manchen wichtigen Berührungen mit Ihren Ueberzeugungen. Auch ich fasste, beeinflußt durch Grassmann's Ausdehnungslehre und Gauss' Einführung der gemeinen complexen Zahlen (1. c.), die Ebene als eine gewisse stetige Doppelreihe, den Raum als eine gewisse stetige 3fache Reihe u.sw. In den gemeinen complexen Zahlen (bezw. auch in der Darstellung re') suchte ich die adäquaten arithmetischen Ausdrücke für die Ordnungsverhältnisse der Ebene nachzuweisen und ebenso in entsprechenden complexen Zahlen höherer Ordnung die arithmetischen Ausdrücke für die ebenen Mannigfaltigkeiten höherer Ordnung. (Hua Dok III/5, 80)

In light of the passages just cited, concepts such as "pure theory of magnitude" (*reinen Grössenlehre*) and "pure theory of manifold" (*reinen Mannigfaltigkeits*) become a clear theoretical background for Husserl in the period 1893-1900. Accordingly, there are two important facts that we will now be discussing in more detail: (1) The first fact that probes a linkage between Riemann's and Husserl's philosophical positions is that, by an appeal to mathematical practice, both theories gain a plausibility. This is important only because in their approaches to mathematical knowledge, the "space lived experience" is the starting point. So, neither Husserl nor Riemann began by asking about the relation between mathematical objects and space, but arrived at this question in an effort to provide an explanation for mathematical knowledge. (2) The second fact, is that Husserl states that the thesis about the Euclidean structure of physical space is an unfounded hypothesis made by natural scientists, which can only be founded empirically. Certainly, Riemann entitled his lecture *On the hypotheses which lie at the foundations of geometry* because he wants to specify that the properties that distinguish space from other conceivable 3-manifolds are only to be established *from* experience. The experience confirms that physical space is Euclidean, but these matters of fact are not necessary, but only of empirical certainty; they are hypotheses and not axioms (Ferreirós 2006, 75). So, Riemann and Husserl look upon the practice of mathematics not as the employment of fruitful techniques but as the collecting of *lived experiences* on space and the unity of sciences.

Accordingly, Husserl explained his views on logic and mathematics in the last chapter of the *Prolegomena zur reinen Logik* in terms that prove the objectivity of mathematics, i.e., of the development of his thought on our mathematical experience. Nevertheless, my reading of the young Husserl suggest that this account will be only understandable through references to Gauss[3] Bolzano and particularly Grassmann; mathematicians whose revolutionary discoveries changed the vision of Husserl in these early years.

3. Grassmann's *Ausdehnungslehre* and Husserl's *Philosophie der Arithmetik*

The purpose of this section is twofold. First, I will provide an explanation on Grassmann's mathematical positions to evaluate Husserl's original mathematical positions. Secondly, I will use this explanation as methodological clue to reconstruct the relationship between Husserl and Riemann in next section.

In *Über der Begriff der Zahlen* and in *Philosophie der Arithmetik*, emerges the idea that all philosophy of mathematics must start with the analysis of the concept of number based on the operations of collecting and counting; the "usual" definition: the "number is a multiplicity of unities" is formalized to a "formal reduction of calculate" (*rechnerischformelle Reduktion*).[4] In the second part of *Philosophie der Arithmetik*, chapters 10 and 11, after 200 pages of detailed *psychological* analysis Husserl realized that:

> Allzu voreilig liessen wir uns von der gemeiniiblichen und naiven Ansicht leiten, die den Unterschied zwischen symbolischen und eigentlichen Zahlvorstellungen nicht beachtet und der fundamentalen Tatsache nicht gerecht wird, dass alle Zahlvorstellungen, die wir über die wenigen ersten in der Zahlenreihe hinaus besitzen, symbolische sind und nur symbolische sein konnen; eine Tatsache, welche Charakter, Sinn und Zweck der Arithmetik ganz und gar bestimmt. (Hua XII, 190)

Indeed, if we allow ourselves to be guided by the common sense, which does not take into account the distinction between *inauthentic* and *authentic* representations of numbers, then we do not make justice to the fundamental fact that all number representations that we possess, beyond the firsts natural numbers series, are *symbolic* and *can* only be symbolic. This is what Husserl found particularly troublesome in the psychological analysis on representations of number. The psychological methodology did not allow to build bases prior to checking the ground of the bigger numbers; of course, the problem was that only small numbers and very easy arithmetical calculations are directly given to us, and thus are analyzable in terms of the first part of the *Philosophie der Arithmetik*, i.e., psychologically. His next approach is based on the distinction between authentic and symbolic concepts:

> Eine symbolische oder uneigentliche Vorstellung ist, wie schon der Name besagt, eine Vorstellung durch Zeichen. Ist uns ein Inhalt nicht direkt gegeben als das, was er ist, sondern nur indirekt durch Zeichen, die ihn eindeutig charakterisieren, dann haben wir von ihm statt einer eigentlichen eine symbolische Vorstellung. (Hua XII, 193)

Husserl's first extensive treatment of the logical problems posed by symbolic knowledge appeared in *Philosophie der Arithmetik*. A *symbolic* or *inauthentic* representation is a representation by means of signs, i.e., a content is not directly given to us but rather only indirectly through signs which univocally characterize it.[5] In fact, most of the numbers are given to us symbolically, thus Husserl proceeds to describe the way in which we perceive symbols and how they represent sets, collections or manifolds. Moreover, there are in fact two variants of this problem, one concerning the justification of the usual algorithms for carrying arithmetical computations, the other with treating the symbols 0 and 1 as proper numerical symbols. "One has to do with "blind" manipulations of meaningful symbols; the other with the use of meaningless symbols as if they had a meaning" (da Silva 2010, 127). In the first case, the algorithmic manipulation of numerals in the usual arithmetical operations is certainly not presided by accompanying intuitions; in that sense, "the symbolic system constituted by numerals and symbolic operations is an *isomorphic* copy of the system of number concepts and conceptual operations" (da Silva 2010, 127). In other words: if one has for numbers, for example, a set of basic principles, and it turns out that a set of basic principles formally coinciding, point by point, with the set of basic principles of arithmetic holds in an entirely different domain then is evident that corresponding to each possible arithmetical proposition is a proposition of the new domain and vice versa, in such a way that, as the basic principles, the inferences, conclusions, proofs and theories are also *isomorphic* (Hua XXIV, 84).

Immediately, Husserl realizes that there is a parallel structure of symbols and concepts. In other words, "Husserl's solution to the problem of extending the number domain by means of symbolic numbers relies on the idea of one-to-one correspondence between the signs (given in so-called normal form) and concepts" (Hartimo 2011, 153). So, calculation (*rechnen*) is a conceptual operation which utilizing the system of number signs, derives sign from sign according to fixed rules, only claiming the final result as the designation of a numerical concept (Hua XII, 257-258). With this definition of "calculation" we have obtained a true and proper characterization of the formal-algorithmic method. Hereby the notion of algorithm is bound up with that of a *mechanical process*. An algorithm is, in fact, a mechanical procedure that operates on configurations of (*sensuous*) signs according to certain formal rules. However, and this is very important, he did not think that operating with the symbols of a system according to prescribed rules constituted knowledge by and in itself. A calculus, he thought, although a useful technique, does not necessarily produce science. Finally, Husserl attributes great importance to this concept of calculation since it makes possible an exact separation of the various "logical" moments that are involved in every derivation of numerals from numerals. Undoubtedly the problem he was struggling with was the so-called *principle of permanence of formal laws*.

There is an important difference between a psychological and logical analysis. To Husserl, the symbolic synthesis and arithmetic analysis had been a constant source of difficulties, whereas no comparable difficulties emerged in dealing with smallest natural numbers. The distinction natural/formal manifested in arithmetic emerged and was developed to a higher level, to be considered not only as a methodological distinction but rather as an ontological distinction. In this sense, the reduction of the concept of number to a psychic act, and his collective connection, has failed, but it has made possible two positive things: (1) to accentuate the constitution (genesis) of the logical and mathematical concepts from the data of consciousness and (2) to separate the arithmetic technique from the conceptual domains making possible the application or extension of the arithmetic technique to any type of domains. To be clear, I am not saying that Husserl forgot and rejected the Brentano's empirical psychology and, in its place, placed another philosophy of mathematics; what I am saying is that he tried to assured the genesis of number in the acts and thinking in accordance with a set of rigorous scientific results to achieve an axiomatization of geometry which requires a principle that maintains the consistency of such extension and/or application in other numerical domains (Hartimo 2011). In other words, Husserl had realized that it should be possible to consider calculation entirely devoid of its conceptual basis. To Husserl, this means that instead of *extending the number domain*, one should rather talk about extending the *arithmetical technique*. To this new "project", Husserl assumes the proposals of Hermann Hankel[6] and Hermann Grasssman,[7] on "Principle of the permanence of formal laws":

> Prinzip der Permanenz: Wenn vermöge der Besonderheit der einen Algorithmus begründenden Begriffe gewisse der algorithmischen Operationen nicht in voller Allgemeinheit ausführbar sind, ohne daß man auf widerstreitende Begriffsbildungen kommt, so erweitert man den Algorithmus, nachdem man ihn von der begrifflichen Grundlage losgelöst und als einen konventionellen gedacht hat, dadurch, daß man jede solche Bildung versuchsweise dem algorithmischen Gebiete adjungiert und die Konvention hinzufügt, daß auch für die durch sie symbolisierten Gegenstande (Zeichen) die alten Gesetze gültig bleiben, also die alten Gesetze unbeschränkt ausführbar sein sollen. Man muß dann in jedem Fall die Konsistenz des erweiterten Algorithmus nachweisen. (Hua XXI, 33)

In other words, according to Husserl's formulation, the principle allows extending the algorithm so that one can use the operations by stipulating that the old laws remain valid. Husserl emphasizes that the extended algorithm has to be shown to be consistent. Furthermore, Husserl investigates the correctness of algorithms by means of term reductions of an equational proof system. Indeed, "Husserl claims that the algorithms produce correct results when every equation for relations between the signs can be, using the definitions of the signs, reduced to an identity" (Hartimo-Okada 2016, 950):

> Wir erkannten schon, dass eine Arithmetik, welche die Zahlbegriffe zum Fundament hat, nicht etwa neben diesen noch andere Zahlformen, diesWort im eigentlichen und begrifflichen Sinn genommen, zulässt. Keine negativen, imaginären, gebrochenen Zahlen lassen sich nachweisen, die als Entwicklungsstufen oder Kombinationsformen der Anzahlbegriffe entstehen könnten. Der Anzahlbegriff lässt keinerlei Erweiterungen zu; was erweitert wird und Erweiterung zulä"t, ist nur die *arithmetische Technik*. (Husserl 1983, 42–43).[8]

The quotes suggest that Husserl realized that extending the numerical domain is only possible indirectly, i.e., by means of symbols. Thus, instead of extending the numerical domain, one should rather talk about extending the arithmetical technique. This "realization suggested him to detach the arithmetical technique from the conceptual domains and thus to allow the possibility of applying the arithmetical technique in any kinds of domains such as that of vectors. For Husserl this is justified by the *principle of permanence of formal laws*" (Hartimo 2011, 156).

The importance of this principle of permanence brought to Husserl the direct influence of H. Grassmann. The philosophical and friendly relationship between Husserl and Grassmann reveals little-known aspects of his philosophical developments, especially that which refers to the theory of manifold and to the principle of permanence of formal laws, of course. An example of this is Grassmann's little-known intervention in Husserl's attempts to generalize arithmetic beyond quantitative domains adopting a structural or purely abstract view of mathematics and logic. Equally unknown is the course that Husserl taught in the winter semester of 1889/90, where it is detailed how Grassmann, altogether with Gauss, represent an unprecedented break in the history of mathematics. Specifically, the description of the theory of parallels and the comparison of Gauss with Abel in terms of the theory of algebraic equations (Hua XXI, 318-322).

That course, actually, to belong to the manuscript K I 36. In unpublished pages of that manuscript, Husserl presents a summary of the introduction of the first edition of Grassmann's *Ausdehnungslehre*. Besides the *principle of permanence of formal laws*, Grassmann's general theory of forms has probably been the most influential aspect on Husserl's thinking and it can be seen as another effort in the line of attempts to combine a rigorous method of proof with a method that aids discovery. In this synopsis, according to Gerard (2010), is possible to observe three points that will define the theory of the "husserlian manifold".

> Seine wesentlichen Ideen, mit Ausnahme des Prinzips der Permanenz, verdankt er (nach seinem eigenen Zeugnis) Grassmann, vielleicht dem genialsten Mathematiker, den Deutschland in diesem Jahrhundert hervorgebracht hat. Bereits seiner im Jahr 1844 erschienenen ersten *Ausdehnungslehre* schickt er eine philosophische Einleitung voraus, in welcher er den Begriff der reinen Mathematik oder reinen Formenlehre aufstellt, welche das besondere Sein als ein durch Denken gewordenes auffasst. "Die Form in ihrer reinen Bedeutung, abstrahirt von allem realen Inhalte, ist eben nichts anderes, als die Denkform". (Manuscript K I 36, 8)

The *Formenlehre* of the *Ausdehnungslehre* of 1844, as the science of extensive and intensive types of connection, is identified with free mathematics. As such it governs all conceivable mental mathematical concepts. In that sense, the *Ausdehnungslehre* is recognized as the abstract foundation of the theory of space in which geometry is a particular application applied to space. Moreover, the general theory of forms describes a hierarchy of operations in terms of their relationship with each other. For example, the distributive property of multiplication over addition, from the right and left, is treated independently of the elements being added or multiplied. Thus, following Grassmann and Husserl, the *Ausdehnungslehre* is the abstract science dealing with methods of our outer intuition and hence it can be said to represent intermediary between transcendental philosophy and pure mathematics.

In the first paragraphs of the *Ausdehnungslehre*, Grassmann discusses the idea of an intellectual mathematics or general theory of forms as a redesign of the concept of pure mathematics. In this sense, Grassmann's general theory of forms is nothing but a set of "symmetry principles" valid in all of pure mathematics and expressed by means of equations. The calculus of extension as a particular mathematical theory results from an interaction of both the general science of forms and the intended applications which suggest important analogies to specify the axiomatic schemata.

So, this definition of pure mathematics results from the classification of sciences in real (*reale*) and formal (*formale*). The first science represents in thought the existent as standing independently over against thought, and have their truth in the correspondence of thought with that existent. The formal sciences, on the other hand, have as their object that which is posited through thought itself and have their truth in the correspondence of the reasoning processes among themselves. (Grassmann 1844, §1). The formal sciences consider either the general principles of thought or they consider the particular which is posited through thought—the former is dialectic (*logic*), the latter pure mathematics (Grassmann 1844, §2). The first is a philosophical science since it searches for the unity in all thought, while mathematics takes the opposite direction since it conceives each thought individually as a particular. According to Grassmann, pure mathematics is therefore the science of the particular existent as something which has come to be through thought. The "*particular existent that has come to be by an act of thought*" is a thought-form or, in short, a form. Thus, pure mathematics is the theory of forms (Grassmann 1844, §3). Each form is determined by its generating elements, which might be equal or different and by its generating act, either continuous or discrete. Forms are thus classified according to opposite concepts: discrete/continuous or equal/different. On the basis of this partition of forms in four kinds, which is dependent on their laws of generation, Grassmann classified mathematics in four branches: Number Theory, Theory of Intensive Magnitudes, Combinatorial Theory and Theory of Extensive Magnitudes (*Grössenlehre*), the latter not applicable to the theory of combinations and only improperly to arithmetic:

> Und durch eine abstr. philosophische Diskussion glaubt er die Spaltung der reinen Formenlehre in vier mathematische Disziplinen: in die Zahlenlehre und Kombinationslehre als die Wissenschaften der diskreten Form und die Lehre von den intensiven und die von extensiven Grössen als Wissenschaften der stetigen Form nachweisen zu können. (Ms K I 36, 8)

To understand the latter, it is necessary to make a distinction about the ways of generating forms. Grassmann says: "Jedes durch das Denken gewordebe kann auf zwiefache Weise geworden sein, entweder durch einen einfachen Akt des Erzeugens, oder durch einen zwiefachen Akt des Setzens un Verknüpfens." (Grassmann 1844, §4). The object generated in the first mode is continuous-form (*stetige Form*) or magnitude (*Grösse*); what is produced in the second way is the discrete-form (*diskrete Form*) or concatenated forms (*Verknüpfungs-form*). The intersection of the forms of generation results in the four main types of forms and, consequently, the four branches of the theory of forms. Correlatively, the sciences of the discrete are divided into number theory (arithmetic) and theory of combinations or theory of collective (*Verbindungslehre*) (Grassmann 1844, §6). The opposition between the two types of discrete forms is expressed in the unique sign that gathers the number, whereas the one gathered to form the combination is gathered in arbitrary letters. As for the continuous form or magnitude, it is divided into an algebraic continuous form (intensive magnitude) and the continuous combinatorial form (extensive magnitude).

Finally, the third point that Husserl quotes from Grassmann is the idea of a general theory of forms. With this point, Husserl concludes his exposition of the introduction of the *Ausdehnungslehre*, but he indicates that the four branches that constitute the theory of the forms must be preceded by a general theory of the forms, since "Diesen vier Disziplinen schickt er eine allgemeine Formenlehre voraus, welche die allgemeine, d. h. für alle Zweige gleich verwendbaren Verknüpfungsgesetze darstellt" (KI 36, 8). The general theory of forms would be concerned with establishing general laws of being insofar as thought develops itself. It is clear, according to Husserl, that such a theory does not yet exist; however, it is essential to theorize about it because it avoids the unnecessary repetition of the same laws in the four particular branches of the theory of forms and in its different sections. In accordance with this, Husserl retains three things from the philosophical introduction of the first *Ausdehnungslehre*: the definition of pure mathematics as the theory of forms of thought, the division of the theory of forms into four branches, and the idea of a general theory of the forms that will later take the name of the theory of magnitudes.

4. Theory of manifold in Husserl's "Early" Writings on Mathematics

Husserl remarks that a purely formal conception on sense of the "Object in general" was developed in the history of pure mathematics, in specific, with the systematic re-introduction of a line of thinking found on *methods of calculation*. Indeed, Husserl here invokes to Hankel and Grassmann on the one hand, as precursors of abstract algebra and "to Riemann on the other, as founder of the specifically so-called theory of manifolds generalizing on problems from (differential) geometry and analysis" (Cortois 1996, 43). So, "Object" here is no longer understood as a specific or determinate something to which we can refer arbitrarily, but rather as a "something" which itself has the sense of arbitrariness thanks to which it is uniquely accessible as a "general something" or a "general magnitude" for the method or procedure of investigation. This conception of Object in the Grassmann and Hankel's position (even the algebra of Vieta and Weierstrass) paves the way for understanding the "theory of arithmetic" as a pure construction of an explicit concept of a "something in general". Husserl reads this as the first emergence of the notion of what he calls a "domain undetermined" interpreted as a formal ontology, the laws of *mathesis universalis* would hold for all structures of meaning, as well as the mathematical manifolds, including the metamathematical manifolds of pure formal axiomatics.

Of course, Husserl still does not have the necessary clarity on the formal terrain and his steps are actually groping on the ground of an *arithmetica universalis* whose formal basis is the concept of manifold. Indeed, in his *Habilitationsschrift* and in the first part of the *Philosophie der Arithmetik*, Husserl maintains the idea that authentic numbers and the everyday conception of arithmetic based on simple operations justify the purely formal extension of it. However, when faced with the problem of the justification of the connection between authentic numbers (or authentic cardinalities) and formal (complex) numbers, Husserl moves towards a position in which numbers are determined according to formal systems. Though some real contribution from *Philosophie der Arithmetik*, where a general definition of manifold is attempted, might have been expected we have rather a general calculus of operations that Husserl identifies with a general arithmetic only after the publication of *Philosophie der Arithmetik*. Indeed, between 1893 and 1901,[9] Husserl considered that the world of formal axiomatics did not have to deal with real possibilities in order to articulate what is given within its sphere of concerns. In any case, the world of the mathematical and purely logical is a world of concepts, where truth is nothing other than analysis of laws. Husserl thinks that if concept of the domain (or field) numerical remain just as undetermined as the object of a concept, then we say any "object" whatever. *The only determining factor is the forms*:

> Ein so unbestimmt und in volliger Allgemeinheit gedachtes und nur durch Formen naher determiniertes Gebiet nennt der moderne Mathematiker eine Mannigfaltigkeit. Und das theoretische System der formalin Folgerungen nennt er die Theorie dieser Mannigfaltigkeit. Besser hiese es Mannigfaltigkeitsform, und dafür ist der korrelative Ausdruck natürlich Theorienform. (Hua XXIV, 86)

We must remember here that this latter, i.e. *Mannigfaltigkeittheorie*, has linkage with Cantor's set theory. Set theory is derived analytically from the concept of set (from a set essence) which is expressed in the relation between a set itself and its elements. The set-essences make it impossible for the members of the relation to be identical. It belongs essentially to the concept of set that no set can contain itself as an element without contradiction. For Husserl, it is part of the idea of set to be a unit (a whole) which comprises certain members as parts in such a way that it is something new that is first formed by them. It belongs essentially to the concept of whole that no whole can contain itself as a part. So, as a kind of whole, a set is subject to the rules governing wholes and parts that stipulate that a whole cannot, without contradiction, be its own part. However, the linkage that holds together the elements of a set (*Menge*) is not based exclusively on the act of collating (psychic). Therefore, Husserl said, in set theory, we make judgments universally about sets that in a certain way are higher-order objects. We do not make judgments directly about elements, but about whole totalities of elements and arbitrary elements, and the whole totalities, the sets to be precise, are the objects-about which. Here ends the possible influence of Cantor.

Instead, Husserl believes that the concept of number has to be something fundamentally different from the concept of collection, which was all that could result from reflecting on acts. Such doubts eventually undermined his confidence in the theories of Brentano, as well as those of Weierstrass and Cantor (Hill and Haddock 2000). Regarding this, Husserl discusses in the *Prolegomena*, how one can understand a modification of the mathematical concept of manifold; same concept that he learned as student in Berlin. It is certainly the case that Husserl adapted the concept of manifold to his philosophical and epistemological needs and took it beyond Riemann and Cantor, beyond mathematics and geometry, and into logic and formal ontology. Husserl is well aware of the differences between Cantor and Riemann. However, in most of his writings, when Husserl is discussing the notion of manifold, he has in mind Riemann, Helmholtz and Hankel and Grassmann as interlocutors, of course (Gauthier 2004). In that period, the notion of manifold was used by Husserl in almost exclusively technical contexts and, only, under a philosophical approach.[10]

Husserl developed his own conception of the theory of manifold that is even more general than the modern geometric or topological conception. Besides, he says that his conception was influenced by Grassmann's conception of an 'extension', Riemann's conception of a manifold. In Hua XXI this is more than clear:

> Cantor versteht unter Mannigfaltigkeit schlechthin einen Inbegriff Irgend geeinigter Elemente. Grundlagen einer allgemeinen Mannigfaltigkeitslehre, Leipzig 1883, S. 43, Anm. I: „Unter einer Mannigfaltigkeit oder Menge verstehe ich nämlich allgemein jedes Viele, welches sich als Eines denken läßt, d.h. jeden In begriff bestimmter Elemente, Welcher durch ein Gesetz zu einem Ganzen verbunden werden kann. (Hua XXI, 95)

In this sense, Husserl begins answering his question by noting that Cantor's *Mannigfaltigkeit* merely meant an aggregate of any elements combined into a whole. Husserl goes on to mention that Cantor's concept does not correspond to Riemann's and other related ones in the theory of geometry: "Was ist das, eine "Mannigfaltigkeit"? Zunächst nichts weiter als, in völliger Unbestimmtheit und Allgemeinheit gedacht, ein "Inbegriff" oder eine "Klasse" von Gegenständen. Nun, das sind doch lauter kategoriale Begriffe" (Hua XXIV, 88). Husserl stresses a *Mannigfaltigkeit* is not only an aggregate of elements that are just combined into a whole, but are ordered and continuously inter-dependent. Indeed, a *Mannigfaltigkeit* is not an aggregate of elements without relations. It is precisely the relations that are essential and serve to distinguish the manifold from a mere aggregate or set. Besides, for Husserl manifolds are not aggregates of elements without any relations. It is precisely the relations that are essential and serve to distinguish a manifold it from a mere aggregate (Hartimo 2011, 2016; Hill 2003). As explained above, Husserl saw manifolds as aggregates of elements that are not just combined into a whole, but are continuously interdependent and ordered so that each member possesses an unambiguous position in relation to any other one. The properties and relations set out in the axioms of a complete manifold in Husserl's sense determine objects unequivocally, bring information and logically eliminate nonsensical conclusions. Husserl said:

> Eine Mannigfaltigkeit ist nicht ein Inbegriff beziehungsloser Elemente. Gerade die Beizehungen sind das Wesentliche und Auszeichnende gegenüber einem bloßen Inbegriff. Nun liegt die Frage doch nahe: Welche systematische Form müssen die Beziehungen haben, welchen Charakter ihre einzelnen Elemente, damit ein System von Sätzen sich ergibt, das der Geometrie entspricht? (Hua XXI, 410)

Cantor used the terms *Menge*, *Mannigfaltigkeit* and *Inbegriff* interchangeably. Right from the beginning Husserl, however, directly confronted the terminological difficulties that one faces when speaking of *Mannigfaltigkeiten*. In his earliest writings, he noted that in place of the word *Vielheit* the practically synonymous terms *Mehrheit*, *Inbegriff*, *Aggregat*, *Sammlung*, *Menge*', etc. (variously translated by "quantity, aggregate, plurality, totality, collection, set, multiplicity") had been used. He acknowledged the ambiguity that comes with trying to define *Menge*. At the beginning of *Philosophy of Arithmetic*, Husserl informs readers that, while recognizing the differences, he would not initially restrict himself to using any one of these terms exclusively. By that time, the manifolds of the *Mannigfaltigkeitslehre* that Husserl himself was developing, and that he ultimately considered to be the highest expression of pure logic, were quite different from Cantor's *Mannigfaltigkeiten* (Hill and Haddock 2000, Chapters 7, 8, 9). From that time on, he strove to distinguish sets from manifolds, multiplicities, totalities, aggregates, etc.

But what exactly does the term manifolds mean? What is a manifold? According to Husserl, manifold is not nothing more than an "aggregate" or a "class" of objects conceived in complete indeterminacy and universality. Simple stated, the concept of manifold in its philosophical sense emphasizes its orderly form. A manifold is a structure, which is defined by its relationships. It is not defined by its own objects but by the set of values of a variable according to its parameters. A manifold is more than a collection of objects that are thought of as completely indeterminate; the manifold is a pure form with no other particular content than that its connections (or laws) that give it its validity. As such it is the basic concept of the theory of manifold. The theory of manifold is the investigation of the forms possible of objects domains as such; that is, when objects of thought have been cleared of the last remnants of intuitive content, which survived even in such notions as set or number, there still remains something to be said about the form of a domain of objects as it appears in all formal mathematical theories alike. In short, the theory of manifolds is for Husserl the theory of science itself. While it may belong, as task, to formal mathematics it is related to the elucidation of all possible forms that any scientific theory may take. So, when Husserl talks of logic or mathematics or ontology, he is referring to a theory of science, responsible for investigating the possible forms, or manifolds, that all deductive systems must adhere to. For Husserl then, a manifold is the form of an "infinite object-province" which can be unified under the exact laws of a nomological science. In terms of higher order, the notion of variety governs the form of a theory and defines its correlates in a relational way. Husserl insists that a variety is an aggregate of elements that do not combine in a whole, but are ordered and continuously interdependent.

Husserl was explicit that he borrowed his concept of manifolds from the contemporary geometry; now, sometimes Husserl also mentions in particular, besides Cantor's theory of sets, Lie's study of transformation groups, Grassmann's theory of extensions, and Hamilton's theory as similar attempts to capture the theory of all theories. In this sense he called the theory of manifolds *a fine flower of modern mathematics*.[11] This means that this new discipline, the theory of manifolds, was not only Husserl's vision. It turns reality in the last years of the nineteenth-century mathematics. Husserl's dream was to extrapolate this discipline to the whole categorical realm of human knowledge:

> Wenn ich oben von Mannigfaltigkeitslehren spreche, die aus Verallgemeinerungen der geometrischen Theorie erwachsen sind, so meine ich natürlich die Lehre von den *n*-dimensionalen, sei es Euklidschen, sei es nicht-Euklidschen Mannigfaltigkeiten, ferner Graßmanns Ausdehnungslehre und die verwandten, von allem Geometrischen leicht abzulösenden Theorien eines W. Rowan Hamilton u.a. Auch Lies Lehre von den Transformationsgruppen, G. Cantors Forschungen über Zahlen und Mannigfaltigkeiten gehören, neben vielen anderen, hierher. (Hua XVIII, 252)

As time goes, Husserl will relegate the problematic or purely mathematical approach to concentrate exclusively on the philosophical field. The establishment of this new course and the attention to logical-formal studies, will make the theory of manifold an essential edge of phenomenology through which various stages ranging from the understanding of complex numbers to pure logic. To minimize its importance is to belittle the philosophical work of Husserl himself. In short, Husserl will have to consider the theory of the manifold of modern mathematics as an embodiment of the ideal of a science of possible deductive systems, but which only partially represented the realization of his own ideal of a science of such deductive systems (Hill, 2003, pp. 173).

> Diese Andeutungen werden vielleicht etwas dunkel erscheinen. Daß es sich bei ihnen nicht um vage Phantasien, sondern um Konzeptionen von festem Gehalte handelt, beweist die „formale Mathematik" in allerallgemeinstem Sinne oder die Mannigfaltigkeitslehre, diese höchste Blüte der modernen Mathematik. In der Tat ist sie nichts anderes, als in korrelativer Umwendung eine partielle Realisierung des soeben entworfenen Ideals. (Hua XVIII, 250)

5. ***Husserl's* idea of *a theory of manifolds* and Formalization**

Husserl's theory of manifolds can be interpreted in three different ways (Milkov 2005): (i) his notion of manifolds was seen as being close to Riemann's theory of varieties; (ii) most often Husserl's concept of manifold was explained referring to the manifold of three dimensions in Euclidean geometry, and

(iii) Husserl followed the general theory of forms or polynomials by Leopold Kroneker's work foundations of an arithmetical theory of algebraic quantities (see Gauthier 2004). I will explore only two aspect (i) and (ii).

Husserl described manifolds as pure forms of possible theories which, like molds, remain totally undetermined as to their content, but to which thought must necessarily conform in order to be thought and known in a theoretical manner. So, we have a new discipline and a new method constituting a new kind of mathematics, the most universal one of all. Here formal logic deals with whole systems of propositions making up possible deductive theories. It is now a matter of theorizing about possible fields of knowledge conceived of in a general, undetermined way and purely and simply determined by the fact that they are in conformity with a theory having such a form, i.e., determined by the fact that its objects stand in certain relations that are themselves subject to certain fundamental laws of such and such determined form. In the previous sense, it becomes a theory of the form of theories whose objective is to investigate the essential concepts and laws inherent in an idea of science. It is also an investigation into the possible forms of object domains as such; that is, when the objects of thought have been eliminated from the last bits of intuitive content, which survived even in notions as a set or number (even in the formal sense), there is still something to be said about the form of such an object domain.

For all of the above reasons, Husserl discusses mathematics as a calculating technique, in specific, how the same technique of calculation can be applied in different domains, i.e., every concept in one domain corresponds to a concept in the other and vice versa or every operational concept corresponds to an operational concept in another domain. But, before is necessary known how the technique of calculation works or rather what is the procedure that follows the theory of manifold:

> In der Mannigfaltigkeitslehre ist z.B. + nicht das Zeichen der Zahlenaddition, sondern einer Verknüpfung überhaupt, für welche Gesetze der Form $a + b = b + a$, usw. gelten. Die Mannigfaltigkeit ist dadurch bestimmt, daß ihre Denkobjekte diese (und andere, damit

> als *a priori* verträglich nachzuweisenden) "Operationen" ermöglichen. (Hua XVIII, 251)

According Husserl, pure mathematics produces "calculation truths" of any kind. In other words, instead of numbers, energies, things, etc., He claims that it is better to think of letters and of rules of calculating. [12] In *Einleitung in die Logik und Erkenntnistheorie*, Husserl said that is incomparably easier to think of *a b c* only as something with which one is allowed to replace the form *a* + *b* or *a* * *b* or *a* – b, etc. Furthermore, letters and rules of calculation are enough. Letters and signs for connectives, it is easier to arrive at the combinations in general possible than with concepts (Hua XXIV, 84). If we accept this, then the problems in mathematics will be resolved in the higher possible completeness and generality. Rather, it is a "mathematics" of an indefinitely general realm of thinking. The only thing that is determined in it is the form. This approach was originally developed by Descartes' analytical geometry i.e., a science that solves geometrical problems reducing them to algebraic equations. In other words, is about to translates the intuitive properties of figures into a formal/algorithmic language that describes space within the quantitative frame of coordinates. Here emerges a connection between what is formal and what is analytical that will be further developed in his *Logische Untersuchungen* (and in *Ideen*). In these works, Husserl defines "formalization" (*Formalisierung*) as the procedure eliminating any material content from the proposition. In the end, we obtain a formal structure such that we can replace all material contents with an empty formal "whatever" without altering the logical form of the proposition. "Despite the early notion of "formal," it still overlaps the notion of "algorithmic" inherited by the *Philosophie der Arithmetik*, and despite the word "formalization" has not been coined yet, Husserl already conceives the first step towards a formal representation as an elimination of any material content" (Caracciolo 2015, 37-38).

The development of the notion of *formalization*, as a procedure of elimination of any material content, results in a formal structure that replaces all material contents with a mere formal void, that is, a *mere something in general* without altering its logical conformation (Caracciolo 2015). Due to the symbolization is mechanic, we can represent concepts through intuitions standing for them: for example, a real point may stand for the concept of point because watching the former we catch a symbolic link to the latter. This connection implies that intuitions and concepts are both different and similar in a way that Husserl does not further clarify. Furthermore, symbolization (whose content *is not* directly given to us) is defined as a mere negation of intuition (whose content *is* directly given to us), and therefore, its representational domain is reduced to what *is not* intuitive. As a consequence, symbolization has not an autonomous representational status.

Indeed, once one discovers that the deductions, series of deductions, continue to be significant and are valid when one assigns another meaning to the symbols, one is free to liberate the mathematical system, which can henceforth be considered as the mathematics of a domain in general, conceived in a general and indeterminate way. It is no longer restricted to operate in terms of a particular field of knowledge, we are free to reason completely on the level of pure forms. Operating within this sphere of pure forms, we can vary the systems in different ways. So, the idea of a theory of manifolds itself seems to draw mainly on the oldest of deductive-axiomatic disciplines. Ii is the "purification" of geometrical thought. By means of this method, Husserl said, people first became fully aware of the role of logical form compared to the content of knowledge, and as a further consequence a new discipline and methodology developed out of this that rose above all particular calculating disciplines and constituted a new mathematics of the most universal kind of all, a supramathematics, so to speak, a higher-level mathematics, a theory of theories as theory of possible theory forms (Hua XXIV, 84). To be more precise, when abstracting from the essentially material directedness of geometry, arithmetic or logic, some core element remains intact: the prototype "deductive theory as such".

About this last line, Husserl claims that we can discover these essential grounds of science by reflecting on *conditions* of science itself. For an investigation of these conditions one must look in two totally different directions.

First, we need to examine some objective logical laws that every science must obey in order to avoid nonsense or contradictions inside of their theoretical architecture (these logical conditions must be fulfilled by any science). In addition to this, we must examine the mental acts of knowledge in which scientific truths are given to us. This examination is directed towards the subjective conditions of knowledge, and it leads to an elucidation of the epistemological conditions of scientific knowledge and knowledge in general. Without such an elucidation of these conditions, Husserl believes that the sciences remain naïve, that is, without an understanding of their origin and essence. According to Husserl, every science is not just a collection of sentences about a certain field of knowledge, but rather a theoretical unity. Its sentences must be interconnected, because otherwise there would be no reason for us to call a mere collection of sentences a theory or a science. From a logical point of view, the unifying elements of this necessary interconnection between the sentences are certain logical laws and rules, e.g., the syllogistic inferences. Thus, the unity of science is based on the logical interconnection of sentences that is made possible by formal-logical rules. These formal structures are the theory-building elements in any science. This purely logical form of a theory can be investigated by logicians, because all these formal elements retain a certain independence from the concrete material content which they combine into a theory. Due to this independence of the logical form, it is possible to investigate all these theory-building elements in a general theory of science. The development of such a theory of science, that is, a theory of the formal structures of any theory, is, according to Husserl, the ultimate goal of theory of theories.

NOTES

[1] The invention of this notion is usually attributed to Riemann. In fact, the term "*Mannigfaltigkeit*", of which the word "manifold" is an English translation, appeared for the first time in the world of mathematics in Riemann's famous *Habilitationsvortrag*. There are other English translations such as "multiplicity" or "variety" in the mathematical literature. In this text

the choice has been made to follow the lead of David Carr, Dorion Cairns, Burt Hopkins and Dallas Willard.

[2] Only recently has it been discussed in a number of essays, Scanlon (1991), Majer (1997), Hill (1995, 2000), da Silva (2000, 2016), Gauthier (2004), Hartimo (2007), Centrone (2010, 2017), Okada (2013).

[3] In the preface to the *Philosophie der Arithmetik*, Husserl acknowledges the influence that Gauss's study on complex numbers exerted on him. Indeed, Gauss plays an important role in Husserl's mathematical formation: "Vielleicht erweckt es von vornherein kein ungünstiges Vorurteil für meine Bestrebungen, wenn ich sage, dass ich die Grundgedanken meiner neuen Theorie dem Studium der vielgelesenen und doch immer nur einseitig ausgenützten Gaussschen Anzeige über die biquadratischen Reste (II) verdanke" (Hua XII, 8). Also *cf.* (Hua XXI, 322–347). Gauss' work is important for Riemann and Husserl in two respects. First of all, it contains a systematic introduction of imaginary and complex numbers as an extension of the real numbers, and secondly, Gauss proceeded to substantiate these impossible numbers by providing a visual and geometrical characterization of them.

[4] Husserl's *Philosophy of Arithmetic* is a dialectical work. "It consists of two parts: the first part focuses on "psychological" investigations of the concepts *multiplicity*, *unity*, and *number*, insofar as they are given to us authentically and not indirectly with a mediation of symbols [...] In the second part Husserl takes up the "logical" and "arithmetical" investigations" (Hartimo, 2011, p. 151).

[5] Besides, there are minor texts of that period published in Hua XII, XXI and XXI where Husserl's treatment presents basically three versions of the problem of symbolic knowledge.

[6] H. Hankel presented the principle of permanence in his *Theorie der complexen Zahlensysteme* (1867); Husserl knew and discussed Hankel's principle at least since his *Habilitation* in July 1887. Indeed, in order to habilitate Husserl defended eight theses in a disputation at University of Halle in 1887. One of the theses is "Das Hankelsche "Prinzip der Permanenz der formalen Gesetze" in der Arithmetik ist weder ein "metaphysisches" noch ein "hodegetisches" Prinzip" (Hua XII, 339). Even, Husserl had attended Hankel's lectures in the University of Leipzig, (*cf.* Schuhmann 1977, 4).

[7] In 1876–1878, before of his studies with Weierstrass and Kronecker in Berlin, Husserl studied mathematics, physics, astronomy, and philosophy at the University of Leipzig (Schuhmann 1977, 4). Of these years in Leipzig, date the friendship between Husserl and Hermann Grassmann's son and Robert Grassmann's nephew, Hermann Grassmann, Jr. It is not so well known that during the winter semester 1877-78, Husserl received the *Ausdehnungslehre* from Hermann E. Grassmann as a gift (Schuhmann 1977, 6) Also *cf.* Hartimo (2011) and Gérard (2010).

[8] Manuscript from around 1889–1890, quotes in (Hartimo 2016, 155).

[9] About the second volume (which was never published), Husserl wanted to provide or to communicate more details on investigations concerning to symbolic representations and the methods of cognition grounded on them.

Also, he wanted to show that arithmetic will appear as one member of a whole class of arithmetic, unified in virtue of the homogeneous character of identically the same algorithm.

[10] According to da Silva (2000).

[11] In *Ideen I*, Husserl said: "Mit anderen Worten, die Mannigfaltigkeit der Raumgestaltungen überhaupt hat eine merkwürdige logische Fundamentaleigenschaft, für die wir den Namen "definite" Mannigfaltigkeit oder "mathematische Mannigfaltigkeit im prägnanten Sinne" einführen. Sie ist dadurch charakterisiert, dass eine endliche Anzahl, gegebenenfalls aus dem Wesen des jeweiligen Gebietes zu schöpfender Begriffe und Sätze die Gesamtheit aller möglichen Gestaltungen des Gebietes in der Weise rein analytischer Notwendigkeit vollständig und eindeutig bestimmt, so dass also in ihm prinzipiell nichts mehr offen bleibt. Wir können dafür auch sagen: eine solche Mannigfaltigkeit habe die ausgezeichnete Eigenschaft mathematisch erschöpfend definierbar" zu sein. Die "Definition" liegt im System der axiomatischen Begriffe und Axiome, und das "mathematischerschopfende" darin, dass die definitorischen Behauptungen in Beziehung auf die Mannigfaltigkeit das denkbar größte Präjudiz implizieren -es bleibt nichts mehr unbestimmt" (Hua III/1, 153)

[12] According Husserl, the rules in general are given for operating with sums, products, quotients in arbitrary combination, etc. All these operations are used *as* mechanical rules of calculation. The letters are manipulated like a *game tokens*. Indeed, one can calculate with concepts and with propositions in the same way as with lines or surfaces. The calculation is not calculation with quantities and numbers, but only belongs being logically deduced. (Hua XXIV, 81-82).

REFERENCES

Caracciolo, Edoardo. 2015. "Formalization and Intuition in Husserl's *Raumbuch*." In *From Logic to Practice: Italian Studies in the Philosophy of Mathematics,* edited by G. Lolli, M. Panza, and G. Venturi, 33-50. Cham: Springer International Publishing Switzerland.

Centrone, Stefania. 2010. *Logic and Philosophy of Mathematics in the Early Husserl*. Dordrecht: Springer.

Cortois Paul. 1996. "From apophantics to manifolds: the structure of Husserl's formal logic." *Philosophia Scientiæ* 1(2): 15-49.

Da Silva, Jairo José. 2000. "Husserl's two notions of completeness, Husserl and Hilbert on completeness and imaginary elements in mathematics." *Synthese* 125: 417–438.

________. 2010. "Beyond Leibniz: Husserl's Vindication of Symbolic Knowledge." In *Phenomenology and Mathematics*, edited by M. Hartimo, 123-145. Vol. 195 of *Phaenomenologica*. Dordrecht: Springer Science+Business Media.

________. 2016. "Husserl and Hilbert on completeness, still." *Synthese* 193(6): 1925-1947.

________. 2017. *Mathematics and its Applications. A Transcendental-Idealist Perspective*. Dordrecht: Springer.

Ferreirós, José. 2006, "Riemann's Habilitationsvortrag at the Crossroads of Mathematics, Physics, and Philosophy." In *The Architecture of Modern Mathematics. Essays in History and Philosophy*, edited by J. Ferreirós and J. J. Gray, 67-96. Oxford: Oxford University Press.

Gauthier, Yvon. 2004. "Husserl and the Theory of Multiplicities ‚Mannigfaltikeitslehre'." In *Husserl and the Sciences. Selected Perspectives*, edited by Richard Feist, 121-127. Otawa: University Ottawa Press.

Gérard, Vincent. 2010. "Mathesis universalis et géométrie: Husserl et Grassmann." In *Philosophy, Phenomenology, Sciences: Essays in Commemoration of Edmund Husserl,* edited by C. Ierna, H. Jacobs, and F. Mattens, 255-300. Dordrecht: Springer Science+Business Media.

Hartimo, Mirja. 2006. "Mathematical Roots of Phenomenology: Husserl and the Concept of Number." *History and Philosophy of Logic* 27(4): 319-37.

________. 2007. "Towards completeness: Husserl on theories of manifolds 1890–1901." *Synthese* 156: 281–310.

________2010. "The development of Mathematics and the Birth of Phenomenology." In *Phenomenology and Mathematics*, edited by Mirja Hartimo, 107-122. Dordrecht: Springer.

________. 2011. "Grassmann's influence on Husserl", In *From Past to Future: Graßmann's Work in Context*, edited by H.-J. Petsche *et al.*, 149-159. Basel: Springer.

Hartimo, Mirja & M. Okada. 2016. "Syntactic reduction in Husserl's early phenomenology of arithmetic." *Synthese* 193(3): 937–969.

Husserl, Edmund. 1950-2012. *Husserliana: Edmund Husserl – Gesammelte Werke*. Bde. 1-41. Haag: Martinus Nijhoff (Bd. 1-26, 1950-1987); Haag: Kluwer Academic Publishers (Bd. 27-37, 1988-2004); New York: Springer (Bd. 38-41, 2005-2012). [Abbreviated *Hua*].

________. 1970. [Hua XII]. *Philosophie der Arithmetik. Mit ergänzenden Texten (1890-1901).* In *Husserliana*, Bd. XII, hrsg. von Lothar Eley. Den Haag: Martinus Nijhoff.

________. 1983. [Hua XXI] *Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass (1886-1901).* In *Husserliana*, Bd. XXI, hrsg. von Ingeborg Strohmeyer. Den Haag: Martinus Nijhoff.

________. 1984. [Hua XXIV] *Einleitung in die Logik und Erkenntnistheorie. Vorlesungen 1906/07.* In *Husserliana*, Bd. XXIV, hrsg. von Ullrich Melle. Den Haag: Martinus Nijhoff.

________. 1994. [*Hua-Dok* III]. *Briefwechsel.* In *Husserliana: Edmund Husserl Dokumente*, Bd. III/I-X, Hrsg. von Karl Schuhmann. Den Haag: Kluwer Academic.

Ierna, C. 2012a. "Husserl's Psychology of Arithmetic." *Bulletin d'analyse phénoménologique* VIII 1 : 97-120.

________. 2012b. "La notion husserlienne de multiplicité: au-delà de Cantor et Riemann." *Methodos* [Online], 12. http://methodos.revues.org/2943.

Majer, Ulrich. 1997. "Husserl and Hilbert on Completeness: A Neglected Chapter in Early Twenty Century Foundation of Mathematics." *Synthese* 110: 37-56.

________. 2004. "Husserl and Hilbert on Geometry." In *Husserl and the Sciences. Selected perspective*, edited by Richard Feist, 101-120 Otawa: University of Ottawa Press.

Milkov, Nikolay. 2005. "The Formal Theory of Everything: Exploration of Husserl's Theory of Manifolds." *Analecta Husserliana. The Yearbook of Phenomenological Research*

Volume 88: 119-35, edited by Anna-Teresa Tymieniecka. Dordrecht: Springer.

Hill, Claire Ortiz. 1994. "Frege's Attack on Husserl and Cantor." *The Monist* 77 (3): 345-357.

________. 1995. "Husserl and Hilbert on Completeness." In *From Dedekind to Gödel*, edited by Jaakko Hintikka, 143-163. Dordrecht: Kluwer.

________. 2003. "Husserl's *Mannigfaltigkeitslehre*." In *Husserl or Frege? Meaning, Objectivity, and Mathematics,* edited by Claire Ortiz Hill and Guillermo Rosado Haddock, 161-177. Chicago/La Salle: Open Court.

Hill, Claire Ortiz and Guillermo Rosado Haddock. 2000. *Husserl or Frege. Meaning, Objectivity, and Mathematics*. Chicago/La Salle: Open Court.

Hill, Claire Ortiz and Jairo José da Silva. 2013. *The Road Not Taken. On Husserl's Philosophy of Logic and Mathematics*. London: College Publication.

Riemann, B. 1876. *Gesammelte Mathematische Werke und wissenschaftlicher Nachlass*. Hrsg. von R. Dedekind, H. Weber. Leipzig: B.G. Teubner.

Rosado Haddock, Guillermo. 1987. "Husserl's Epistemology of Mathematics and the Foundation of Platonism in Mathematics." *Husserl Studies* 4(2): 81-102.

________. 2006. "Husserl's philosophy of mathematics: its origin and relevance." *Husserl Studies* 22: 193–222.

________. 2017. "Husserl and Riemann." In *Essays on Husserl's Logic and Philosophy of Mathematics*, ed. by Stefania Centrone, 229-244. Dordrecht: Springer.

Scanlon, John. 1991. "Tertium non datur": Husserl's Conception of a Definite Multiplicity." In *Phenomenology and the Formal Sciences*, edited by Seebohm, Føllesdal and Mohanty, 139-147. Dordrecht: Kluwer.

Schuhmann, K. 1977. [Hua-Dok I]. *Husserl-Chronik (Denk- und Lebensweg Edmund Husserls).* In *Husserliana: Edmund Husserl Dokumente*, Bd. I. Den Haag: Martinus Nijhoff.

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Husserl's Logic of Probability: An Attempt to Introduce in Philosophy the Concept of "Intensive" Possibility

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Abstract

Husserl's insisting reflections on the question of probability and the project of a logic of probability, although persisting throughout his work, published and unpublished, from the *Prolegomena* to later works (the *Krisis*), has not received any serious attention. While exposing the main lines of his project, this article aims at listing some of the reasons explaining this paradoxical situation. 1) The logic of probability is not conceived by Husserl as an extension of formal logic and especially of an already made logic, but as a reform of logic (from the recensions of Schröder to *Formal and Transcendental Logic*, and beyond). 2) This entails a revised notion of *proposition*, enlarged to every forms of "positions" or "thesis" and, extended, following the correlation of intentionality, to the noematic side. 3) The very notion of the "possible" at the basis of any logical, algebraic, arithmetic and geometric treatments of probability is enlarged and modified accordingly. 4) As a consequence, his position is rather singular and very hard to locate in the battle field among mathematicians, logicians and philosophers around the question of "foundation of probability" and the interpretation of probability calculus (a priori probability *vs* a posteriori probability, subjective *vs* objective probability, logical *vs* psychological probability, etc.).

Keywords: possibility, modality, range, logic, von Kries

In *Formal and Transcendental Logic*, Husserl declares explicitly that, in the *Prolegomena*, he committed a double fault in his presentation of "pure logic", the second failing being a restriction of the scope of "formal logic", by the exclusion of "modal modifications of judgement". Instead of restricting "pure

* In the following, I reformulate and complete a thesis that I have exposed for the first time in a Husserl Circle Meeting in Paris, in 2009.

logic" to the sole "logic of certainty and truth", Husserl argues that "in connection with the concept of truth" "the modalities of truth are not mentioned, and *probability* is not cited as one of them"[1] – "as universal formal possibilities, modal variants of judging and of judgments enter into certainty or truth logic." The reason for such exclusion was that ontological and axiological modalities, with their correlative act-modifications, were wrongly "regarded as *extra-formal*". According to Husserl's view on logic and history of logic, this limitation, inherited from the tradition, is explicitly considered as unacceptable. Modalities do belong to the content or "*matter*" (Materie) of judgement and contribute to the constitution of the object referred to, to the "*something*" with its specific determinations. Since Husserl's understanding of "formal semantics" must be seen in the frame of the "correlation" between apophantic and ontology, this means that without those modal determinations, the whole sphere of logic would be drastically amputated, and semantics would remain empty. But what does semantic mean here?

What is at stake here is *not* the proposal of a *modal extension* of formal logic such as it has been promoted at the same time by his former student Oskar Becker (1930 and especially 1952, §2)[2] or others like Lewis (1928 and 1932) nor a prefiguration of a semantical approach *à la* Kripke, but something that has to do with a deeper understanding of formal logic, formalization and formal semantics. Since "probabilities" are themselves modalities, in order to understand what is the profile of the new and enlarged formal logic resulting from the inclusion of modalities strictly understood into the "formal content" of logic, and what motivates this enlargement, I would like to examine more closely Husserl's evolution on the delineation of formal logic, his previous project of a "logic of content", in connection to his project of reform of formal logic (Lobo 2017a; 2017b).

Considering, on the one hand, that, roughly speaking, probabilities are a kind of possibility, which, in contrast with empty formal possibilities, should be called "loaded" or "intensive possibilities" and, under some conditions, countable or measurable possibilities; considering on the other hand, that *transcendental logic* deals with another kind of possibilities:

"real possibilities", or "possibilities of objectivity" or else "possibilities of a full meaning" of concepts and judgements, ruled by certain *a priori* conditions; we are entitled to infer that the inclusion of probability as a modality within the ground framework of formal logic is essential to understand why and how this change of the logical status of probabilities led Husserl to rehabilitate *transcendental logic* and redefine in new terms its essential tasks, as well as those of a critique of logical reason[3]. Furthermore, this helps us to understand why the first task of this transcendental logic is to promote and justify a deep reform of formal logic, by surveying the larger domain of formal disciplines such as axiology, practice, choice, etc. [4]. This should explain also why the project of transcendental logic takes place in the frame of a fully new discipline: transcendental phenomenology. Last but not least, if we take into consideration that the turning point occurred around 1909 (while the outburst of probabilities in contemporary mathematics and physics was not already achieved nor fully acknowledged), this illustrates – if it does not demonstrate it – the epistemological relevance of phenomenology.

In the following, I shall propose a brief overview of the evolution of Husserl's conception of probability; a presentation of the problem of the logical foundations of probability calculus following Husserl (following Von Kries, and others like Boole); a sketchy presentation of Husserl's logic of probability and the foundations it provides to probability calculus, and some prospective views on the larger theory of manifolds obtained by considering "probability spaces" as manifolds of (equally or unequally) loaded possibilities, as fields of possibilities varying in intensity; before concluding with some "snapshots in the twilight".

1. Brief overview of the evolution of Husserl's conception of probability

As long as probabilities were at the periphery of pure logic and pure mathematics, all epistemological issues touching the application of mathematical concepts or of formal forms of reasoning to empirical contexts were relegated or ascribed to the much looked for, but informal "logic of induction" or "inductive logic."[5] The most difficult and epistemologically promising aspect of Husserl's logical approach is the promotion of probabilities thus understood as the *core* and *fundament* of formal logic, and *not* as a secondary extension. As soon as probability becomes part of the ground structure of formal logic and pure mathematics, this hierarchy as well as the distinction between applied and pure mathematics loses their pertinence. Such was the position of Husserl in 1901 until the shift afore mentioned occurred at the turn of 1909.

This shift has not been sufficiently noticed, if not fully ignored or vigorously repressed for various reasons.

The first important reason is that Husserl's position on probability is quite difficult to locate in the battle field of interpretations around probability calculus: *a priori* vs *a posteriori* probability, subjective *vs* objective, logical *vs* psychological, etc. Going beyond some discordances, Husserl seems to belong to a tradition known as "range theory of probability", which is a sub-division of the logical approach to probability, whose major figure are Boole, Stumpf, Peirce, Ramsey and Keynes[6]. Probabilities are attributed in the first instance not to events, processes or things in general, but to propositions (Keynes 1921, 10-19). More precisely looking at the logical concepts and principles promoted by Husserl and Keynes, we could pinpoint the influence of the neo-Kantian Johannes von Kries (Fioretti 1998, 2001; Heidelberger 2001; Rosententhal 2010). But, as we shall see, because of the inclusion of modalities and probabilities in the "formal matter" of terms and propositions, the widened notion of "formal truth" and the theory of judgment and proposition are accordingly deeply reshaped.

A second reason, strictly connected to the former, is that, as long as we identify Husserl's conception of formal logic with one of the former or contemporary conceptions of so-called classical logic, and its theory of multiplicities, with that which prevails as the foundation of modern mathematics, i.e. set-theory, we remain inevitably blind to its modal core and, consequently, Husserl position either remains invisible, or appears as inconsistent within the set theoretical frame promoted by Kolmogorov's axiomatization of probabilities in 1931 (see Kolmogorov 1956).

For these reasons, Husserl's theory of probability has remained almost completely unnoticed up to now, among mathematicians as well as logicians and philosophers, even those interested in the question of the "foundation of probability."[7]

Husserl's conception of probability is clearly exposed for the first time in the *Lessons on Logic and Theory of Science* from 1909 (Hua 30). It belongs to a tradition which goes back to Leibniz (Couturat 1901, 239-240), according to which the foundation and the correct interpretation of probability calculus requires a *logic of probability*, i.e. the consideration of the form and the content of a special kind of propositions. This position, which is that of Keynes too, was strenuously rejected by the dominant figures of the modern theory of probability, such as Borel[8]. In the *Prolegomena*, the name of Von Kries is not mentioned in reference to probabilities, especially when Husserl refers to the status of probabilities as *derived logical forms* compared to plain logical propositions, but in connection to the distinction between deductive sciences and descriptive sciences, i.e. *nomological (deductive) sciences* and *ontological (descriptive) sciences*, which is clearly taken from von Kries and his book on probability (Hua 18, § 64)[9]. But when considering the "ideal conditions of possibility of science in the most general manner", Husserl admits the existence of "ideals elements and laws even in the field of empirical thinking, in the sphere of probabilities", as an "*a priori* basis", as pure conditions of possibility of empirical science in general. Yet Husserl, on one stroke, rejects probabilities outside the sphere of pure logic and, consequently, the possibility of a transcendental logic is brushed aside (Hua 18, § 72). They represent the second fundament of the technology of science at work in empirical sciences to approximate, through successive revisions, the pure form of a scientific theory. Consequently, the manifolds on which probability measures are implemented remain outside the field of pure (definite) manifolds, so to speak, mathematically outlaw.

Probabilities are primarily subjective modifications of judgment which can become objects for new judgements of second order, expressing thus mere asymptotic approximations of truth judgement in the strict sense of the term (Hua 18, § 6). The content of probability judgement does not enter the formal realm of logic, and the wider form of knowledge does not imply any enlargement of pure logic. We can talk of a "logic of empirical sciences", only in a derived sense, as a "technic of evaluating probabilities and founding probability" (Hua 18, § 64). In the lessons from 1906/07, despite recent development in the mathematical treatment of probabilities[10], the constitution of a formal theory of probability seems still dubious. The quantification and numerical determination of "degrees of justified conjectures", built up under the form of a deductive discipline is not enough. What is measured remains something subjective and ambiguous. As in 1901, Husserl's refers to Laplace fundamental notion of "equipossible cases" as "state-of-affairs" of which we have no knowledge, or rather of which we are in a state of "no-knowledge" (*Unkenntnis*). The underlying principle is known as the indifference principle. According to Laplace here rephrased by Husserl, the meaning of probability equations is nothing more than a subjective mixed state of ignorance (*Unwissenheit*) and knowledge (*Wissen*)[11].

In order to eliminate the thread of "probabilism" (Hua 18, § 22, [65]), – a variant of Psychologism and Skepticism – the first reaction of Husserl is to expel probabilities from the sphere of pure logic. The mathematics of probability are not formal mathematics ("*aber formale Mathematik ist das nicht*"). Probability is nothing more than a modalized certainty[12] and the correlative "state-of-affaire", each time at stake, appears as such, *just as it would be posited in an apodictic evidence as certain, if we could reach the level of evidence for such a judgement*, i.e. *ideally*, — but haloed or fringed by subjective modifications of the "holding-for-true", namely that of presumption. In other words, the "logical content" or "matter" is exactly the same; i.e. nothing of the modification of belief enters the content of judgment and the probability apparatus remains outside of the logical sphere of judgement. The change of modality or modalization is presupposed to be parallel to that of fulfilment, of making-evident, but fully independent of the process of identification and determination[13]. Probability calculus remains just a sophisticated device, among many others epistemological technologies and a substitute (or surrogate) for an evident and certain judgment. At any rate, "probability cannot rival truth, nor can any presumption rival intellectual evidence (*Einsicht*)" (Hua 18, 75); the more that probability can do is to strive asymptotically toward such a full and complete evidence, to approximate it (*ibid*. 29).

The knowledge, broadened through inclusion of a wider range of belief "distinguishing reasonable from unreasonable, better from worse-founded assumptions, opinions and surmises", is not knowledge in pregnant sense. Correlatively, the ontological domain of probabilities relies ultimately on empirical facts, on individual existences. Since *every probability*, even the highest and most valuable, refers back to an "existential content" (Hua 18, 84), even a deductive theory of probability must be expelled from the sphere of *Mathesis pura*, which is, by definition, freed from any existential supposition, or, which amounts to the same, the demonstration or the postulation of possibility is tantamount to that of existence, while mathematical impossibility is identical to inexistence[14].

But as we said above, the relation of formal mathematics to "material mathematics" is regulated by ideal laws, which are epistemologically relevant for the understanding of the applicability of mathematics and the progressivity of knowledge. The modal (probable) characters of the facts on which empirical theories rely, impregnate, so to speak, the theories and the knowledge themselves. On the other hand, *these epistemological situation and relations* must be logically exposed and explained. And indeed, a "pure theory of probability" should study the different ways in which an empirical theory is modified, and occasionally enlarged, as well as the ways in which a formal domain (such as that of arithmetic or any definite axiomatic system) is modified and enlarged (Hua 12, 452 *et seq*.). Nonetheless, Husserl maintains that this pure theory of probability is not part of pure logic.

The *Prolegomena* conclude with a programmatic "pure theory of probability", whose status and tasks remain ambiguous and rather undetermined, because of the amphibology of the very notion of probability, which reaches here its climax. Following the classical definition (at least until its absorption into measure theory), probability designates sometimes the rational number resulting from the comparison of favorable cases over the totality of equipossible cases ; sometimes the substitute of evidence for empirical knowledge as such, or for the application of mathematical models, or for the degree of approximation to the postulated ideal theory obtained through this modelling; sometimes the ideal hypothesis or subtractions underpinning this application. In this later sense, the pure theory of probability should be the logic posing the ideal laws governing the production and the validation of such idealizing fictions, and explaining why a theory is enlarged in order to cope with experimental facts contradicting it, or why another theory is rejected although no experimental fact invalidates it. But rather equivocally, Husserl concludes that the "ideal elements and ideal laws" founding the "possibility of empirical sciences in general" and the "idea of the unity of empirical explanation" belong to "pure logic" *only in a correspondingly extended sense* (Hua 18, 258]) — an external extension which constitutes the second theoretical fundament of logic normatively converted: i.e. technology properly speaking (Hua 18, § 72).

In the *Introduction to Logic and theory of knowledge* from 1906-1907, the idea of a pure logic of probability is not yet clearly defined. These lessons provide an exposition of a noetic as a "pure theory of law (*Rechtslehre*) of knowledge", i.e. as a theory of validity and possible validity of knowledge in general and an exploration of the eidetic frame of logic, i.e. the eidetic underpinnings of its pretension to set the norms of valid knowledge. The mathematical treatment of probabilities is thus promoted to the rank of *fundamental* technology of knowledge[15]. And this is a real promotion indeed, since the vast domain of effective mathematics is equipped with powerful symbolic technics and tools. And beyond the mathematical discipline which deals quantitatively and numerically with degrees of presumption and belief, Husserl acknowledges the right of a new deductive discipline *closely linked* to "formal logic as *mathesis*" (Hua 24, 132). Overall, the reduction of probability to a numerical calculus nourishes a reasonable hope of bringing the theory of probability inferences, which play an essential role in all empirical sciences, closer to the theory of science. Yet this deductive theory of probability remains outside formal logic and is not part of the noetic; since it is itself in need of justification and formalization.

In the lessons on *Logic and theory of science*, which started in 1909, the delimitation of logic proposed in the *Prolegomena* becomes controversial as well as the division of labor between mathematician, logician and philosopher (Hua 18, 255). As in 1901, Husserl still denies any polemical intention against logicians and mathematicians, and any revisionist intention[16]. Mathematicians remain the "only competent engineers for effective constructions", while the philosopher's task "resides in a totally different direction": that of "proposing a complementary reflection in the essence and the meaning of fundamental concepts and prevailing fundamental laws, and not, at least, in the considerations of the internal relations of those disciplines to all other disciplines." (Hua 24, 163) The aim of such a philosophical consideration entails immediately a redefinition of the relations of logics to mathematics. We are invited even to "skip traditional syllogistic, which is only a piece of pure mathematics, i.e. pure mathematics of possible propositions and possible predicates in general, and the whole field of theoretic analysis".

But since the main goal is precisely defined as a "deepening and enlargement of the idea of the theory of science", we cannot escape the conclusion that, at least, the delimitation of pure logic and of pure mathematics will not keep untouched.

Since the culminating point appears under the heading of "logic of probability" it is of interest to grasp what Husserl understands under this expression.

2. The problem of the logical foundations of probability calculus following Husserl (following Von Kries)

The turning point leading to a logic of probability as a fundamental part of pure logic is placed under the influence of von Kries. Before Husserl, the opposition between *a priori* and *a posteriori* probability was already criticized, displaced and complicated by Johannes von Kries's investigations on the *Principles of probability calculus* (Von Kries 1886).

This reference, as it is better known by recent publications, although discrete, is important for the subsequent historical development of the logical theory of probability. As we already saw, Husserl borrows from him the distinction between nomological sciences and ontological sciences, but above all the concept of "range" ("range of play" or *Spielraum*) (Heidelberger 2001; Fioretti 1998; Keynes 1921, chap. VII, 97 *passim*)[17]. This notion will become a central concept in the description of the subjective constitutive manifolds, of the horizon structure of consciousness, of the processes of determination, and the dynamical relation between intention and fulfillment. But before that, this notion provides the fundamental concept of Husserl's logic of probability and the fundamental principle for a logical interpretation of probability which be illustrated by Keynes.

Von Kries's investigations on the logical principles of probability calculus help us to understand Husserl's subsequent evolution. Von Kries is aware that a purely subjectivist interpretation of probability theory reaches an impasse and that a logical foundation of the calculus could not depend on ambiguous or arbitrary principles such as the *indifference principle* or the *principle of insufficient reason* (Laplace 1921, 2). He criticizes also the interpretation of "probability propositions" in terms of practical expectation. Because probability calculus does not measure a mental state of belief, but expresses a logical relation between two quantities[18], the transition from the logical concept of probability to measure needs itself to be logically justified and clarified. This way of setting probability calculus, if not circular, as it is frequently argued (Poincaré 1896; von Mises 1957, 67)[19], is at least arbitrary and apply only to countable sets of cases[20]. Even if we put aside the important problem of continuous (or geometrical) probabilities, this amounts to an assumption which holds only for a limited number of cases and limit-cases.

If probabilities are grounded on a set of equipossible *items* (cases, events, experiments, etc.), then "all the secret of probability calculus" lies in the *construction* or the *setting* on a manifold of equiprobable cases, or which amounts to the same of an homogeneous distribution. The principle of indifference or insufficient reason states: two or several cases must be considered as equally possible if, whatever our state of knowledge, we cannot find any reason to hold one for more probable than the other[21]. This presupposition requires a cautious investigation, since otherwise and in case its consistency is fully demonstrated, probability calculus will amount just to a useless symbolic game. This certainly justifies that we *do not* attribute *more weight* to one of the possibilities than to the others, but that surely does not justify the *positive attribution* to each one of a strictly equal weight. The calculation is thus groundless.

That does not mean that we should dismiss any logical approach in favor of an *a posteriori* or frequentist approach. Since we do not want either to dismiss the requirement of a logical clarification and foundation, in favor of an *a posteriori* or frequentist approach or to depend on psychological investigations, we must look for a sound and non-arbitrary logical principle, fundamental enough to account for discrete and continuous, but also homogenous as well as non-homogenous manifolds of possibilities, of possibilities equally and stably loaded and possibilities unequally and unstably loaded (Von Kries 1886, 15). What is at stake here is the enlargement of mathematics to inhomogeneous fields, i.e. fully random spaces, where homogenous distributions are just an important but limit-case, or to put it in modal terms, where possibilities are diverse in intensity or variably loaded. The parallel between the geometrical starting point of the theory of manifolds (with Riemann) and the one dealing with random or stochastic manifolds emerges here[22].

Von Kries investigation is motivated not only by a purely philosophical interest on the so-called foundation of probability, but rather because of persisting difficulties and paradoxes which have accompanied the birth and the development of probability calculus since the time of Fermat and Pascal. As it has been observed from the beginning of its development, this calculus sets innumerable problems of interpretation in its "pure" form, as well as in its so-called "applications", with a constant shift and confusion between statistics and probabilities. It has given way to famous paradoxes (such as D'Alembert's or Bertrand's paradoxes, without mentioning those of quantum physics). The conflict between the combinatorial starting point (enumeration of possibilities) and the continuity character of many probabilities has been partially tamed by Kolmogorov's axiomatization (1931). But such a taming presupposes, as Pierre Cartier notices, rather strong assumptions, those of measure theory and Lebesgue's integral, generalized to abstract spaces (Cartier 1985, 15).

One of those paradoxes, quoted by von Kries, stems directly from the equivalence between "the principle of ignorance" or "insufficient reason" and the equipossibility or equiprobability — to hold a proposition *A* for true or false. Von Kries alludes to other paradoxes such as Bertrand's or to d'Alembert's objections[23]. I follow here Zabell:

> "Before we possessed any means of estimating the magnitudes of the fixed stars, the statement that Sirius was greater than the sun had a probability of exactly 1/2; it was as likely that it would be greater as that it would be smaller; and so of any other star" (212). Using the very same example of Sirius (making it clear that Jevons is von Kries's target), von Kries showed (10–11) how this type of reasoning could be used to arrive at contradictory results. Thus, arguing one way, the probability that there is gold on Sirius is 1/2, that there is iron is similarly 1/2, and therefore that there is neither is 1/4. (Of course there is an— unargued — assumption of *independence* being made here.) Taking the 68 elements known at the time and arguing in similar fashion gives a very small probability that none are present, or equivalently a very large probability that at least one is present. On the other hand, starting immediately from the proposition "Sirius has an earthly element", one immediately arrives at a probability of 1/2." (Zabell 2016, 135-136)

The experiment is the following. In throwing a coin twice, calculate the probability of showing Heads. The classical enumeration of cases (HH, HT, TH, TT) answers 3/4. D'Alembert (1784, 471) argues that if I got H with the first throw, the game is over and a second throw is pointless. The *order* is thus essential and the present order of outcomes produces, so to speak, *a reduction or a collapse of the manifold of possibilities*[24]. This objections seems itself pointless and has been indeed unanimously rejected, to the exception of Bayes in his *Essay* from 1764 (See Zabell 2016, 136). Nonetheless nobody could demonstrate where the argument went wrong.

Von Kries gives other examples which indicate clearly that any probability depends on the assumption that the fundamental manifold is complete – one would say, in Husserl's sense, *definite*. Targeting Stanley Jevons reasoning, Von Kries develops a new paradox, taken from Jevons: "If A and C are wholly unknown things, we have no reason to believe that A is C rather than that it is not C; the antecedent probability is then 1/2." (Jevons 1874; cf. Keynes 1921, 46)

In order to avoid similar paradoxes, Von Kries (1886, 36-37) introduces the principle of ranges (of play) (*das Prinzip der Spielräume*), which states "that assumptions are in a numerically probability relation, if and only if they include *mutually indifferent (original and comparable in size) ranges of play* (*Spielräume*)", and "that certain probability values arise, *where the totality of all possibilities can be exhausted by a number of such assumptions*." (ibid. 36)

This principle is fundamental and logically sufficient. It will be rephrased by Husserl under the title of *fundamental field principle*. If the "fundamental field of equal possibilities is not defined univocally" or if "it is mixed up with a different field", we fall then inevitably into wrong inferences and paradoxes. Husserl considers these paradoxes as "deceiving inferences" (or paralogisms) stemming from the lack of clarity and soundness of the logical foundations of probability calculus. In order to resolve them all on one stroke, he adopts a principle equivalent to that of Von Kries, although disguised under new terms, the "fundamental field" ("*Grundfeld*") principle[25]. "All the deceiving inferences in probabilities and in the theory of probability itself, so much dreaded but still non cleared, are based on the fact that either the fundamental field of equal opportunities has not been defined exactly or univocally, or, in spite of rigorous definitions, that, in the course of reasoning, the initial field has been confused with another one. In this interweaving of probabilistic inferences there are, as a rule, different fields, but always one field is the fundamental field so much so that all the other fields are extracted from it exactly or loosely." (Hua 30, 253-254)

If the expression of "*Spielraum*" is eclipsed as a mathematical or logical term, designating a more subtler form of manifolds, it will reappear as a fundamental phenomenological term, as a fundamental character of any subjective constitutive manifolds, i.e. the fact that it is *open* and never fully *saturated*, i.e. as a constitutive moment of the horizon structure. All the subsequent difficulty lies thus in the conceptualization and formalization of such non-definite (i.e. incomplete) manifolds.

3. From the project of a logic of probability to the project of a reform of formal logic

We must now explain how this principle leads Husserl, contrary to von Kries, to a reform of logic and an enlargement of the formal theory of manifolds. Here phenomenology as such steps into the game.

The major change is the *phenomenologically* enlarged notion of *proposition*, which covers all forms of "positions" or "thesis". These modalities that Husserl names "doxic" along with "axiological and practical" modalities must be understood as noetic as well as noematic determinations. Their explicit thematization opens the larger fields of formal disciplines (including formal axiology, formal theories of action, decision, choice, collective choices, games, etc.).

This enlargement is necessary to fill a gap which persists throughout the historical development of formal logic, as Husserl repeatedly says, in 1923 (in *Erste Philosophie* (Hua 7, 21-22) and in *Formal and Transcendental Logic*, in 1929, as we saw above). The default of formal logic (old and new) resides in the fact that every material and intuitive content is eliminated, because it is wrongly assumed that any intuition and any content are necessarily empirical. Blindness to categorial intuition goes obviously on a par with deafness to modalizations. Nevertheless, it is a disastrous mistake for the very understanding of formalization to eliminate the very possibility of a formal content, a "formal matter" that is conveyed by qualities (i.e. modalities) of acts, in general, and of judgement in particular. Hence Husserl argues that traditional formal logic does not "include amongst its theoretical elements neither the concept of truth nor its derivatives and modalities", i.e. concepts such as "possibility, necessity, probability etc. and their negations". And this represents an "inadmissible restriction" (*unzulässig Beschränkung*) (Hua 7, 26), which has hindered the development of an efficient logic of truth, describing formally "how judgements can reach material adequacy" and "how their truth and falsehood are decidable" (Hua 7, 25). From this "very important lack" (*sehr bedeutsamer Mangel*), ensued serious imperfections of logic, especially in its "methodological procedures" and in the constitution and understanding of formal mathematical fields, and more precisely regarding the definition of probabilities.

The controversies around probabilities, and especially between subjectivist and objectivist, *a priori* and *a posteriori* stem from the fact that modalities (on each side of the battle field) are considered as exclusively subjective, psychological and empirical modifications. From this psychologist prejudices stem also false analogies, such as that between degrees of sensation and degrees of belief (with Wundt, Fechner and Meinong). The discovery of intentionality should prevent from such misleading analogies. If any act has its sensuous and emotional substrate, objectifying acts as well as axiological acts (acts of feeling and willing) have among their inner intentional constituents modalities in the broad and the narrow sense of the term.

This enlargement and deepening, following the intentional correlation, goes on a par with an enlargement of the noematic thesis or "propositions". Moreover, considering that each position and proposition is produced by a kind of modification or "function" that Husserl calls either "qualitative" modification (or "modalization"), the notion of predication and predicative function should be enlarged accordingly. Last but not least, acts of reflection, whatever their kind, for instance those underpinning an act of nominalization (Husserl 2006, 75, 97-105; Husserl 1969, 113-118), are themselves such modifications, and eventually combinations of modal modifications or neutralizations. But to restrict ourselves to probability modifications, phenomenologically speaking, probabilities are *relational modalizations*, comparisons and evaluations of the respective "weight" of manifold possibilities (and within samplings of such manifolds) emerging from spontaneous "thematizations" of specific intentional modifications, "modalizations" of the moment of "belief"[26].

Without dwelling on the details of the theory of science thus promoted, we must insist on the deeper and larger concept of science, resulting from the inclusion of modalities. Instead of being restricted to the "sole knowledge of apodictic truth" i.e. to demonstrative knowledge, the methodology and theory of science must "explore the immense variety of the concrete life enfolding in man's mind, during his intellectual work" in "which he lives without noticing it" (Hua 7, 39-40)[27]. These investigations are not purely informal, and without inputs in the determination of the tasks of formal logic. One of the most noticeable consequences is precisely the proposal of a logic of probability as a fundamental part of formal logic. The inclusion of probabilities in the sphere of formal logic entails thus a reframing of formal logic and mathematics.

The turning point to my view, despite visible hesitations, can be dated from the lessons on *Old and New Logic* from 1909, in which Husserl explicitly mentions the possibility of an enlargement of formal logic through inclusion of probabilities[28]. I can here but give some spot checks on the lessons on *Logic and Theory of Science* given from 1909 onward.

3.1. An enlarged and deepened concept of proposition

First of all, what is the wider formal concept of proposition which gives way to "intensive" or "loaded" possibilities understood as specific propositional functions?

Let us start with judgments and their predicative propositions. The traditional view point considers propositions such like "it is certain that *p"* or "the certainty that *p* is justified" as well as "it is doubtful that *p*", or "there is a doubt whether *p* is valid", etc. as belonging to logic in an enlarged sense. But they are excluded from the description of the basic forms of propositions, i.e. from morphology of meaning or logical grammar defining what are well formed formulas. It is even possible to examine the conditions under which a proposition expressing a doubt, a question, is valid, i.e. is rational. But "they have no place in the frame of the theory of judgement, understood as meaning of acts of judging". "The same thing holds for judgements of possibility and necessity". "As long as their meaning includes, contains, a subjective and empirical content about he who judges, on his opinions, his knowledges, conjectures etc.", "these distinctions have no room in formal logic."[29]

But as soon as we get rid of psychologist assumptions, we must admit that every judgement contains a certain "quality", i.e. a certain modality (inclusively plain "assertions") and that an "unqualified" (or "non-modalized") judgement is nothing but an *abstract constituent* obtained through a *sui generis* modification (precisely that of "bracketing" or neutralization), which instantly displaces outside the brackets the original mode of assertion (of certainty) — Frege's famous assertion stroke. Yet a universal bracketing remains ideally possible.

The fact that currently probability judgements appear as *secondary forms* of judgments, as judgments *about previous* judgements, does not entail that elementary (or "first order") judgements be deprived of any quality; more precisely, that the content of original judgments or even of primary representations should be deprived of any modal component[30]. The primitive and fundamental form of judgement is always a compound of modalities (characters of positionality) and a mere "as such" (*als Was*), a mere something which, without those modal characters, remains formally an empty "something whatever" (*eine leere beliebige Etwas*) (Hua 30, 106, 140). Consequently, we must admit as original propositional forms: the "proposition of truth" (in the narrow sense of the term), the proposition of probability, the proposition of question of knowledge (*Wissensfrage*), the proposition of doubt but also propositions of will, of wish, and their corresponding sub-modalities etc. (Hua 28, 119 *et seq.*)[31]

Moreover, and generally speaking, against the common prejudice at the basis of the so-called "linguistic turn" in philosophy, the proposition in the narrow sense of the term, i.e. as *expression* of a predicative act presuppose the later one as such, i.e. as an *expressed* act, which involves or at least presupposes pre-linguistic and pre-grammatical acts. By limiting our consideration to the series of propositions characterized by their doxical character, as "holding-for-true-something" and correlatively "holding-for-being-something" in the various modes of "holding-for-certain", " for-possible", "probable" etc. these acts are presupposed by their expression, and exist, at least ideally, be they expressed or not[32].

3.2. Consecutive enlargement of formal logic and constitution of logic of probability

The domain thus delineated is "nomological" in the proper and deeper sense of the term, since logic is fundamentally an examination of all the modes and essential laws governing the "position-of-truth" (*Wahr-Setzung*) as a quality of act. Husserl not only admits a "formal logic of qualitative modalities (*eine formale Logik der qualitative Modalitäten*), of possibilities and probabilities as well as a formal logic of problematicities (*eine formale Logik der Fraglichkeiten*)", as a discipline belonging to the same ideal sphere (that of pure logic), but he asserts that "pure logic" is two-fold, and that we must admit as a first and fundamental group of logical laws, "the laws of probabilities, of presumed possibilities", i.e. the rules that are at the basis of the rational norms of validity of probabilistic inferences, of questioning, problematizing, doubting, etc. (Hua 30, 79). This norms (and the *Normierung* modification as such) are not grounded on psychological empirical findings, but on an eidetic analysis of intentional essences, and the delineation of the central constituent of every intellectual activity, the "sphere of doxic positionality", of "acts of belief", of "holding-for-true" in the larger sense of the term.

The perspective of the theory of justification of acts of knowledge is not limited to the connection of assertive judgments, nor to proof theory, but extends to the all sphere of acts partaking in the process of justification (*Rechtsausweisung*) of judgements (such as perception, memories, etc.). Husserl describes this as an enlarged theory of epistemological norms encompassing a wide range of "pure disciplines" still to be constituted, which, if they were constituted, would enable us to reduce to ideal principles and decide according to those principles, any epistemological situation, any actual case of judging in all its form (presumptive judgement, probable judgement, etc. as well as any actual case of founding, justifying, inferring deductively, explaining, or inferring inductively, etc.).[33]

We understand better why the critic of logical reason started here entails a reform of logic and an interventionist conception of the epistemological role of phenomenology, for this enlargement underpins a new settings of the norms of validity, starting from those of the holding-for-true. These norms don't apply exclusively to the "lived experience" of assertive judgements, but to every act of judgement "in the widest sense of the term", i.e. "lived experiences of holding-for-possible and holding for probable, of questioning and doubting". And since those norms are intimately connected to mathematical forms and norms, "a new perspective of interpretation of many norms of the pure mathematics is thus opened up."[34]

3.3. Logic of probability and manifolds of intensive or loaded possibilities

This Idea of a logic of probability leads to that of a formal logic in a deeper sense, to "an enlargement of the idea of pure logic into a pure logic of probabilities and possibilities", and, correlatively, intertwined with it "to an enlarged pure arithmetic and pure theory of manifolds."[35]

Probability fields as modal manifolds. The pure formal manifolds must be classified following deeper principles that the usual one (discrete *vs* continuous, measurable, countable, signature and degree of curvature, with or without torsion, connected or not, etc.), i.e. the *forms of possibility*. Beyond the distinction between physical and logical and mathematical possibilities in the usual sense of the term, we must distinguish between analytical possibilities, synthetic *a priori* possibilities. But the latter ones must be divided in turn into "extensive" possibilities and intensive possibilities.

Probabilities are "intensive" i.e. founded possibilities, in as much as they are "loaded", so to speak, because something "talks in favor" of them. In contrast, classical mathematical possibilities are "empty possibilities", "mere imaginations", in as much as nothing talks in favor of them, except the general fact that they can be deduced or constructed. *They are modally flat, of nil curvature*, to speak analogically. A probability as an empirical conjecture or a presumption (*Vermutlichkeit*) is a founded possibility. A likelihood or a plausibility is the same thing than a "probability": a *founded possibility*, a possibility implying and presupposing fundaments of plausibility — a possibility loaded with diverse fundaments, variable in number and weight. Contrary to a misleading analogy, these possibilities are not necessarily discrete and independent, nor "continuous" and extensive, but they must be connected. They represent a primitive form of connected manifold, maybe more fundamental than the connected manifolds developed in the wake of Riemann's prophetic conference by Weyl and E. Cartan (Cartan 1923, 326).

Measure. What is usually expressed in terms of *degrees of intensity*, or in subjective terms, of degrees of belief[36], are an improper expression of the *number of foundations of probability*. This point is very important for a justification and a setting of the fundamental algebraic operations of addition and multiplication, before secondary distinctions such as that between discrete and continuous probabilities.

The subjective expression talks of: "*more or less strong or weak presumption*". In contrast Husserl says: "the *presumption is reinforced by the number of foundations* of *probabilities*: the more things speak against a probability and the more the probability decreases." (Hua 30, 252) As a founded mode of possibility, an intensive probability implies a countable sub-manifold of fundaments of possibility.

Husserl goes on analyzing this modal field. He who has learned, through the analysis of intentionality, to separate what is on the side of consciousness (or *noesis*) and what is on the side of meaning (or *noema*), will recognize, without difficulty, in the present instance, an "*objective expression*", i.e. that the founded possibilities as well as their fundaments are the correlates, contents or "significations" of new acts.

These founded or loaded possibilities are originally ruled by a *relation of preference and intensification*, and consequently by *laws of increase*. The manifolds of possibilities are ordered following these relations of intensification, with a simple possibility (without negative or positive fundaments), positive and negative intensities of possibility, comparable to positive and negative quality in the sphere of assertions. A difference remains, for in the sphere of assertions, "there are no preferences and intensifications", whereas in the former, "in the domain of probability", "they play such an important role." (Hua 30, 253)

Negation and quantification. A negative presumption, something speaks against "A is B" is equivalent to "something speaks in favor of the fact that 'A is not B'". For this reason, we have a rich variety of negatives possibilities. Before any partition, we have here an original domain of additive positive and negative magnitudes. Two possibilities with equal weights are indifferent in a totally different sense than the empty indifference ("nothing speaks in favor of *p*"). The first indifference is an equality of weight between negative and positive fundaments, an "absolute problematicity". In case there is no indifference between two possibilities, one is necessarily heavier than the other one. It is then strictly more probable that *p* than *non p*. Probability in the strict sense is thus the *relative overweight* of the positive motives – compared to the negative motives, if any. In some circumstances, vague expressions "strong", "weak", "very restricted" probability can be converted into exact ones (numerical or measures), but often it is not possible (Hua 30, 253).

4. Snapshots in the twilight[37]

On the footsteps of Husserl, the mathematician and phenomenologist Gian-Carlo Rota suggested in different papers, that Husserl's phenomenology was aiming at providing logic with new fundamental concepts, new constants. He insisted simultaneously on the limits of the logical syntactical approach to probability and even expressed strong doubts that probability theory, despite its axiomatization (after Kolmogorov), possessed any true syntax. Rota in his incentives investigations on the foundations of probability theory and statistics aimed at bridging the gap between probability theory and other mathematical theories, including some parts of first order logic and algebra. Rota mentions, as a possible application and as "the most promising outcome", the translation of "the notion of quantifier on a Boolean algebra" into that of "linear averaging operator": "in this way, problems in first order logic can be translated into problems about commuting sets of averaging operators on commutative rings" (Rota, 1973).

The semi-formalized analyses of phenomenology anticipate thus quite strikingly Rota's suggestion that the general form of conditional probability is similar to Reynolds operator (Lobo 2017b, 156-170). The fact that this operator is used in what is considered as belonging to "applied mathematics" is not a reasonable objection. Historically, most of the formal mathematical theories (Euclidian geometry, vector calculus, graph theory, etc.) have emerged from semi-formal fields, and have been only secondarily "purified", that is detached from their empirical or "material" (*sachhaltig*) clothing.

My guess is that by symbolizing the relations and the laws exposed by Husserl in this text and later writings, especially *Ideas I*, we get very close to a form of operator. Mathematically: the logical expression and formalization of this system of modifications gives way to a *linear functional* which constitute the hidden hypothesis of the so-called axioms of probability calculus (in Kolmogorov). By formalizing it, we obtain an operator which is similar to Reynolds operator, which is known in algebra as *Reynolds operator* and in other fields as *averaging operator,* and writes: $Au = u$, $Az = z$. Would not it be possible, by introducing "belief" functions, under the form of modal functions, to obtain, via an adequate formalization, an operator of the type $A(fAg) = AfAg$?

4.1. Phenomenology, algebraic logic and logic of probability

But this requires more generally to understand better Husserl's position toward algebraic logic. Beyond the current characterization of formalization as algebraic transformation, that is an emptying of any material reference or content, Husserl's reception of algebraic logic (as promoted by Boole, Peirce, and subsequently by Halmos and others) has been concealed by the focus of the dominant debate (formalism *vs* logicism *vs* intuitionism; Hilbert, Frege and Heyting). Yet, Husserl constantly considered Boole as "an outstanding technician in logic", although "a very mediocre philosopher of logic" (Husserl 1994, 59; Hua 22, 9)[38]. One must consequently not use the philosophical occasional nonsenses in which he fell as an excuse to reject his "splendid" logical construction. Husserl's enthusiasm of the early years is still perceivable in later texts from 1913 onward (Hua 24,162; Hua 17, 83; Hua 30, 271-272). Against the critiques from the side of logicians, as well as the attacks from philosophers (such as Lotze or Windelband) (Hua 24, 162, trans. 160; cf. Hua 30, 248-249), he praises Boole for having achieved "at one stroke" almost miraculously (Husserl 1994, 88; Hua 22, 40), a logical calculus. The "reduction", i.e. "ingenious transference of the arithmetical algorithm over the domain of class, through which the class calculus stood forth at one stroke, is almost a miracle" (Husserl 1994, 88; Hua 22, 40; Husserl 1994, 441), showing convincingly that class calculus and arithmetic were but two provinces of the same country (Hua 17, 203; Husserl 1969, 78). The interpretations of the 0 and 1 (as meaning respectively the logical universe or the total class and the null class) may lead to absurdities, as demonstrated convincingly by Schröder (Husserl 1994, 84; Hua 22, 35-36)[39]. But Schröder's argument is itself considered by Husserl as "sophistical": it rests on a confusion between "subordinate class" and "element", and correlatively between two separate relations (inclusion and membership). This demonstrates only that Boole's method must not be applied blindly, but this does not concern the "technical" as well as the "mathematical" presentation, which Husserl considers "exemplary" (Husserl 1994, 88; Hua 22, 40) and of indubitable "superiority" over the old methods of inferring (Husserl 1994, 90; Hua 22, 42). Similarly, Schröder and Venn's substitution of the concept of "identity relation" to the Boolean concept of "exclusive addition" demonstrates rather the superiority of the later concept, since this concept alone enabled "the ingenious transference of the arithmetical algorithm to the domain of classes" and the miracle of a "logical calculus".

Above all: Husserl adopts the same attitude towards Venn's critiques of Boole's choosing the "exclusive addition" instead "identity addition" and so-called improvements (Husserl 1994, 88; Hua 22, 40). Against Venn, Husserl argues that the importance of the calculus does not lie in its practical applications and value (for instance, spare of time in reasoning). Boole's algorithms are worth logically and mathematically, not practically, although Husserl does not exclude that future epistemological situation requiring such sophistication and other "fruitful applications". It is true that, from a theoretical point of view, the current scientific forms of reasoning (in mathematics and physics) do not enter into inferential complexity such as to require the sophisticated apparatus provided by Boole[40]. But once again this is no objection against Boole main theoretical goal, among which the application of its logical algebra to the calculation of probabilities, and its analysis and elucidation of probability. For, this "application" presupposes the distinction between "conceptual extensions" and "sets", and of the formally defined identity relation over sets[41]. Boole has thus shown that the mathematical treatment of probability "*is itself a part of logic*" (Hua 17, 203; Husserl 1969, 78). This "application" is absolutely central in the project of Boole[42] and has survived through the contemporary branch of algebraic logic (see Rota 1973; Ellerman & Rota 1978). The brilliant insight of Boole of the deep formal analogy between arithmetic and syllogistic (Hua 30, 272) has given the first sample of successful formalization in the field of logic, so frequently misunderstood by philosophers in their polemics against "mathematizing logic", as well as by the mathematicians, unaware of this essential distinction within the field of arithmetic (Hua 30, 271) and of the relation from purely formal theories (which are merely hypothetical theories) to mathematical theories (which without being applied nor material are nonetheless true, i.e. categorical) (Hua 30, 273)[43]. Such is the meaning of Boole's application to probability calculus. Its purpose is to logically clarify the logical underpinnings of probability inferences as a special form of deductive inference.

Nonetheless, Boole's logical analysis of probability calculus appears incomplete to Husserl from a logical point of view viz. from a noetic point of view. As we already saw, one of the turning points dates 1909[44]. A systematic exposition of the forms of judgements stemming from the modal modification of belief (of the "holding-for-true") and their noematic correlates is missing. By restricting the focus on one form of judgment and proposition (the categorical form), the noematic notion of proposition has been mixed up with the "apophantical" one, and, subsequently, the full extension of both has been narrowed (Hua 3, §§ 133-134; see Lobo 2011).

In order to formulate the most general formal laws of thought, it is crucial to start from the larger sphere of "judgment forms" and grasp the relation of categorial form to the other apophantical forms. There is also a need of a closer definition of the "qualities" (i.e. modalities) of judgement ("possibility, probability, problematicity"). Husserl's logic of probability appears as an extension and mutation of its "formal logic of content". But the "formal content" is a "modal content" gained in "a modalized intuition" or "*a modal intuition*" (sic) (Hua 23, 418) which is, as Husserl suggests, a "*categorial intuition*", and the larger theory of proposition shall cover all forms of propositions those expressing modes of the "holding-for-true" (such as possibility, probability, etc.) which enter the sphere of formal logic (Hua 30, 250-251; Hua 17, *n*. § 35, *n*. § 50; Lobo 2018), but also, axiological and practical propositions, entailing the diversity of modes of the "holding-for-worth" and, correlative, manifolds of forms of values. This enlargement is required in order to distinguish between different forms of possibility (analytical possibility, synthetic *a priori* possibilities, conditioned possibilities, rationally motivated possibilities, ordered or not, diversely loaded, etc.), which are usually mixed up in mathematics and in probability theory, and ignored or misinterpreted by the logicians.

> "The up-to-now unresolved philosophical and factual difficulties connected with the founding of probability theory are all based on the fact that, on the one hand, the distinction between the psychological and the theoretical side of meaning has not be carried out, and, on the other hand, one does not understand the concept of possibility, which is fundamental for probability theory, which must be distinguished from other logical concepts of possibility. This must, from the outset, relate to and recall us of our previous remarks on modalities." (Hua 30, 250)

This analysis refers back to a phenomenological analysis and its noetic-noematic distinctions, and within it, to the analysis of the central sphere of modalities (i.e. of positionality)[45].

In the battle field of probability, Husserl takes a stance, on the footsteps of Boole (but also Von Kries)[46], on the side of the "logic of probability". This position goes on a par with his theory of modalities, rooted in the phenomenological investigation of the "correlational a priori" (i.e. intentionality), and forbids any assignment of his projected formal logic neither on the side of subjectivist interpretation of probability, nor on the naively objectivist side[47]. Meanwhile, there is room for a non-naive objective theory of probabilities and a transcendentally rooted subjective interpretation of probabilities as modal modifications of the holding-for-true, of intentional *meaning*.

For Husserl, the study of belief and its modifications belong in the theory of judgment and its forms (Lobo 2018a; 2018b; 2017). In the light of the "correlational a priori" (i.e. noetic-noematic) analysis, to each mode corresponds a mode of sense of being, i.e. a new mode of being. The task of formal logic is to express both, and the investigation of modalities shall appear in this light as the fundamental and larger basis of formal apophantic and formal ontology. In other words, modes of belief are the correlates of a dependent moment of any objectifying intentionality, not exclusively of judgment. A judgement, which is a "holding-for-true" (in any of its modes) and would not *opine* (without a *Meinen*) is just as absurd as the denial of the intentional character of perception. The *Psychological Studies in the Elements of Logic* [or *Elementary Logic*] from 1894, represent already a breaking point with those who deny any modal (qualitative) content to representations such as perception (*Wahrnehmung*), whereas any perception entails intentionally a holding-for-true (*Für-wahr-halten*)[48].

In the review of W. Jerusalem's "Glaube und Urteil" (1894) which aims at clarifying "the rather confused relationship between belief and judgment" (Hua 22, 135; Husserl 1994, 181), Husserl's pithy objection to the main thesis ("Belief is nothing other than a *feeling which accompanies the* *judgment's holding of something to be so*") is unambiguous: "One can hardly expect advancement of the theory of judgment from such fictions as these". In the recension of J. Bergmann's *Die Grundprobleme der Logik*, (Hua 22, 180-245) Husserl assumes that the proposition which supports the negation is not the affirmative judgment, but "only to the *signification content* of the judgment, *abstracted from the belief character*, that talk of containing or being contained could have any reference – and even then it must not be taken literally." (Hua 22, 185; Husserl 1994, 230) But further on, against Brentano and Bergman, he considers that the meaning of the proposition, deprived of this moment of belief, must at the same time retain something of the belief. Their position and the correlative classification of acts (of psychical phenomena[49] (in three classes: representations, judgments, feelings and volitions) is criticized and, in the case of judgment, the main argument presented above rests on a confusion between "two essentially different relationships: 1. the relationship between the mere representation underlying the judgment and the *belief-Moment consummating the judgment,* and 2. the relationship between a plain and simple judgment and the judgment on it. Belief is not something "added" to "representation" in order to convert it into a judgment. Husserl claims: "in my opinion, we have to include under 'representation' the total signification content of the judgment, *the whole of the meaning of the assertion*". Consequently, "we cannot, *as in Brentano and Bergmann*, restrict 'representation' to the (nominal) representation of the object taken as subject, even though it were to include in one content the representations of the determining properties predicated of the object." (Hua 22,186; Husserl 1994, 231). It seems that we should divide accordingly the notion of "belief": (1) "the 'belief," as "characteristic of certainty or conviction" and (2) "the belief belonging to the *content or matter* of the judgment as such." (Hua 22, 186; Husserl 1994, 231) Moving away from Brentano's lesson (distinction between matter and quality of the act of judgement, i.e. meaning and belief), Husserl holds that "*the matter [i.e.* "*what* is believed"] *is not the representation*, possibly as it existed prior to predicative articulation, and it is no representation expressible by a name";

the quality of judgment is to Husserl's view "no acknowledgement or rejection directed upon such a representation", but belongs, as a constituting part, to the content, to the meaning. The copula "the 'is' […] is nothing less than an expression of 'belief,' and much less then is the "*is not*" an expression of a co-ordinate 'unbelief.' *Rather, the positing and 'certainty' characteristic belongs to the matter as a whole*, regardless of however it may further articulate itself into parts. The usual expressions for this characteristic – '*holding to be true,' 'believing,' 'consciousness of validity*,' and the like - all *suggest the erroneous view that we have here a predication of truth, validity or correctness upon the matter, and moreover, that we must here distinguish two co-ordinate* qualities: a *holding-to-be-true and a holding-to-be-false***.** Even this latter point does not seem to be absolutely beyond all doubt. Every (normal) assertion expresses a judgment, but every judgment also finds its expression in a possible assertion". And the final conclusion: "incorrect". "In each case the expression of the rejection, of the non-belief or the untruth, *pertains to the matter of the assertion*; and what makes it an assertion is not the non-belief predicated, but rather the *character of conviction or of 'believing' which as it were animates the matter*. *Every asserting is a believing*." (Hua 22, 185-186, emphasis mine)

To reject the foundation of validity upon belief considered as a feeling or a habit as well as its defence (as we find in von Kries) amounts to rest on symmetric confusions. One does not take into consideration that the modes of belief "belong primarily to the content: more precisely, to its logical forms" (Hua 22, 226; Husserl 1994, 280) This is true for each categorial form and their validation. Each one has a "type of intuitive realization which is just its own." But these differences effect the "validity feeling," the belief, only insofar as it is belief with a content of this or that form. They belong primarily to the content: more precisely, to its logical forms.

4.2. Formal operators stemming from pure phenomenological reflection

In order to understand more precisely how logical forms related to probability emerge from the phenomenological analysis, let us follow here some indications given by Husserl, which are concentrated on the forms of belief, their correlates and their modifications. As every phenomenologist knows, any lived experience is intentionally structured and, roughly speaking, constituted of two components: a real (reell) and a unreal or ideal one, called also respectively, *hyletic* and *intentional* components.

Let us denote this lived experience: *e* and express its composition by $e = \{r; i\}$ or $e = \{e_r; e_i\}$. And we can, by mere abstraction, explore either sides, and for a start, in a purely static manner. The analytic of real components and their "combinations" amounts to the analysis of the connections between productive (*erzeugende*) modifications, in other words, of their syntax or synthesis, which, by *modifying modifications of real components, modify the whole lived experience*. This analysis represents the new path of the phenomenological transcendental aesthetics or "hyletic" phenomenology, which covers diverse groups, systems or structures of modifications (temporal, spatial, kinesthetic, not to mention impulses, passive affectivity and tendencies). Historically Husserl started with the constitution of time and space, *as systems of modifications*. The combinations of these groups of modifications are called "*continuous syntheses*" by Husserl.

In *Ideas I*, §§ 84 and following, the analysis of the group of modifications qualified as real (*reell* in contrast with *real*) are characterized as "productive" (*Erzeugende*), since they give way to new phenomenological unities. Through such synthesis, we obtain, for instance, a temporalization of lived experiences, and eventually their insertion into one unique flow. Such is the case of retention, which apply to a former retention and transitively to the whole lived experience just retained. This is, for example, the case for "consciousness of delight". It is given "in a continuum of consciousness, which forms remains firm". It presupposes an "impressional phase", which is just a "limit-phase with regard to the continuum of retentions". The following analysis indicates precisely the way this continuum takes on the form of *a flow*. Since the impressional-phase and the retentional-phase do not belong to the same level, the retentional flow is "conjugated" to the continuous import of new impressional phases as limits and starting points of ever new retentional-phases; "they are related to each other continuously and intentionally under the form of a continuous embedding of retentions of retentions." The former retentional phase "combines" (*fügt sich*) with the new impressional phase, and the new retentional phase applies not only to the new impressional phase, but to the whole conjunction; "continuously the impression converts into a retention, and the latter continuously in modified retention, and so forth". All this is, of course, described through eidetic variation and under transcendental reduction.

This amounts to build up a kind of linear operator of the type $Au = u$, $Az = z$ and more precisely of the type $A(fAg) = Af\,Ag$. In order to justify this symbolic transcription, let us repeat and abbreviate the former analysis of retention. Any *e* (lived experience or *Erlebniss*) is submitted to a retentional modification, which is a continuous synthesis of retentions of retentions. For any *e,* holds the following proposition:

$$r(e) = e$$

And as each new retention-phase is conjugated, so to speak, with a new impressional-phase and its new retentional phase, we get:

$$r(e)\ r\ (e') = r\ (e'\ r\ (e))$$

This is an analog of a "averaging operator", or else, Reynolds operator used in fluid dynamics, and in functional analysis or in invariant theory. A Reynolds operator is, algebraically, a linear operator acting on algebraic functions. If any *e* defines an intentional function, as Husserl suggests, the retentional modification as a real modification applies as a similar operator. And as we write Reynolds operator $R(\varphi)$, $P(\varphi)$, or $\rho(\varphi)$, we may write the retentional operator under the form R and the every time new *e* using prime numbers as indices or the prime symbol ʼ. And as, for every two functions φ, ψ, Reynolds operators satisfy the following condition:

$$R(R(\varphi)\psi) = R(\varphi)R(\psi) \text{ for every } \varphi,\ \psi.$$

The linear retentional modification should be written:

$$r(e)\ r\ (e') = r\ (e'\ r\ (e))$$

And, as for the Reynold operator, here too the following condition holds:

$$R(\varphi\psi) = R(\varphi)R(\psi) + R((\varphi - R(\varphi))(\psi - R(\psi)))$$

for all φ, ψ.

We should consequently find for the retentional operator, for every two lived experiences *e, e',* the following condition:

$$Rét(e.e') = Rét(e)Rét(e') + Rét\ (e - Rét(e))\ (e'\text{-}Rét(e')))$$

Which is to be interpreted: *the retention of two lived experiences* (for instance the hearing of sound *a* and the sound *b* or *a'*) *is the product of the retention of* a *and the retention of* b *to which is added* (fügt sich) *the retention of the product of the difference of* e *minus the retention of* e *and the difference of* e' *minus the retention of e'.*

Other condition, which holds also:

$$R(R(\varphi)\psi) = R(\varphi)R(\psi) \text{ for every } \varphi, \psi.$$

Phenomenologically:

$$R(R(e)e') = R\ (e)R(e') \text{ for every } \varphi, \psi.$$

In words: *Every retention of the product of a retention of an* e *and an* e' *is the product of the retention of* e *and the retention of* e'.

Last, the condition :

$$R(R(\varphi)) = R(\varphi),$$

which states that for every φ *R* is an averaging operator if and only if it is a Reynolds operator. Similarly, we have for retentions: *R(R(e) = R(e).* A retention of a retention of a lived experience is itself a retention of a lived experience. We are her at the starting point of a *chronometry* on which is rooted the constitution of the consciousness of the *etc.*

In order to understand why and how, we must shift to the other great group of modifications and the correlative components, those which we named "unreal" or "ideal" components.

Among those components we find all the constituting elements of the noetic and noematic structures, analyzed as meaning (noetic and noematic). And in order to go straight to the nucleus which is directly concerned by the present issue (that of foundation of a logic of probability), let us focus on the group of *modal modifications*, i.e. of modalizations, and more specifically the subgroup of *doxic modifications*.

As we learn from Husserl, the sphere of modal modifications does not restrict itself to the sphere of judgement in the strict sense of the term, nor to that of predication. All lived experiences (nomination, perception, memory, and even imagination) have a kinship with judgements understood as predicative certainty. A first enlargement concerns the sphere of derived modal forms of certainty: *suppositions, conjectures, doubts, refusal* as well the correlates corresponding to them. The noema nucleus is just the invariant through a series of different characterizations. "The same S *is* P which represents the noematic nucleus can be part a certainty, the supposition of a possibility or of a conjecture, etc." (Hua 3, 196-197).

Let us denote the set of modal modifications by Greek capitals with indices M_1, M_2, ... M_n in order to remind that those *modifications* (of the acts of taking for true, or for real, for being), are operators rather than mere functions.

But with that reservation in mind, we may use small letters $\mu_1, \mu_2, \ldots, \mu_n$. If we specify, these modification as *thetic* modifications, we should write: $\theta_1, \theta_2, \ldots, \theta_n$.

Last, in order to distinguish within the sphere of thetic modifications, the two major subgroups of axiological and doxic modifications, let us use respectively write*:* $a_1, a_2, \ldots, a_{n1}$ and $\delta_1, \delta_2, \ldots, \delta_n$.

This subgroup is a sub-sphere of the "sphere of positionality", which obeys certain laws, from which stem the so-called laws of logic – at least if we admit the goals assigned to transcendental logic by Husserl, which is to describe the emergence and growth of logical forms from the soil of the most primitive forms of synthesis (passive synthesis, continuous synthesis, kinesthetic synthesis, etc.)

What is particularly new in the way phenomenology conducts this investigation into the original soil of logic is that they lead to the structural fundamental laws which are at the same time laws of the "fundaments" of new logics. Those fundaments involve precisely the laws of "combinations" exposed by the newer and enlarged logical grammar promoted by Husserl, which is undoubtedly larger than that which was examined by the traditional or current logical grammars.

Once again this enlargement stems from the implementation of the *epokhè*, i.e. the bracketing of the natural thesis. The phenomenological laws define well-formed acts, whether those acts be performed psychologically, or by human consciousness, or not. For instance, we know that it is *a priori* possible to reflect on a reflection, *in infinitum*, even though no human mind has ever actually done it, for obvious reasons.

Among the original constants or modifications introduced by phenomenology which opens this larger field of possible syntaxes, we must count the modification of neutrality, or neutralization. *Every lived experience can be "expressed" under the form of a combination of modal or thetic modifications and neutralization.* And more especially: every objectifying intentionality as a doxic form can be expressed under the form of a combination of neutrality character and doxic characters. Following the correlational *a priori*, this holds for the noetic as well as for the noematic side. And since the modalization apply to both sides, as in real modification, noetic modalities have their noematic counterparts; but each noetic modification produces a new modified correlate in which the former noetic modal character is infused. A doubt about a perception of A transforms the "*perceived A*" into a "*maybe perceived A*", or a "*possible illusion or semblance of A*", or a "*misidentified non-A*", etc. Or to stand by the sphere of judgment, let us think of a nominalization of *S est P*, though which the assertive force is not excluded but "thematized", i.e. incorporated to the "matter" of the new consciousness, that of the nominal form *"the S which is P"* or "*the Sp"*, which in turn can be transformed (as we have just done it by quoting this nominal form into inverted comas, as an example).

The sphere of modalizations forms a group, in as much as they form a "monoid" of the type $\{M, *, \mu\}$ where every *e* (*Erlebniss*) is, among other things, but essentially, a combination of the type: $\mu_1 * \mu_2$. Thanks to the neutrality modification, the internal law of composition, and associativity,

1. $\mu_1 * \mu_2 = \mu_3$
2. $(\mu_1 * \mu_2) * \mu_3 = \mu_1 * (\mu_2 * \mu_3)$
3. $\mu * \nu = \mu$

the modal group of modification is a monoid. But in order to get a full group structure, a sub-sphere of symmetrical elements should be added, corresponding to *opposite* elements, denoted $\mu^{-1}$and such that $\mu * \mu^{-1} = \nu$. But this seems rather artificial. Moreover we must observe a peculiarity of the neutrality, which causes some perplexity about this attempt of notation.

The first complication touches the ambiguity of the counterparts. The neutrality operation, which could be that of an explicit *épochè*, can also take the form of the *quasi-modification* (i.e. a neutrality modification of the "pure fantasy" type) which produces an imaginary counterpart. Of course, we can admit as many actual replicas of any lived experience as possible, all the more if we remember that any actual *e* is just an arbitrary instantiation, an example taken out of an eidetic extension of the type to which *e* belongs. The same holds for the modal components of *e*. Through the neutrality modification, the first manifold faces a manifold of imaginary counterparts.

The second complication comes from the use of the parenthesis, as neutrality operators. If we admit that the parenthesis themselves are a form of neutralization, this renders even more problematic our attempt, since the associative propriety expressed above would amount to a mere tautology:

$$\mu * \nu = \nu * \mu = (\mu)$$

In order to avoid such collapse, we should write:

1. $\mu_1 * \mu_2 = \mu_3$ with $\mu_1, \mu_2, \mu_3 \in \mathrm{M}$
2. $\nu * \mu = (\mu)$ for all $\mu \in \mathrm{M}$
3. $(\mu_1 * \mu_2) * \mu_3 = \mu_1 * (\mu_2 * \mu_3)$

But maybe, this collapse is *significant* and *useful*. It would unable to understand (and solve) many paradoxes, such as d'Alembert's or a heavier ones, such as the renormalization in quantum mechanics known as wave packet collapse.

Associativity appears in this case a special form of synthesis, corresponding to that which Husserl calls « polytheses », combination of theses of the same level, without any foundation (*Fundierung*) relation or other kinds of modification producing differences of levels.

The sub-group of doxic modifications is essential to understand the emergence of probability. The doxic theses satisfy the following conditions:

1. $\delta_1 * \delta_2 = \delta_3$ with $\delta_1, \delta_2, \delta_3 \in \mathrm{M}$
2. $\nu * \delta = (\delta)$
3. $(\delta_1 * \delta_2) * \delta_3 = \delta_1 * (\delta_2 * \delta_3) = \delta_1 * \delta_2 * \delta_3$

But

$$\delta_1 * \delta_2 * \delta_3 \neq (\delta_1 * \delta_2 * \delta_3)$$

i.e. a *combination of doxic thesis is different from the neutralization of this combination*. (2) must be read: *a doxic thesis combined with a neutralization equals the same thesis*. (3) is a more fundamental form of associativity, in comparison with which the usual associativity appears as a particular and derived case. This associativity states: *two combined neutralized theses combined with a third one, which is not neutralized, equals the first one not neutralized combined with the neutralized combination of the other two*. Or else, *a neutralization inserted in a combination of the same level but not fully neutralized does not change the doxic compound form*.

The linearization of those doxic modalities is obtained by the distinction of two levels of combination, with neutrality ν and doxic thesis δ, and the introduction of a modalization of any of the two, or of both. Let us call δ_1 and δ_2 two doxic thesis, M any modalization. The following operator

$$M(\delta_1 M\delta_2) = M\delta_1 M\delta_2$$

means that *the modalization of the product of* δ_1 *and the same modalization of* δ_2 is tantamount *the product of the modalization of* δ_1 *and the same modalization of* δ_2. As an approximation: "the doubt about (the certainty) δ_1 and the doubt of the certainty δ_2 is equal to the product of the doubt on

δ_1 and the doubt of the certainty δ_2". The substitution of ν to the neutralization-parenthesis () gives the following formula:

$$M\nu\delta_1 \, M\nu\delta_2 \, = \, M\delta_1 \, M\delta_2$$

and means exactly the same thing: "the doubt about the certainty δ_1 and the doubt of the certainty δ_2 is equal to the product of the doubt on δ_1 and the doubt of the certainty δ_2" , as long as the whole doxic compound is not neutralized, any compound is *absorbed* into that modalization.

From the application of the parenthesis or neutralization to the whole doxic compound results a neutral compound, an "imaginary" counterpart. And we understand why and in which very precise sense the *imaginary or fictitious represents the element of phenomenology*, here under the elementary form of doxic syntaxes giving and founding the meaning of probabilities.

NOTES

[1] "A more serious fault of the *Prolegomena* is, by the way, the following: In connection with the concept of truth the modalities of truth are not mentioned, and probability is not cited as one of them. When they are taken into account, an enlargement of formal logic becomes necessary: to the effect that, as universal formal possibilities, modal variants of judging and of judgments enter into certainty- or truth logic - because any such variant can enter into the predicational content of the judgment and, when it does it must not be regarded as extra-formal. In other words, only the content that goes beyond anything-whatever is the « matter » of judgments, in the sense proper to formal logic; all the forms in which one judges _ not only with certainty but also in the mode of possibility, or in other modalities - belong to anything-whatever. A kindred enlargement results from taking into consideration the fact that emotion, and volitions also bring modalities of anything-whatever', which are introduced in the same manner into the dox sphere. (On this last point ct. *Ideen* . pp. 243f1. [English translation, pp. 531f1.]: also § 50 pp, 135 ff., *infra*.)." (Husserl [Cairns] 1969, 101)

[2] Starting with a S5 reduced system of modalities, Becker proposes a "statistical model" or interpretation for the modal calculus of first degree, then for composed modalities (§2). This statistical interpretation is founded on an analogy between modal calculus and classical probability calculus. The quotient 1 for necessary; the quotient 0, for impossible or necessary not. For the interpretation of the range of possibilities, Becker proposes that a proposition holds as true if at least superior to 1/2 otherwise the proposition must hold as not true.

[3] This is the subtitle of *Formal and Transcendental Logic* (Husserl 1969).
[4] Against Kant's conviction as well as that of its modern enemies, that it was *almost* achieved from the beginning; Husserl thinks that it is only at its very beginnings (see Hua 28, 244-245).
[5] Before the influential contribution of Keynes, and the later works of Carnap (1945), Husserl had in view Boole's *Laws of Thought* (1854) and John Venn's *Logic of Chance* (1876).
[6] Keynes (1921, *nn.* 2, 5) notes that the first who took notice of that was Ancillon, in *Doutes sur les bases du calcul des probabilités* (1794), before being « emphasised by Boole », Czuber (in his *Wahrscheinlichkeitsrechnung*,) and Stumpf (1892).
[7] To the exception of Albino Lanciani (2012) and more recently Carlos Vargas (2018).
[8] Émile Borel (1939, §§ 18-19) refers to his recension from 1921 of Keynes "nice book", and to Jean Nicod's critique in "Le problème logique de l'induction" (1924).
[9] This is attested by Husserl's recension of von Kries's "Zur Psychologic der Urteile", *Vierteliahrsschrift fur wissenschaftliche Philosophie*, 23, 1899, S. 1-48, (Hua 22: 224- sq.).
[10] The Russian school (Tchebychev, Liapounov et Markov) is apparently ignored by Husserl. No mention of the French tradition (Poincaré, Borel, Lebesgue) either. But he knows the English logical tradition (Boole, Venn), and could not ignore the development of statistical physics (Gibbs, Boltzmann, and Einstein). He was well acquainted with the logical and philosophical investigations on probability calculus (Wundt, Stumpf, Meinong). The contribution of Per Martin-Löf on random sequences (1966) and on logic are a typical example of a cross-over influence (between Kolmogorov, Frege, Brouwer and Husserl).
[11] "Was bei ihr beirren kann, ist nur der Umstand, daß die Theorie der Wahrscheinlichkeiten als eine mathematische Disziplin konstituiert ist. In gewissen Sphären, die genau zu umschreiben sind, sind die Grade berechtigter Vermutung zahlenmäßig bestimmbar, und die zugehörigen Grundsatze ermöglichen eine rein deduktive Disziplin und eine in quantitativ-mathematischen Formen sich entwickelnde. Aber formale Mathematik ist das nicht, wie schon die Grundsatze und Grundbegriffe lehren. So erklärt Laplace den Grundbegriff der Wahrscheinlichkeit mittels des der *gleichmöglichen Falle*, und diese erklärt er als so1che, über die wir in gleichem Maße in Unkenntnis sind. Von der Wahrscheinlichkeit sagt er, sie beziehe sich zum Teil auf unsere Unwissenheit, zum Teil auf unser Wissen usw." (Hua 24, 132)
[12] "It is in this latter sense, with an eye to degrees of probability. that one speaks of a greater or lesser degree of knowledge. Knowledge in the pregnant sense , – its being quite evident that S is P – then counts as the absolutely ideal limit which the graded probabilities for the being-P of S asymptotically." (Hua 18, 30 ; Husserl 2001, 18)
[13] Husserl will discover in the Fifth Logical Investigation, § 27, that the processes of modalization intersects with and is connected to that of determination, and reciprocally. Not only with expressed judgements, but

with acts of perception (such as the perception of a person and/or a mannequin). Pointed at but not yet clearly analyzed in Lobo (2000, 246-270),

[14] The essentiality (*Wesenshaftigkeit*) of a mathematical constructs amounts to its logical possibility and its mathematical existence. It is diametrically opposed to the "essencelessness" (*Wesenslosigkeit*), which means "impossibility" (*Unmöglichkeit*) or "imaginarity" (*Imaginarität*) (Hua 18, 242).

[15] "The principal part of the art of logic that governs the sciences of matters of fact is the art of judging probability and providing grounds for probability. It plays the greatest role everywhere, even where it is not expressly a question of probability" (Hua 24, [17] 12).

[16] I fully agree with M. van Atten's suggestion that Husserl should have been a strong revisionist in mathematics, and that, after many hesitations, he was. (Van Atten 2007, 59-67).

[17] Felix Hausdorff has developed an incredibly original and profound theory of probability through a reworking of von Kries's notion. See, Carlos Lobo, "Espace, espace de jeu, jeu de hasard. *Position philosophique du problème de l'espace et des probabilités chez Felix Hausdorff*" (Forthcoming)

[18] Measure presupposes a fundamental operation setting the equipossibility of a set of cases (possible events) (figuring as denominator) and a propriety discriminating within that set a portion (of favorable cases) (the numerator).

[19] For an answer to this so-called vicious circle, see Borel (1924, 19): "En réalité, il n'y a pas de cercle vicieux à supposer l'on a la notion vulgaire du sens des mots 'également probable', lorsqu'on veut définir le sens mathématique précis du mot probabilité. Les logiciens qui prétendent construire des systèmes entièrement logiques, sans cercle vicieux, oublient qu'il est impossible de ne pas utiliser le langage usuel, ne serait-ce que pour définir les termes scientifiques que l'on emploie et pour construire les phrases dont on se sert; or, le langage usuel doit être considéré comme une acquisition globale de chaque individu, acquisition qui suppose un grand nombre de cercles vicieux".

[20] "The last *provisio* makes the definition circular, for the concept of probability then is dependent upon the concept of equiprobability. From the purely technical point of view, Laplace's definition reduces calculation of probabilities to counting" (Kac and Ulam 1968, 36).

[21] See Laplace 1921, 10.

[22] This parallel was clearly evidenced by Poincaré (1896). Despite formal imperfections (up to modern mathematical standards and those of the German school of the time), P. Cartier (2006) considers that Poincaré's *conceptual analysis* is the right one ("*C'est encore un ouvrage du 19-ième siècle pour les méthodes analytiques, mais c'est un des premiers ouvrage à faire percevoir les enjeux de la physique statistique, bien au-delà des habituels exercices de combinatoire liés aux jeux de hasard. En ce sens, ce livre appartient déjà au vingtième siècle, frayant la voie à Einstein, Ehrenfest, Wiener, Landau, et tant d'autres…* "). Among the treasures of this book "*à la fin du chapitre 12, une définition très claire de l'ellipse de dispersion, c'est-à-dire le fait que la loi de Gauss à plusieurs dimensions dépend du choix d'une forme quadratique définie positive sur l'espace de configurations*". Normal distribution or Gauss's curve of errors represents a special case amongst a

rich variety of possible "abstract random spaces", i.e. manifolds (with negative curvature).

[23] In *Doutes et questions sur le calcul des probabilités*, 1770, Mélanges de littérature, d'histoire et de philosophie, Tome V. Amsterdam, quoted by Von Kries (1886, 278-279).

[24]Which works as a renormalization comparable to the wave-packet collapse of quantum mechanics.

[25] "Alles kommt dabei darauf an, in objektiv gültiger Weise ein Feld von gleichen Möglichkeiten herzustellen von Möglichkeiten, für die in exakt nachweisbarer Weise genau dasselbe spricht, die gleichen Gewichte, positiv wie negativ - und nun jeden geschlossenen Kreis von Wahrscheinlichkeits-erwägungen auf dieses Grundfeld zurück zubeziehen." (Hua 30, 253).

[26] For a phenomenological exploration of the "central sphere of positionality" (or "modal core") see Lobo (2017).

[27] To the exception of great scientists such as Maxwell who declares: "The actual science of logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore the true logic for this world is the calculus of Probabilities, which takes account of the magnitude of the probability which is, or ought to be, in a reasonable man's mind." (Keynes 1920, 172; quotation from J.C. Maxwell)

[28] "Es ist nun leicht einzusehen, daß Für-möglich- und Für-wahrscheinlich-Halten und auch Fragen und Zweifeln Bewußtseinsarten sind , die mit dem setzenden, behauptenden Urteilen gattungsmäßig verwandt sind, so daß eine Reihe von Akten von propositionalem Inhalt unter einer höheren Gattungs-idee, unter der Idee des prädikativen Urteils oder, noch allgemeiner, eines prädikativen Aktes in einem außerordentlich erweiterten Sinn stehen. (Ob das Sich-denken in diese selbe Reihe gehört oder ob es nicht eine eigene Behandlung erfordert, wollen wir hier nicht entscheiden.)" (Hua 30, 59).

[29] "Ebenso wenig aber auch Sätze, die, statt vom urteilenden Ich auszusagen, vielmehr objektiv hinstellen: „Es ist gewiss" („Die Gewissheit ist berechtigt"), „Es ist zweifelhaft" = „Es besteht ein berechtigter Zweifel", ebenso: „Es ist wahrscheinlich" usf. Es mag innerhalb der Logik im weiteren Sinn eine objektive Wahrscheinlichkeitslehre, objektive Lehre von vernünftigem Zweifel, von vernünftigem Fragen etc. geben: in der Lehre von den Urteilen im Sinn von Urteilsbedeutungen ist nicht ihre Stelle. Dasselbe gilt für die Möglichkeits- und Notwendigkeitsurteile, sofern ihr Sinn mitbeschlösse irgendeinen subjektiven und empirischen Gehalt, der Beziehung hätte auf den Urteilenden, auf seine Meinungen, Kenntnisse, Vermutungen u.dgl. Solche Unterschiede gingen die formale Logik nichts an." (Hua Mat 6, 2003, 230).

[30] It is by a reflection on the content of perception that Husserl discovered that the "perceived" (*wahrgenommen*) as such entailed a qualitative character as part of its content: and makes in case of a "perfect trompe-l'oeil" , the whole difference between a mannequin (*Wachspuppe*) and a lady (*eine Dame*), i.e. between two different noemas (of empathy on the one hand and of picture or sculpture consciousness on the other), with their own essential modal characters. Fifth Logical Investigation, § 27 (Hua 19/1, 176-178)

[31] About the analogon of doxic doubt in the sphere of will (simple and disjunctive doubt-of-will distinct from question-of-will properly speaking). My comment Lobo (2006, 35-68).
[32] The question whether thinking in general and rationality in particular cannot be worked without any symbolic activity is another question, which is not addressed here.
[33] "Alles nun, was wissenschaftlich zu leisten ist unter dem Gesichtspunkt der Rechtsnormierung der Erkenntnisakte, d.h. der Akte, die entweder selbst Urteile im weitesten Sinn sind oder bei der Rechtsausweisung von Urteilen als rechtsverleihende Wahrnehmungen, Erinnerungen usw. eine wesentliche Rolle zu spielen berufen sind, das weisen wir der noetischen Normenlehre zu, und sie hat, nach dem Gesagten, es nicht im eigentlichen Sinn mit Akten als menschlichen oder sonstigen Erlebnissen, sondern mit den entsprechenden Aktideen zu tun. Wäre diese reine Disziplin ausgeführt, so wären wir also in der Lage, jeden Fall aktuellen Urteilens (im engeren Sinn, des aktuellen Vermutens oder Für-wahrscheinlich-Haltens usw.), ebenso jeden Fall aktuellen Begründens, aktuellen deduktiven Schließens und theoretisierenden Erklärens, induktiven Schließens usf. auf ideale Prinzipien zurückzuführen und nach seiner Normalität prinzipiell zu beurteilen. / Von vornherein wollen wir dabei den wissenschaftstheoretischen Charakter unserer Untersuchungen zur Geltung bringen; also wir wollen uns von vornherein in jedem Schritt deutlich machen, daß das Gebiet reiner Erkenntnis, das wir jetzt wissenschaftlich begrenzen, ein Grundstück einer allgemeinen und reinen Wissenschaftslehre sein muß. Wir verstehen darunter eine Wissenschaft, welche in systematischer Weise die zur Idee echter Wissenschaft gehörigen Wahrheiten erforscht, somit alles, "'Das notwendig gelten muß, wenn eine Wissen." (Hua 30, 38)
[34] "So, wie wir Urteilserlebnisse als Erlebnisse des 'So ist es!' unter Idee bringen und in der Einstellung der ldeensetzung nach Geltung und Nicht-Geltung normieren können, so (auch) Urteilserlebnisse in einem erweiterten Sinn, ich meine hier Erlebnisse des Für-möglich und Für-wahrscheinlich-Haltens, des Fragens und Zweifelns. Auch in dieser Hinsicht gibt es Formen und Normen der Gültigkeit, die mit diesen Formen zusammenhängen, und man kann mancherlei Normen der rein mathematischen Wahrscheinlichkeitslehre in dem angedeuteten Sinn interpretieren. Und schließlich kann man auch alle Erkenntniserlebnisse überhaupt, nach allen immanenten Charakteren unter Idee bringen und mit Bezug auf die Rechtsprechungen der Wahrheit, Möglichkeit, Wahrscheinlichkeit, Fraglichkeit durchforschen." (Hua 30, 39)
[35] "Ziehen wir neben dem Urteilen als In-Gewißheit-Behaupten, Behaupten, Aussagen, Für-wahr-Halten auch in Erwägung die mit ihm wesentlich verflochtenen Modalitäten, so das Vermuten, das Für-möglich Halten, so ist das darin Bewußte nicht vermeinte Wahrheit oder Satz, sondern vermeinte Wahrscheinlichkeit oder Möglichkeit. Das gibt Anlaß zur Erweiterung der Idee einer reinen Logik um eine reine Logik der Möglichkeiten und Wahrscheinlichkeiten. Mit der Logik der Behauptungen, der apophantischen Logik, zeigt sich aber aus wesentlichen Gründen auch verflochten, obschon in ganz anderer Richtung, die reine Arithmetik und weiterhin die gesamte

formale Mathematik oder Mannigfaltigkeitslehre. Diese Disziplinen bilden sozusagen ein höheres Stockwerk der Apophantik, und es ist von größter philosophischer Bedeutung, sie in diesem Zusammenhang zu erkennen und zu charakterisieren." (Hua 30, 29)

[36] See Recension of J. Bergmann's *Die Grundprobleme der Logik, zweite, völlig neue Bearbeitung* in Hua 22 (186 *et seq.*); Wundt standpoint exposed in 1894 (Hua 22, 129.); compared in both case to Brentano's theory of judgement.

[37] I shall repeat here the last subtitle of Weyl's magnificent paper "The Ghost of modalities" dedicated to Husserl (Weyl 1940, 278-304).

[38] "And, again, one can be an outstanding mathematician, while being a very mediocre philosopher of mathematics. Boole provides an outstanding example of both.)" (Husserl 1994, 59; Hua 22, 9).

[39] Similar objection by Venn and similar answer from Husserl. "Also, that the Boolean method so frequently utilizes senseless symbols does not yet in itself serve as the basis for a logical objection. We can only object, rather, that that method does not adequately justify the use [399] of such symbols" (Husserl 398-399).

[40] "As a rule they are of such a simple type that to solve them by means of the calculus would be the most laughable of detours." (Husserl 1994, 90; Hua 22, 42)

[41] "This latter, which coincides formally with the former, can indeed be profitably applied in many particular fields of mathematics- for example, in the theory of functions, where manifolds of values of arguments frequently come into consideration. *Likewise in the calculation of probabilities, where sets of chances make the application possible. Beginnings have already been made in these matters, but here too we do not have sufficient results definitively to decide the question about practical value*. But I would in no case wish to cast doubt upon the extraordinary theoretical interest that belongs to the algorithmic treatment of the theory of pure deductions, as well as of pure set theory." (Hua 22, 43; Husserl, 1994, 90)

[42] See Boole (1952, 239) which reformulates retrospectively the purpose of the *Laws of Thought* (Boole 1854).

[43] This represents a strong opposition to Peirce, who considers *all* mathematical reasoning as hypothetical. (See, for instance, Baldwin's Dictionary).

[44] *Lessons on Logic and epistemology*, published under the title *Vorlesungen über Logik und Wissenschaftstheorie*, Husserliana 30, … and *Alte und New Logik*, Husserl, Mat. 6.

[45] For further development of those points, I must refer once to Lobo (2018; 2017a; 2017b).

[46] This discrete and discreet reference in the historical development of probabilities and philosophy of probability is better known now (see Keynes 1921; Rosenthal 2010; Zabell 2016; Lobo 2018).

[47] To be compared with Hausdorff, *Beiträge zur Wahrscheinlichkeitsrechnung*, from 1901, in Gesammelte Werke, Vol. 5, pp. 531-532, and compare to Husserl (Hua 30, § 51-52) and Peirce (Probable Inference, Baldwin's Dictionary, 354).

[48] Rather than a "holding-to-be-so" (Husserl 1994, 149; Hua 22, 102): "we have the inconvenience of denying its name to the perceptual representation [*wahrnehmende Vorstellung*] as it naturally presents itself, for what we in

fact have in such cases is a holding-to-be-so [*Fürwahmehmen*] of what is represented (even if only inauthentically) in the perceptual 'representation'."

[49] Not to be confused, as this frequently occurs, with "physical phenomena" in Brentano's sense, i.e. sensuous and emotional data.

REFERENCES

Becker, Oskar. 1930. "Zur Logik der Modalitäten." *Jahrbuch für Philosophie und phänomenologische Forschung* 11: 497-548. [Halle: Max Niemeyer].

Becker, O. 1952. *Untersuchungen über den Modalkalkül*. Meisenheim am Glan: Westkulturverlag Anton Hain.

Boole, George. 1854. *The Laws of Thoughts*. New York: Dover.

________. 1952. *Studies in Logic and Probabilities*. London: Watts & Co.

Borel, Émile. 1924. *Éléments de la théorie des probabilités*. Troisième édition. Paris: Hermann.

Borel, É. 1939. *Valeur pratique et philosophie des probabilités*. Paris: Gauthier-Villars.

Carnap, Rudolf. 1945. "On inductive logic." *Philosophy of Science* 12: 72–97.

Cartan, Élie. 1923. "Sur les variétés à connexion affine et la théorie de la relativité généralisée (première partie)". *Annales scientifiques de l'E. N. S. 3e série* 40 : 325-412.

Cartier, Pierre. 1985. "Analyse non standard: nouvelles méthodes infinitésimales en analyse. Application à la géométrie et aux probabilités". *Les rencontres physiciens-mathématiciens de Strasbourg - RCP25*. Tome 35. "Conférences de: P. Cartier, A. Guichardet et G.A. Viano", exp. n°1, 1-21. Strasbourg : Université Louis Pasteur.

Cartier, P. 2006. "Le *Calcul des Probabilités* de Poincaré". Bure-Sur-Yvette: Institut des Hautes Etudes Scientifiques IHES/M/06/47.

Couturat, Louis. 1901. *La logique de Leibniz d'après des documents inédits*. Paris: Félix Alcan.

Fioretti, G. 1998. "John Maynard Keynes and Johannes von Kries." *History of Economic Ideas* 6: 51-80.

Fioretti, G. 2001. "Von Kries and the other "German logicians": Non-numerical probabilities before Keynes." *Economics and Philosophy* 17: 245–273.

Heidelberger, M. 2001. "Origins of the logical theory of probability: Von Kries, Wittgenstein, Waismann." *International Studies in the Philosophy of Science* 15 (2): 177-188.

Husserl, Edmund. 1950-2012. *Husserliana: Edmund Husserl – Gesammelte Werke*. Bde. 1-41. Haag: Martinus Nijhoff (Bd. 1-26, 1950-1987); Haag: Kluwer Academic Publishers (Bd. 27-37, 1988-2004); New York: Springer (Bd. 38-41, 2005-2012). [Abbreviated *Hua*].

________. 1950. [*Hua* 3]. *Ideen zu einer reinen Phänomenologie und Phänomenologischen Philosophie, Erstes Buch, Allgemeine Einführung in die Reine Phänomenologie*. In *Husserliana,* Bd. III, hrsg. von Walter Biemel. Den Haag: Martinus Nijhoff.

________. 1956. [Hua 7]. *Erste Philosophie (1923/4). Erste Teil: Kritische Ideengeschichte*. In *Husserliana*, Bd. VII, hrsg. von Rudolf Boehm. Haag: Martinus Nijhoff.

________. 1969. *Formal and Transcendental logic*. Translated by Dorion Cairns. The Hague: Martinus Nijhoff. [Also mentioned in the text as FTL]

________. 1975. [Hua 18]. *Logische Untersuchungen. Erster Teil. Prolegomena zur reinen Logik*. In *Husserliana*, Bd. XVIII, hrsg. von Elmar Holenstein. Haag: Martinus Nijhoff.

________. 2001. *Logical Investigations,* Trans. J-F. Findlay, New York/London: Routledge.

________. 1979. [Hua 22]. *Aufsätze und Rezensionen (1890-1910).* Hrsg. von B. Rang. The Hague: Martinus Nijhoff.

________. 1984. [Hua 19]. *Logische Untersuchungen. Zweiter Teil. Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*. In *Husserliana*, Bd. XIX, hrsg. von Ursula Panzer. Haag: Martinus Nijhoff.

________. 1985. [Hua 24]. *Einleitung in die Logik und Erkenntnistheorie. Vorlesungen 1906/07.* In *Husserliana*, Bd.

XXIV, Ullrich Melle. The Hague, Netherlands: Martinus Nijhoff.

________. 1988. [Hua 28]. *Vorlesungen über Ethik und Wertlehre. 1908-1914*. In *Husserliana*, Bd. XXVIII, hrsg. von Ullrich Melle. The Hague, Netherlands: Kluwer Academic Publishers

________. 1994. *Early writings in the philosophy of logic and mathematics*. Vol. 5 of *Edmund Husserl's Collected Works*. Translated by Dallas Willard. Dordrecht: Springer.

________. 1995. [Hua 30]. *Logik und allgemeine Wissenschaftstheorie*. Vorlesungen 1917/18. Mit ergänzenden Texten aus der ersten Fassung 1910/11. In *Husserliana*, Bd. XXX, hrsg. von Ursula Panzer. Haag: Kluwer Academic.

________. 2003. [Hua Mat 6]. *Alte und neue Logik. Vorlesung 1908/09*. In *Husserliana: Materialien*, Bd. VIII, hrsg. von Elisabeth Schuhmann. Dordrecht: Kluwer Academic Publishers.

Jevons, William Stanley. 1874. *The Principles of Science*. London: Macmillan.

Kac, M. & Ulam, Stanley. 1968. *Mathematics and Logic*. New York: Dover.

Keynes, John Maynard. 1921. *A treatise on Probability*. Cambridge: MacMillan.

Kolmogorov, A. N. 1956. *Foundations of the Theory of Probability*. Transl. N. Morrison. New York: Chelsea Publishing Company.

Lanciani, Albino. 2012. *Analyse phénoménologique du concept de probabilité*. Paris: Hermann.

Laplace, Pierre-Simon. 1921. *Essai philosophique sur les probabilités.* Tome 1 et 2. Paris: Gauthiers-Villars.

Lobo, Carlos. 2000. *Le phénoménologue et ses exemples*. Paris: Kimé.

Lobo, C. 2005. "L'a priori affectif (I). Prolégomènes à une phénoménologie des valeurs." *Alter* 13: 35-68.

Lobo, C. 2017a. "Husserl's Reform of Logic. An Introduction." *New Yearbook for Phenomenology and Phenomenological Philosophy* 15: 16-48.

Lobo. C. 2017c. "Le projet husserlien de réforme de la logique et ses prolongements chez G-C. Rota." *Revue de synthèse* 138: 105-150.

Lobo, C. 2018. "Some Reasons to reopen the question of foundations of probability theory following the Rota way." In *The Philosophers and Mathematics. In Honour of Prof. Roshdi Rashed*, edited by Hassan Tahiri, 145-188. Dordrecht: Springer.

Poincaré, H. 1896. *Le Calcul des Probabilités*. Paris: Gauthier-Villars.

Rosenthal, J. 2010. "The natural-range conception of probability." In *Time, chance, and reduction: Philosophical aspects of statistical mechanics*, edited by G. Ernst & A. Hüttemann, 71–91. Cambridge: Cambridge University Press.

Rota, Gian-Carlo. 1973. "The valuation ring of a distributive lattice." In *Proceedings of the University of Houston, Lattice Theory Conference (Houston, Tex. 1973)*, Dept. Math., Univ. Houston, 574-628.

Ellerman, David P. & Gian-Carlo Rota. 1978. "A measure-theoretic approach to logical quantification." *Rend. Sem. Mat. Univ. Padova* 59: 227-246.

Stumpf, Karl. 1892. "Über den Begriff der mathematischen Wahrscheinlichkeit." *Sitzungsberichte der philosophisch-philologischen und historischen Classe der Königlich Bayerischen Akademie der Wissenschaften* 20: 37–120.

Van Atten, Mark. 2007. *Brouwer meets Husserl: On Phenomenology of Choice Sequence*. Synthese Library 33: 43-52.

von Kries, J. 1886. *Die Principien der Wahrscheinlichkeitsrechnung*. Tübingen: Mohr.

von Mises, Richard. 1957. *Probability, Statistics and Truth*. New York: Dover.

Vargas, C. 2018. *Origens da fenomenologia. Desenvolvimento Inicial da Filosofia do Edmund Husserl*. Rio de Janeiro: Multifoco.

Weyl, Hermann. 1940. "The Ghost of Modalities." In *Philosophical Essays in Memory of Edmund Husserl,* edited by Marvin Farber, 278-303. Cambridge (MA): Harvard University Press.

Zabell, S. 2016. "Johannes von Kries's Principien: A Brief Guide for the Perplexed". *Journal of General Philosophy of Science* 47: 131-150.

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Ideation und Idealisierung: Die mathematische Exaktheit der Idealbegriffe und ihre Rolle im Konstitutionsprozess bei Husserl

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Abstract

Ideation and idealization: The mathematical exactness of the ideal concepts and their role in the constitution process in Husserl

In this article I trace the origins of the process of idealization and compare it to the process of ideation at Husserl. I argue that Husserl's statements do not keep – as originally intended – a clear distinction between both. On the contrary, ideation and idealization build up a continuing process whose telos is the exact determination of the experienced object. Hence, the idea of the Kantian thing-in-itself as the particular actuality of a thing embracing an open infinity of possible adumbrations – a transfinite infinity – becomes an ideal of exact determination comprising a closed infinity of the infinite ways of givenness – an actual infinity. This leads to an *aporia* insofar as the infinite perfection of the idea has to be presupposed in order to conceive of the infinite imperfection of the actuality of the thing. The *aporia* may be resolved by considering the historical contingency of ideas and the open essence of a thing, which in conjunction lead to a conception of an idea allowing both for openness and indeterminacy. Nevertheless, although the telos of adequate knowledge is not the exact, but the optimal givenness, the exact idea still constitutes a telos insofar as it guarantees the pursue of the maximal possible fullness of perception.

Keywords: Husserl, ideation, idealization, Kantian thing-in-itself, infinity, openness

1. Einleitung

Eine der Aufgaben der Vernunft bei Kant besteht in ihrer gegenüber der Sinnlichkeit begrenzenden Funktion, da sie die

„Anmaßungen der Sinnlichkeit einschränken“ (Kant 2016, AA, A 255/B 311) soll. Umgekehrt verlässt aber die Vernunft die Schranken der Anschaulichkeit, indem das lebensweltlich Konstituierte idealisiert wird. In seiner *Krisis*schrift führt Husserl diese Überschreitung der Sinnlichkeit auf die Leistung der Vernunft zurück, infolge derer eine „Idee einer absoluten, exakten Bestimmbarkeit der Dinge“ entsteht (Hua VI, 361). Husserl fragt nach der Motivation für diese Bestimmung „der Dinge des universalen offen unendlichen Welthorizonts, den wirkliche Erfahrung in der Endlichkeit ihres Fortschreitens nie durchmessen kann“ (Hua VI, 357–359). Im „Galilei“-Paragraphen verortet Husserl den Ursprung der Institutionalisierung dieser Bestimmung in der wissenschaftlichen „Praxis“, die eine exakte Objektivität leistet. Die Idealisierung entsteht dadurch, dass aus der offenen Endlosigkeit der erfüllten Antizipation ein Grenzübergang in die Idee einer „unendlichen Totalität“ der Dingeigenschaften entsteht, die in die auf Anschauung beruhende normal lebensweltliche Praxis integriert wird. Eben dieses „Einströmen“ (Hua VI, 115, 141 Anm. 1, 213, 466) der Idealisierungsprodukte in die normale Praxis aber ermöglicht die Lebensweltvergessenheit. Auf diese Weise werden die Idealisierungsprodukte zu Selbstverständlichkeiten, die als „Ideenkleid“ (Hua VI, 51) die ursprünglichen lebensweltlichen Selbstverständlichkeiten überdecken und verdecken. So bildet sich eine „unendliche Welt von Idealitäten“. Die ‚Idealisierung’ besteht darin, dass wir etwas ‚Ideales’, das den Bereich der Anschaulichkeit transzendiert, in die auf Anschauung beruhende normale lebensweltliche Praxis einfügen, und in diesem Gang das Bewusstsein vom Unterschied zwischen lebensweltlichem und gedachtem Optima verlieren, wie Klaus Held erklärt (Held 2003).

Obwohl die exakten Idealbegriffe auf die morphologischen aufbauen, schließt einerseits die geometrische Reinheit das Typische der sinnlich anschaulichen Gegebenheiten aus; umgekehrt ist alle echte Deskription morphologisch und für eine exakte Bestimmung unzugänglich (Hua V, 132). Daher muss die Objektivierung, die aus mathematischer oder geometrischer Bestimmung stammt und Idealbegriffe ergibt, von der Objektivierung, die durch Abstraktion morphologischer Wesen erfolgt und Gattungsbegriffe ergibt, aufs schärfste unterschieden werden. Folglich, wenn die Regelstruktur eine Idealgestalt als ihre Grenze vorzeichnet, die exakt im Sinne der Genauigkeit und Vervollkommnung ist, dann ist dieses Wesen nicht als Resultat einer Abstraktion, sondern als Resultat einer „Idealisierung“ zu verstehen. Wie Husserl in den *Ideen II* erklärt, „alle erscheinende Raumgestalten lassen eine Idealisierung zu, sind in geometrischer Reinheit zu erfassen und ‚exakt‘ zu bestimmen“ (Hua IV, 82). Die Ideation, wie Held hervorhebt, hält sich dagegen in den Grenzen der lebensweltlichen Brauchbarkeit des Optimums von Gegenständen oder ihren Eigenschaften. Das in der Ideation gegebene Optimum von Rund, von Rundheit einer Tischplatte ist also nicht dasjenige Optimum, das ein Idealisierungsprozess erreicht, der zum bloßen Gedanken der Rundheit, zu ihrer abgelösten geometrischen Vorstellung führt (Held 2003, 133*ff*).

Husserls Aussagen entsprechen aber nicht immer diesen deutlichen Unterscheidungen, vielmehr unterliegen sie Umdeutungen, die auch Zweideutigkeiten zulassen. Um sie in ihrer Entwicklung zu beleuchten, werden wir erstens der husserlschen Umdeutung des Kantischen Begriffs des „Ding-an-sich“ und seiner Rolle in der Konstitution des Dinges und zweitens, dem Problem der Idealisierung nachgehen. Dabei wird insbesondere in die unterschiedlichen Auffassungen einerseits von den Ideen, die den Fortgang der Anschauung regeln oder leiten und andererseits von den ‚Weisen‘ der Unendlichkeit eingegangen.

2. Das Kantische Ding an sich als Idee einer Einzelwirklichkeit

In der Welt der Erfahrung wird von einem beliebigen Einzelding als ein Beispiel irgendeines Dings ausgegangen, um die „offen endlose Mannigfaltigkeit seiner immer unvollkommenen aber zu vervollkommnenden subjektiven Vorstellungen als durchlaufen“ (Hua VI, 359) zu denken. Das Vermögen, diese Reihe fortzuführen, ist durch die wirkliche Anschaubarkeit des erfahrenen Dinges begrenzt, obwohl eine in der Anschauung unerfüllbare leere Antizipation eines vollkommeneren Dings – das Ding in seiner partikulären Wirklichkeit als Ding-an-sich, gemäß Husserls Aussagen in den früheren Schriften – stets mitgegeben ist. Die Endlichkeit der anschaulichen Reihe ist insofern noch offen, da ihre Erfüllung, obwohl antizipiert, noch leer ist. Es handelt sich dementsprechend um einen „leeren Vorentwurf der Reihe" und zugleich um den „leeren Gedanken" ihrer Erfüllung (Hua VI, 359), da diese in der alltäglichen Erfahrung ausgeschlossen bleibt – eine Leistung, die noch keine Idealisierung bedeutet.

In der sinnlichen Erfahrung ist nicht nur eine absolute Gegebenheit bzw. eine „gesättigte Gegebenheit" eines Dings, d.h. eine Wahrnehmung, bei der das Ding „allseitig zu sehen" wäre (Hua XVI, 114), sondern auch der endgültige und daher vollbestimmte Gegenstand als Grenze oder Ziel des Erfahrungsprozesses nie adäquat gegeben – dies zum einen deshalb, weil jede Abschattung einer Wahrnehmungsphase einseitig ist und sich in einer Erscheinungsreihe zugleich entleert und bereichert (Hua XVI, 114f), d.h. immer eine unvollständige Gegebenheit ist (Hua XX/1 194; Hua XVI, 136), zum anderen deshalb, weil dieser Prozess der kontinuierlichen Wahrnehmung unendlich ist und daher nur eine stets unvollendete Synthese des Gegenstandes erbringen kann. Der *realen* Unmöglichkeit einer allumfassenden und adäquaten sinnlichen Anschauung entspricht somit die Möglichkeit einer inadäquaten Vorzeichnung dieser Anschauung, d.h. die „Gegebenheit in Form einer Idee" (Hua III/1, 332). In einem ersten Schritt trennt Husserl diese Idee von dem unendlichen Prozess, in dem es sich kontinuierlich-einstimmig bestimmt, wie Rudolf Bernet (Bernet 2004, 160; Hua XX/1, 198) bemerkt. Wie Husserl in den *Ideen I* erklärt,

> „(a)ls ‚*Idee*' (im Kantischen Sinn) ist *gleichwohl die vollkommene Gegebenheit vorgezeichnet* – als ein in seinem Wesenstypus absolut bestimmtes System endloser Prozesse kontinuierlichen Erscheinens, bzw. als Feld dieser Prozesse ein a priori bestimmtes *Kontinuum von Erscheinungen* mit verschiedenen aber bestimmten Dimensionen, durchherrscht von fester Wesensgesetzlichkeit." (Hua III/1, 332)

So bezeichnet Husserl diese Idee des Dings als eine einsichtige „apriorische Regel" (Hua III/1, 332) für den Fortgang der Erfahrung. Sie ist das „Ideal der adäquaten Gegebenheit" (Hua III/1, 332) eines Dings, d.h. die Idee einer „unendliche(n) Gesamtheit", die einsehbar dem Vervollkommnungsprozess vorliegt (Hua XX/1, 198*ff*.). Hier ist die

Adäquatheit der Wahrnehmung durch die gesamte Reihe inadäquater Wahrnehmungen als *Ideal* gebildet. Adäquat gegeben ist also nur die Gesamtreihe, da die verschiedenen inadäquaten Wahrnehmungen sich gegenseitig ergänzen. Diese einzelne Wahrnehmungen, die sich in einem geregelten, „synthetisch-einheitliche(n)" Fortgang der Wahrnehmung als fortschreitende „Reihen" bilden, werden von einer „ganz einzigartige(n) Intuition, also auch Evidenz" erfasst. Der Fortgang der Wahrnehmung eröffnet einen „offenen Horizont oder ‚Spielraum'", so das diese „offene" mit einem „‚und so weiter'" behaftete unendliche Folge „in ihrem Sein" vollkommen von dieser Intuition erfasst und in einer inadäquaten Wahrnehmungsevidenz gegeben wird (Hua XX/1, 199*ff*).

In einem zweiten Schnitt beschreibt Husserl den Ursprung der Idee im Kantischen Sinn, sie entsteht nämlich, als ein ‚Ausschnitt' der aktuellen Erfahrung eines Dinges. So erklärt Husserl, dass aus diesen vieldeutigen und unendlichen Möglichkeiten bzw. aus diesem unendlichen Umfang eines möglichen Dinges die „aktuelle Erfahrung", die „einzige Wirklichkeit ‚des Dings', des ‚an sich' völlig bestimmten" herausschneidet. Diese Wirklichkeit eines Dings darf aber weder mit dem wirklichen, transzendenten Ding noch mit seinem Wesen verwechselt werden. Sie ist das „Seinsmoment" eines wesenhaft identischen Dings, das durch aktuelle Erfahrung gewährleistet wurde (Hua XX/1, 197), und „ist *genau so weit gegeben wie das Wirkliche selbst*" (Hua XX/1, 198). So definiert Husserl die

> „*Wirklichkeit eines Dinges (als) eine ‚Idee' im Kant'schem Sinn,* Korrelat der ‚Idee' eines ‚gewissen', aber im voraus nie vollbestimmten, vielmehr unendlich vieldeutigen Wahrnehmungsverlaufs, eines ins Unendliche erweiterungsfähigen und […] nur einem Typus nach festgelegten." (Hua XX/1, 197)

Zwar verwendet Husserl hier mit dem „an sich" eine Kantische Terminologie, doch dabei wird das „An-sich" den jeweiligen subjektiv relativen Erfahrungen zugeordnet. Mit dem „an-sich"-Sein des Dings ist das gemeint, was das Ding wirklich ist. Weit davon entfernt eine Wesensbestimmung zu sein, ist *die Idee im Kantischem Sinn die Idee einer gewissen partikulären Wirklichkeit eines Dings*, die den Wahrnehmungsprozess leitet: Sie ist das Ideal der adäquaten Gegebenheit eines Dings in seiner *partikulären Wirklichkeit;* ein Ideal, das unerreichbar ist, obwohl es das Horizont bildet, in dem die aktuellen Erfahrungen des wirklichen Dings sich einschreiben, wie Bernet (Bernet 2004, 161) hervorhebt. So stellt sich heraus, dass die Wirklichkeit eines Dings nur in einer aktuellen Erfahrung festgestellt werden kann, d.h. *a posteriori*, während die Möglichkeit eines fortschreitenden Erfüllungsprozesses nach der Regel ihres idealen Einsichttypus – das Ding an sich – *a priori* gegeben ist (vgl. Breuer 2018).

3. Ideation und Idealisierung

Die Idee im Kantischen Sinn meint aber nicht nur eine partikuläre Wirklichkeit eines Dings: „Exakte Begriffe haben ihre Korrelate in Wesen, die den Charakter von ‚Ideen' im Kantischen Sinne haben." (Hua III/1, 155) Die Idee im Kantischen Sinn ist also das Korrelat von zwei abgrundtief verschiedenen Gegenständen, wie László Tengelyi bemerkt: Denn, wenn das Ding an sich sich auf ein individuelles, einzelnes Ding bezieht, erweist sich das Ding selbst als ein unendliches Ganzes, das innerlich gegliedert ist und keinen Totalaspekt bietet. So ist das Ding an sich selbst ein offenes Ganzes. Dagegen, wenn die Idee im Kantischen Sinn sich auf exakte bzw. geometrische Wesen bezieht, stellt sie eine Grenze dar, die auf einmal erfasst werden kann. Sie verwandelt diese Offenheit in einem geschlossenen Ganzen (Tengelyi 2007, 84*ff*.). Hieraus folgt, dass Ideen im Kantischen Sinn sich auf unterschiedliche Wesensarten beziehen können: Einerseits können sie „anschauliche(n) Dinggebenheiten in ihren anschaulich gegebenen Wesenscharakteren", andererseits aber auch „Idealwesen" entsprechen. Aus diesem Grunde unterscheidet Husserl zwischen zwei „Arten" der „Ideation":

> „Diejenige Ideation, welche die Idealwesen ergibt als *ideale ‚Grenzen'*, die prinzipiell in keiner sinnlichen Anschauung vorfindlich sind, denen sich jeweils morphologische Wesen mehr oder minder ‚annähern', ohne sie je zu erreichen, diese Ideation ist etwas grundwesentlich anderes als die Wesenserfassung durch schlichte ‚Abstraktion', in welcher ein abgehobenes ‚Moment' in die Region der Wesen erhoben wird als ein prinzipiell Vages, als ein Typisches. Die *Festigkeit und reinliche Unterscheidbarkeit der*

> *Gattungsbegriffe* bzw. Gattungswesen, die ihren Umfang im Fließenden haben, *darf nicht mit der Exaktheit der Idealbegriffe verwechselt werden* und der Gattungen, die durchaus Ideales in ihrem Umfange haben." (Hua III/1, 155).

Das anschauliche Ding gibt sich als fließendes, und an ihm ist sein typisches Wesen intuitiv, d.h. anschaulich, zu erfassen: „Die anschaulichen Dinggegebenheiten in ihren anschaulich gegebenen Wesenscharakteren" (Hua III/1, 155) können in deskriptiven, morphologischen Begriffen von „vagen Gestalt-Typen" erfasst werden, wie z.B. „gezackt, gekerbt, linsenförmig, u. dgl." (Hua III/1, 155). Diese morphologische, typische Wesen dürfen nicht mit den exakten Wesen bzw. geometrischen Begriffen verwechselt werden, denn die letzten können laut Husserl nur in Idealbegriffen verstanden werden, die – ungleich der morphologischen Wesen – nicht der schlichten Anschauung entstammen. Die Typenbegriffe stammen aus den unmittelbaren Gegebenheiten der Anschauung oder Erfahrung, während die exakten Idealbegriffe der reinen Geometrie mittelbar gegeben sind durch eine auf die anschaulichen Gegebenheiten gegründete, ideell-limetische Objektivation. Die Anschaulichkeit wird verlassen, sobald das in einstimmiger Erfahrung sinnlich qualitativ Charakterisierte einen objektiven Index, eine physikalische Bestimmung bekommt. Dies würde bedeuten, dass das sinnlich Qualitative in einem beliebigen Punkt des Prozesses der Näherbestimmung nicht länger als Vages oder Typisches aufgefasst wird, sondern als Index für Objektives in Betracht kommen kann; es handelt sich hier m.E. um ein Idealisierungsverfahren, das bei der Mathematisierung bzw. Geometrisierung des Qualitativen einsetzt.

4. Die idealisierende Leistung – phänomenologische und mathematische Auffassung des Dings an sich

Hier setzt der erste Schritt der „idealisierenden Leistung" (Hua VI, 359) ein: Er besteht in der „Konzeption des ‚immer wieder'", bzw. das wiederholte Entwerfen möglicher mit jeweils eigener Erfüllung gedachten Reihen; eine Leistung, die anstelle der offenen Endlosigkeit der Dingerfahrung, anstelle der endlichen, weil von der möglichen Erfahrbarkeit begrenzten Wiederholung, eine „evident mögliche Unendlichkeit" der

Wiederholbarkeit des „Immer-wieder“ hervorbringt. Dieses Immer-wieder richtet sich auf den „leeren Vorentwurf der Reihe“ und denkt ihn aber „mit der möglichen Erfüllung“ (Hua VI, 359). Diese Wiederholung „in infinitum“ der Denkakte bezüglich der möglichen Erfüllung bzw. Erfüllungsreihen zeugt von der *Kontingenz* dieser Erfüllungen. Dieser Vorentwurf ist nach Marc Richir von einem phänomenologischen Gesichtspunkt aus leer, weil er in einem „symbolischen Vorentwurf [*pré-projet symbolique*]“ besteht, eine in Worten Husserls „anschauliche Fiktion“ (Hua VI, 359) die keine andere sein kann als die „aktuale Unendlichkeit der Bestimmbarkeit“ (Richir 1990, 223). Dieser Vorentwurf erschaut die unendliche Reihe als *aktual gegeben*: „die offen endlose Mannigfaltigkeit… [wird] als durchlaufen gedacht“ (Hua VI, 359). Dies bedeutet, dass erstens die Unendlichkeit *im Denken aktual* wird, und zweitens, dass die Erfüllung – die in einer phänomenologisch-anschaulichen Hinsicht leer bleibt, weil sie die mögliche Anschauung verlässt – einen *symbolischen Charakter* aufweist: *Wie jedes Symbol vertritt dieser Vorentwurf den Gegenstand in seiner Abwesenheit.*

Während der erste Schritt die Idealisierung einer Tätigkeit betrifft, besteht der zweite Schritt in der Idealisierung eines Gegenstandes: Aus der beschriebenen endlichen Iteration der Erfüllungsleistung, wonach das im Endlichen liegende Ding an sich die anschauliche Reihe bestimmt, erwächst eine im Unendlichen liegende *Idee,* die einen doppelten Charakter aufweist: Sie ist die Idee, zum einen „der in unbedingter Allgemeinheit wiederholbaren Fortsetzung“, zum anderen der idealen „Eigenschaftlichkeit des exemplarischen Dings als solchen“ (Hua VI, 361). Aus der offenen Endlosigkeit der von einer leeren Antizipation erfüllbaren Reihe (1) und der von dem Ding an sich erfüllten Antizipation (2) entsteht durch einen *Grenzübergang* in die Unendlichkeit die Idealisierung des Dings selbst „als Seiendes seiner Eigenschaften“ (Hua VI, 359). Das aktual Unendliche kann jetzt in seiner unendlichen Exaktheit erfasst werden (Richir 1990, 224). Wie Husserl sagt:

> „Es entspringt als erstes die Idee der in unbedingter Allgemeinheit wiederholbaren Fortsetzung, in einer eigenen Evidenz als frei denkbare und evident mögliche Unendlichkeit,

> anstelle der offenen Endlosigkeit, anstelle der endlichen Iteration die Iteration im unbedingten Immer-wieder, dem in idealer Freiheit zu Erneuernden.“ (Hua VI, 359)

Es handelt sich um die „’Entdeckung‘ des mundanen Apriori“, wodurch die tatsächlich anschauliche Erfahrungswelt „homogenisiert“ und ins Unendliche erweitert wird. Dabei wird diese Unendlichkeit von jedem Ding wirklicher Erfahrung in der Weise von inneren und äußeren Unendlichkeitshorizonten in sich getragen und mit ihnen apperzipiert (Hua XXIX, 142). „Nicht das bloße Bewusstsein von der offenen Endlosigkeit“ der möglichen Erfahrung (des möglichen immer-weiter-gehen-Könnens) und die damit verbundene Iteration erzeugt diese „Verunendlichung“, sondern die „Entdeckung des mathematischen Kontinuums“ (Hua XXIX, 143). In Worten Husserls:

> „Die evidente Idee (nicht das Ideal) der Erfahrbarkeit in *infinitum* der Welt als Welt möglicher Erfahrung, der Möglichkeit der Erkenntnis der Welt, impliziert selbstverständlich die Idee der Erkennbarkeit ins Unendliche, und damit wird die wesensmäßig zur Lebenswelt gehörige Induktivität idealisiert. Sie wird in ihrer Weise der Verunendlichung unterzogen, und zwar innerhalb der abstrakten Wesensschicht der Welt, innerhalb deren die Mathematisierung erfolgt war […] Die Idealisierung ergibt hier: alle Dinge, abstrakt auf Körper reduziert […] stehen in der Einheit einer universalen Naturkausalität. Das ist die ‚Evidenz des Kausalgesetzes‘ als Apriori der unendlichen Welt.“ (Hua XXIX, 143)

An die Stelle also des auf das Endliche bezogene Ding an sich als Idee sowohl einer partikulären Wirklichkeit eines Dings als auch einer unendlichen Folge, tritt hier die in der Unendlichkeit liegende Idee einer „unendlichen Totalität“ der Dingeigenschaften in der Endlichkeit auf, um sie mit M. Richir gesagt, „zu vereinheitlichen [„*uniformiser*]“ (Richir 1990, 224). Im Gegensatz also zum Ding an sich, der in seiner *Offenheit* potentielle Unendlichkeiten sinnlicher Vorstellungen in sich umfasst und dem Vervollkommnungsprozess *immanent* ist, ist diese Idee als *exakte* Einheit einer „*aktualen* Unendlichkeit“ dem Prozess *transzendent*. Die Unendlichkeit kann aktual werden, nur indem sie als *Idee,* d.h. die Sinnlichkeit überschreitend, verstanden wird. Diese Erfüllung, als *aktual im Denken* gegeben, deutet auf den *geschlossenen Charakter der Reihe*, die durch eine bestimmte „Idee“ des Dings begrenzt ist. Daher können wir zwischen einer *‚phänomenologischen*' und einer *‚mathematischen*' Auffassung des Ding an sich unterscheiden: Im Gegensatz zur ‚phänomenologischen' Auffassung des Ding an sich bzw. zur Idee einer Einzelwirklichkeit eines Dings, die leibhaftig gegeben ist und die offene Endlosigkeit der Reihe zulässt und gleichsam führt, stellt die ‚mathematische' Auffassung des Ding an sich als exakte Idee eine Grenze dar, die diese Offenheit in eine begrenzte Unendlichkeit verwandelt. Dies bedeutet, dass das ‚phänomenologische' Ding an sich eine *offene Unendlichkeit* von möglichen Apperzeptionen zulässt, während das ‚mathematische' Ding an sich als Telos einer *geschlossenen Unendlichkeit* unterschiedlicher Modi diese nicht nur begrenzt, sondern durchaus bestimmt. Den offenen Reihen der variablen Abschattungen, die durch das phänomenologische Ding an sich geleitet werden, wird eine exakte Idee auferlegt, die somit die Offenheit der Apperzeptionsreihen aufhebt.

Das Ding wird jetzt als „Einheit der konzipierten Unendlichkeit" (Hua VI, 359) von genauen Darstellungen verstanden. Das idealisierte Ding erweist sich als eine Limesfigur, die den unendlich möglichen Dingdarstellungen eine Grenze setzt: Es handelt sich m.E. nicht mehr um eine offene, sondern um eine *geschlossene* Unendlichkeit. Es handelt sich um das Durchlaufen von „doppelten Unendlichkeiten, die der Erscheinungs-mannigfaltigkeiten, in denen sich ein und dasselbe Ding darstellt, und die Unendlichkeit der Dinge" (Hua VI, 361). Im „idealen Durchlaufen dieser unendlichen Totalität" – was m.E. eine aktuale Unendlichkeit der Bestimmbarkeit und der Gattung impliziert – wäre eine „ideale Erkenntnis des Dinges" erreicht, als Dinges nicht nur der wirklichen, sondern ebenso der möglichen Erfahrungen. „So erobert – folgt Husserl – das idealisierende Denken die Unendlichkeit der Erfahrungswelt" (Hua VI, 360): Exakte Objektivität wird als eine Erkenntnisleistung betrachtet, die diese Idealitäten auf die Erfahrung evident anwendbar macht.

5. Erfahrungs- und Idealisierungsprozess. Transfinite und aktuale Unendlichkeit

Diese Aussagen stellen einerseits Husserls frühere Unterscheidung zwischen Ideation und Idealisierung in Frage:

Während in den *Ideen I,* diejenige Ideation, die Idealwesen ergibt, von derjenigen Ideation, die nur typische Wesen hervorbringt, scharf unterschieden werden mussten, legen Idealitäten in der *Krisis*schrift „den Dingen der faktischen Welterfahrung je ein Ideal ein", das die „Brücke" zwischen lebensweltlicher und „absolut vollkommener" Erkenntnis" bildet (Hua VI, 360). Diese Idealfiguren sind jetzt das Telos des Erkenntnisprozesses, der mit der wirklichen Erfahrungsbekanntheit beginnt. Es handelt sich also nicht um zwei verschiedene Leistungen (Ideation, geleitet durch die ‚phänomenologische' Auffassung des Ding an sich, und Idealisierung, geleitet durch die ‚mathematische' Auffassung des Ding an sich), sondern um einen kontinuierlichen Prozess, der Erfahrbarkeit und Nicht-Erfahrbarkeit bzw. Ideales umschließt und dessen Telos die exakte Bestimmbarkeit des Erfahrungsdings ist.

Andererseits implizieren m.E. diese Aussagen eine gewisse Änderung der Ding-an-sich Auffassung: Das Ding-an-sich ist jetzt nicht das Ideal einer partikulären Wirklichkeit eines in der Erfahrung stets unvollkommen gegebenes Dinges, sondern das Ideal eines exemplarisch vollkommenen Dinges, das nie in der Erfahrung gegeben sein kann. Denn die mathematische Methode konzipiert ein „Vollkommenheitsideal aufgrund einer Konzeption der Unendlichkeit von Unvollkommenheit […denn sie] idealisiert die Eigenschaftlichkeit der Dinge […] auch die unvollkommene Erfahrbarkeit, in der unsere Erfahrung von bekannten zu unbekannten Dingen fortschreitet" (Hua VI, 361 Fn. 1). Das Ding-an-sich erweist sich somit als ein Vollkommenheitsideal, das die Anschaulichkeit verlässt, insofern es das Exemplarische des Dings als geschlossene Unendlichkeit einerseits der unendlichen Unvollkommenheit der Gegebenheitsweisen, andererseits der Unendlichkeit der in der Erfahrung gegebenen Dinge darstellt. Dieses Ding an sich kann nur im „reinen Denken" (Hua VI, 362) erfasst werden – eine idealisierend-geistige Leistung, die auf die Anschaulichkeit fundiert ist.

Die Unendlichkeit kann aktual werden, nur indem sie *als Idee,* d.h. die Sinnlichkeit überschreitend, verstanden wird. Diese Erfüllung, als *aktual im Denken gegeben*, deutet auf den *geschlossenen Charakter der Reihe*, die durch eine bestimmte „Idee" des Dings begrenzt ist. Offene Endlosigkeit und aktuale Gegebenheit der Grenze schließen sich eigentlich aus: Es handelt sich hier um den Gegensatz zwischen dem Transfiniten und dem aktual Unendlichen im Sinne Georg Cantors. Eine *transfinite* Reihe ist „eine unbegrenzte Stufenleiter von bestimmten Modi […], die ihrer Natur nach nicht endlich, sondern unendlich sind, welche aber ebenso wie das Endliche durch bestimmte, wohldefinierte und voneinander unterscheidbare Zahlen determiniert werden können" (Cantor 1966, 176). Während das potentiell Unendliche eine veränderliche wachsende Größe ist (das Transfinite), ist das aktual Unendliche ein „in sich festes, konstantes", unvermehrbares Quantum (das Absolute), das jede endliche Größe übertrifft (Cantor 1966, 372–375). Somit ergibt sich folgende Unterscheidung: Während innerhalb des endlichen Erfahrungsbereichs die unendliche Vielfalt der sinnlichen Erfahrung synthetisiert wird und somit eine wesentliche *Offenheit* gegenüber den subsumierten Gegenständen besteht, ist die exakte Idee eine „*vollkommene*" Einheit, die als *genau bestimmte Objektivität* die Identifizierung des Dings entlang der erdachten Unendlichkeit von *unvollkommen* sinnlichen Darstellungen *allererst ermöglicht*. Dies bedeutet, dass die exakte Idee vorausgesetzt werden muss, wobei das Problem eines Begründungszirkels entsteht, was Husserl selbst anerkannt hat. Wie Richir hervorhebt, weist diese Idee absolut keinen phänomenologischen Ursprung auf insofern sie „unbedingt" ist: Sie begründet sich selbst, jenseits jedweder Motivation und phänomenologischer Begründung (Richir 1990, 225).

6. Idealisierung und Aporie. Geschichtliche Kontingenz der Idealwesen

Der Idealisierung des Dings folgt die Idealisierung der Welt, wonach der offene Welthorizont in seiner möglichen Erfahrbarkeit überschritten wird. Im idealen Durchlaufen der relativ vollkommenen Darstellungen erwächst die Erkenntnis der Dinge und der Welt, indem das idealisierende Denken die Unendlichkeit der Erfahrungswelt erobert. Dingen und Welt wird ein Ideal auferlegt, das aber keine Möglichkeit einer Brücke zwischen dem vorgegebenen Ding und dessen individuellem idealen Sein zulässt. Husserl versucht, diese Brücke *a tergo* zu bauen: „(D)as radikale *Problem der historischen Möglichkeit ‚objektiver' Wissenschaft*" besteht darin, die aus der Idealisierung der Erfahrung erwachsene exakte Objektivität als Erkenntnisleistung auf die Erfahrungswelt zurückzuführen und in ihrer „*Sinnhaftigkeit*" evident zu machen (Hua VI, 360):

> „*Objektivierung* ist Sache der *Methode*, fundiert in vorwissenschaftlichen Erfahrungsgegebenheiten. Mathematische Methode ‚konstruiert' aus anschaulicher Vorstellung ideale Gegenständlichkeiten und lehrt, diese operativ und systematisch zu behandeln. Sie erzeugt nicht handelnd Dinge aus Dingen, sie erzeugt Ideen; Ideen entspringen durch eine eigenartige Geistesleistung: durch Idealisierung." (Hua VI, 361)

Um der Arbeit der Objektivierung nachzugehen ist es sinnvoll, auf die bereits erwähnte Stelle bei Husserl zurückzukommen. Die Objektivierung nämlich,

> „konzipiert ein Vollkommenheitsideal aufgrund einer Konzeption der Unendlichkeit von Unvollkommenheit, durch eine ihr eigenwesentliche Gradualität motiviert. Sie idealisiert die Eigenschaftlichkeit der Dinge. Sie idealisiert damit korrelativ ihre Identifizierbarkeit, andererseits idealisiert sie auch die unvollkommene Erfahrbarkeit, in der unsere aktuelle Erfahrung von bekannten zu unbekannten Dingen fortschreitet; so wird einem Gang iterativer Vervollkommnung eine schlechthinnige Unendlichkeit der Iteration substruiert – als Ideal." (Hua VI, 361 Fn. 1.

Aus dem jetzt vollständigen Zitat wird ersichtlich, dass die Idee der unendlichen Vollkommenheit bzw. der unendlichen Bestimmbarkeit vorausgesetzt werden muss, damit die Idee der unendlichen Unvollkommenheit der Erfahrung erkannt werden kann. Obwohl diese exakte Idee aus der sinnlichen Erfahrung erwächst und nachträglich auf sie angewandt wird, muss sie im Voraus vorliegen, um das Ding in seiner Identität und Unvollkommenheit zu konzipieren (Vgl. Richir 1990, 228). Beide Ideen der Unendlichkeit – einerseits der idealen und vollkommenen Bestimmbarkeit und Erfahrbarkeit und andererseits der reelen und unvollkommenen Erfahrbarkeit – sind *koextensiv* und bedingen einander.

Dennoch setzt Husserl die Bestimmung der Unerreichbarkeit einer idealen Perfektion m.E. als eine *Begrenzung* und zugleich als eine *Bedingung* der Idealisierung:

Die Idealisierungsvorgänge sind trotz ihrer Vollendung nie ein für allemal ‚fertig' oder festgelegt, sondern *provisorisch*: Die unendliche Iteration der Idealisierungsarbeit deutet korrelativ auf die *Kontingenz* ihrer Produkte; trotzdem muss diese *kontingente*, aber *vollkommene* Idee als Telos dem Prozess unterliegen, um die *Identität* der Darstellungen zu gewährleisten. Es ist die „Iteration der Potentialitäten" (Hua XV, 670) bzw. die unendliche Iteration der horizonthaften Erfüllungsmöglichkeiten der Horizonthaftigkeit der Welt, aus der die Unendlichkeit erwächst: Unendlichkeit bzw. die *unendliche Potentialität* entsteht aus der Iteration der Idealisierungsarbeit.

Es handelt sich darum, die „Sinnhaftigkeit" der „geistigen Motive", nämlich der Mathematik und der mathematischen Naturwissenschaft, in ihrer Ursprünglichkeit aufzudecken und zu „verstehen". Dazu gehört als „Sinnesfundament" die „Welt der Sinnlichkeit" sowohl in ihrem „historischem Wandel" wie „in ihrer invarianten Allgemeinheitsstruktur" (Hua VI, 360*ff*.) der „Raumzeitlichkeit" (Hua VI, 362). Diese Ausführungen Husserls lassen darauf schließen, dass die Iteration der Idealisierungsarbeit durch die geschichtliche Wandelbarkeit der Sinnbildungen bedingt ist und umgekehrt. Wenn dies zutrifft, dann sind korrelativ dazu die aus diesem Vorgang entstandene, unterschiedliche Tele nicht absolut festgelegt, aber auch nicht willkürlich: Die *geschichtlich bedingte Wandelbarkeit* und zugleich *strukturelle Invarianz* der Welt lässt sich auf die ihr subsumierten Ideen übertragen, so dass eine wesentliche *Korrelation* entsteht.

In diesen Aussagen modifiziert Husserl – so scheint mir – seine Position aus den *Logischen Untersuchungen* und den *Ideen I*: Das *Telos* ist, obwohl vollkommen bestimmt, *kontingent* und unterliegt dem historischen Wandel bzw. der sich geschichtlich entwickelnden Sinnbildungen und bestimmt diese zugleich. Das *Telos* weist eine *Doppelstruktur* auf: *Wandelbarkeit durch wiederholte Erzeugbarkeit bei festgelegt-Sein auf eine allgemeine Struktur*. Das *Telos* unterliegt also einem *genetischen Prozess*, dessen Ursprung ‚nicht sogleich' zugänglich ist. Dies bedeutet, dass die *außerweltliche* und vollkommene Idee eine *Form-prägende* bzw. *Form-vorbestimmende Funktion* auf die *innerweltlichen* und empirischen Darstellungen der Dinge ausübt. In dieser

Funktion ähnelt das *Telos* der aristotelischen Form, aber es unterscheidet sich davon durch seine *Transzendenz*; denn, wie Richir bemerkt, die Idealisierung „bleibt" Idealisierung, sie wird nicht von der Erfahrung „aufgesogen", wie bei Hegel, sondern wird unter sie „substruiert" (Richir 1990, 229).

Aber zurück zum Text: Husserl unterscheidet zwischen einer ersten und einer zweiten Idealisierung: Während die erste „exakt identifizierbare Ideen" erzeugt, konstruiert die zweite operativ „Ideengebilde aus vorgegebenen Ideen" (Hua VI, 361), die auf die raumzeitliche Struktur der Welt beschränkt sind (Hua VI, 362). Hieraus ergibt sich die Frage nach dem Unterschied zwischen Mathematik und Physik: Mathematik „konstruiert" aus der anschaulichen Vorstellung „ideale Gegenständlichkeiten", die danach operativ in die Praxis eingesetzt werden können, während die Physik durch die Mathematik als „reines Denken" und die von ihr vollzogenen operativen Konstruktionen das Feld ihrer Leistung schon vorgegeben bekommt (Hua VI, 361*ff.*). Im Prinzip scheinen sich beide Disziplinen zu decken, jedoch beschränkt sich die mathematische Leistung auf die „bloßen raumzeitliche(n) Gestalten bzw. auf die zur Welt universal gehörige Struktur der Raumzeitlichkeit" unter Abstraktion vom nicht rein Körperlichen. Die Physik scheint hier die Aufgabe zugeteilt zu bekommen, die von der Mathematik erzeugten Objektivitäten auf die wissenschaftliche Praxis anzuwenden – eine zunächst rein *instrumentelle* Funktion, die zu befragen wäre.

Nach Husserl ist nicht nur jede Praxis *a priori* bestimmt, sondern auch die Faktizität der Geschichte setzt das *Apriori* der Geschichte, die auf das Sein der Menschen bezogen ist, voraus. Dieses *Apriori* wird dem Menschen nicht nur auferlegt, sondern zugleich von ihm, „in" ihm ausgebildet (Hua VI, 362). Aus diesen Gründen fragt sich Husserl, ob „nicht alle Wissenschaft […] aus einer Idealisierung, die selbst im historischen Raum ist, das Apriori der Geschichte voraus[setzt], das selbst aus einer Idealisierung ist?" (Hua VI, 363) Zu dieser Problematik eines *aporetischen Zirkels* zwischen ein und derselben Objektivität, die ursprünglich entstanden, *a priori* im historischen Raum eingesetzt wird, gesellt sich noch ein Wesensmerkmal der Husserlschen Phänomenologie: nämlich, wie Richir hervorhebt, das einer Teleologie der Vernunft, die – stark von der phänomenologischen Praxis imprägniert – mit der „unmöglichen Verwirklichung [*réalisation*]“ des vernünftigen *Telos*, dem keine „vollbestimmte *Arche* [*plein de soi*]“ entspricht, konfrontiert ist (Richir 1990, 236). Es handelt sich also um das Problem der *Unzugänglichkeit des Ursprungs* sowie der *Unerfüllbarkeit bzw. Unerreichbarkeit des Telos*, eine ‚*Insuffizienz*‘, die gerade das Wesentliche der Phänomenologie ausmacht.

Aus diesem von Husserl anerkannten *Zirkel* gibt es m.E. einen Ausweg, nämlich durch die Anerkennung der oben herausgestellten *Kontingenz der Ideenbildungen*: Einerseits,

1. indem die Ideen als Produkte einer *historischen Genesis* verstanden werden und somit auch dem Wandel unterliegen, wodurch sie ihre Absolutheit, jedoch nicht ihre Exaktheit verlieren würden, oder andererseits,
2. indem die reinen Objektivitäten nur als *allgemeine, absolut gültige Strukturen* verstanden werden, die eine *kontingente Mannigfaltigkeit von Variationen* umfassen und zulassen.

In dieser Hinsicht scheint Husserl eine Vermittlung zwischen der lebensweltlichen Praxis und den Gestalten einerseits und den aus ihr entstandenen absoluten Objektivitäten andererseits zu suchen, indem er betont, dass diese „idealisierende geistige Leistung“ ihr Material an den subjektiven „Dingerscheinungen“ bzw. „Dingvorstellungen“ hat und dass das „Ich“ ihr Vollzieher ist (Hua VI, 357–359). Doch die *Kluft* bleibt bei Husserl bestehen: Die Vollzugsweisen der konkreten Deckung der ganzen Intentionalität von Erscheinungen und Horizonten und derjenigen des idealisierenden Denkens unterscheiden sich; denn die letztere ist nicht sinnlich, sondern *logisch* bestimmt: „zunächst das kontinuierliche Veranschaulichen der unbestimmten kommenden Erscheinungen als möglichen, dann das Exemplarische, dann die Konzeption der Unendlichkeit, etc.“ (Hua VI, 362). Diese Welt ist, obschon sie ursprünglich eine subjektiv geistige Leistung ist, „objektiv“, insofern ihre Erkenntnisse der Idealisierung dienen, welche aus der Anschauung entspringt, aber sie verlässt. So vollzieht sich der Übergang zwischen *doxa* und *episteme*“ (Vgl. Aguirre 2010, 167–190).

7. Offenheit des Wesens und der Unendlichkeit

Obwohl Husserl in späten Manuskripten die Idealisierung im Bereich der Lebenswelt verneint – „die ursprüngliche lebensweltliche Erfahrung […kennt] noch nichts von diesen Idealisierungen“ (EU, 43) – scheint Husserl in seiner *Krisis*schrift durchaus an den Gedanken festzuhalten, dass eine „raumzeitliche Unendlichkeit“ zur Lebenswelt gehört, denn „(d)as Kategoriale der Lebenswelt hat die gleichen Namen“ aber deutet auf keine Exaktheit hin (Hua VI, 142). Trotzdem markiert eine Stelle in den *Ideen II* eine gewisse Änderung in Husserls Auffassung von der Unendlichkeit. Er erwägt darin die Möglichkeit einer „Offenheit“, die sowohl das Wesen eines individuellen Dings als auch die Unendlichkeit kennzeichnet. In Husserls Worten:

> „Im voraus – a priori – ist [das Ding] durch sein eigenes Wesen vorgezeichnet. […] Aber hat jedes Ding […] überhaupt ein solches Eigenwesen? Oder ist das Ding sozusagen *immer auf dem Marsch*, ist es gar nicht in dieser reinen Objektivität zu fassen, vielmehr vermöge seiner Beziehung zur Subjektivität prinzipiell nur ein relativ Identisches, etwas, das nicht im voraus sein Wesen hat bzw. hat als ein für allemal erfassbares, sondern ein *offenes Wesen* hat, das immer wieder je nach den konstitutiven Umständen der Gegebenheit neue Eigenschaften annehmen kann? Aber da ist das Problem, den Sinn dieser Offenheit […] zu präzisieren.“ (Hua IV, 299, hervorgeh. von mir)

Die Erwägung dieser „Offenheit“ führt Husserl dazu, die in den frühen Schriften behauptete Abgeschlossenheit der Welt und der Dinge für den Geist in Frage zu stellen:

> „Besagt die ‚Unendlichkeit‘ der Welt statt einer transfiniten Unendlichkeit (als ob die Welt ein in sich fertig seiendes, ein allumfassendes Ding oder abgeschlossenes Kollektivum von Dingen wäre, das aber eine Unendlichkeit von Dingen in sich enthalte), besagt sie nicht vielmehr eine ‚Offenheit'?“ (Hua IV, 299)

Husserl bricht hier eindeutig mit dem aristotelischen Glauben an einer an sich immer schon feststehenden und unveränderlichen Substanz oder Essenz des individuellen Dinges. Hier richtet sich Husserl gegen die aristotelische Ontologie der Formsubstanz, der Essenz. Offenheit besagt hier, dass die Eigenschaften der „Dingsubstanzen“ sich offen gestalten können, so dass sie an keinem Variationsumfang mehr begrenzt sind: Ein Ding kann sich qualitativ unendlich bis sogar zur Unkenntlichkeit verändern, da es an keine feststehende Essenz mehr gebunden ist. Es ergibt sich ein schwerwiegendes Problem: Ohne ein Subjekt, das die Dinge *hic et nunc* bestimmt, kann das Ding nicht vollständig individuiert werden. Dies bedeutet, dass die Identifizierbarkeit der Dinge eine Iteration des Immer-wieder-bestimmens voraussetzt, was in Husserls Verständnis wiederum eine idealisierende Leistung bedeutet. Ein Problem, das sich m.E. nicht mit Hinweis auf den Text schlichten lässt.

Diese Stelle aber markiert eine wesentliche Änderung der Idee des Ding an sich, wie sie noch in den *Ideen I* aufgefasst wird. Dieser Schrift zufolge bleibt im Wahrnehmungsprozess prinzipiell immer ein Horizont „bestimmbarer Unbestimmtheit"; denn „*in infinitum unvollkommen zu sein, gehört zum unaufhebbaren Wesen der Korrelation Ding und Dingwahrnehmung*" (Hua III/1, 91f.). Diese Unvollkommenheit ist durch den ‚Sinn von Ding' gefordert: „Bestimmt sich der Sinn von Ding durch die Gegebenheit der Dingwahrnehmung […], dann fordert er solche Unvollkommenheit, verweist uns notwendig auf kontinuierlich einheitliche Zusammenhänge möglicher Wahrnehmungen" (Hua III/1, 92). Aber der Sinn dieser Unbestimmtheit ist „vorgezeichnet" durch das allgemeine Wesen dieses Wahrnehmungstypus. Die *Unbestimmtheit* bedeutet hier keine *offene* Möglichkeit, sondern eine Möglichkeit, die man als ‚von *geschlossenem* Umfang' bezeichnen könnte. Denn die Wahrnehmungen erstrecken sich in „unendlich vielen Richtungen in *systematisch fest geregelter Weise*" (Hua III/1, 92). Es handelt sich hier um eine Unbestimmtheit, die nicht vollkommen in Bestimmtheit verwandelt werden *kann*, weil der Wahrnehmungsgegenstand eine unendliche Abschattungs-mannigfaltigkeit ist. Unbestimmtheit und Unvollkommenheit werden hier gleichgesetzt; denn es ist die *verbleibende Unbestimmtheit des Wahrnehmungsprozesses, welche die konstitutive Unvollkommenheit des wahrgenommenen Dings bedingt*. Diese Unbestimmtheit ist jedoch keine, die Beliebiges zulässt; denn erstens sind die Erfüllungsmöglichkeiten durch den Wesenstypus vorgezeichnet und motiviert, und zweitens wird der Wahrnehmungsprozess geleitet und genormt durch die Idee des

Dings als einer einsichtigen, „feste(n) Regel“ für den Fortgang der Erfahrung (Hua XX/1, 200).

Diese „Unbestimmtheit“ muss also von der „Offenheit“ streng unterschieden werden: Wenn das Ding eine offene Essenz hat und dementsprechend neue Qualitäten erwerben kann, dann ist der Wahrnehmungsprozess von einer Idee geleitet, die selbst diese Offenheit zulässt. Diese Idee stellt weder eine feste und unveränderliche Norm, noch eine Variabilität innerhalb eines gewissen Umfangs dar; denn sie sprengt jedwede Normativität. Die Idee im Kantischen Sinn könnte als eine kategoriale Form bzw. als ein allgemeiner Gattungsbegriff verstanden werden, der sich wie jede kategoriale Form in der sinnlichen Anschauung des Dings erfüllt. Eine Erfüllung, die jedoch hier bei jeder kategorialen Wahrnehmung, d.h. bei jeder aktuellen Wahrnehmung vom Subjekt wiederholt zustande gebracht werden muss, um den Gegenstand – provisorisch, da er sich kontinuierlich verändert – zu identifizieren. Es entsteht hier eine unendliche Reihe verknüpfender Akte, die den erfüllenden Anschauungen zugeordnet werden können – eine Erfüllung, die jedoch wegen der Offenheit des Dingwesens nicht gesichert ist, d.h. nur eine mögliche sein kann.

8. Schlussbemerkung: Ziel adäquater Erkenntnis als optimale Gegebenheit des Gegenstandes

Das Ziel adäquater Erkenntnis ist nicht die eigentlich unmögliche „absolute Gegebenheit in absolut erfüllter Weise“ (Hua XVI, 125), sondern die „optimale Gegebenheit“ des Gegenstandes; sie ist das *erreichbare Telos* des Wahrnehmungsprozesses, das „Ideal der letzten Erfüllung“. „Optimale Gegebenheit“ meint hier die Erfüllung einerseits der vollen und gesamten *Intention* – d.h. der Gegenstand ist wirklich gegenwärtig oder gegeben, wenn es das ist, was intendiert ist – und andererseits der *Bedeutung*, wenn der Gegenstand das ist, was gemeint und gedacht wird. Diese Adäquation des „Gedankens“ an die „Sache“ ist eine doppelte: Die Synthesis der Erfüllung wird als Anpassung der „Anschauung“ verstanden, erstens an die Bedeutung – das Gedachte ist vollständig gegeben –, zweitens an „die Sache selbst“, d.h. wenn die

Anschauung keiner Erfüllung mehr ermangelt (Hua XIX/2, 648). *Optimale Gegebenheit* wird also als *optimale Anschauung* verstanden, deren Evidenz den Charakter einer „maximal gesteigerten Erscheinung“ gemäß unseren Interessen hat. Diese Steigerungsgrenzen sind variabel, sie sind von der Befriedigung der unterschiedlichen Interessen abhängig (Hua XVI, 126). Sie sind also nicht absolut gegeben, sondern *subjektiv-relativ*. Dieser Begriff des Ding an sich als Ideal der „adäquaten“ Gegebenheit scheint sich somit bei dem der „optimalen“ Gegebenheit zu erübrigen, besonders wenn die Unerreichbarkeit des ersteren als Mangel der Wahrnehmung oder der Erkenntnis verstanden wird. Im Gegenteil, die optimale Wahrnehmung setzt m.E. die adäquate Wahrnehmung voraus; denn die adäquate Wahrnehmung bietet der optimalen ein Ziel, das sie nicht zu erreichen „braucht“, wie Husserl sagt (Hua XIX/2, 613*ff.*), aber dennoch eine Richtung, die sie einschlagen ‚muss‘, um eine *bestmögliche* Fülle der Wahrnehmung sowie ein *maximales* Zufriedenstellen der Interessen zu gewähren. Der *Zweck* des Erfahrungsprozesses innerhalb der Lebenswelt liegt also in dem Erfüllen unserer Erkenntnisinteressen und nicht in dem Erreichen der exakten Bestimmbarkeit des Gegebenen. So ist der dynamische Erfahrungsprozess durch eine gewisse ‚Offenheit‘ gekennzeichnet. Sie besagt hier nicht nur einen variablen Umfang der Erfüllung, sondern auch die Kontingenz der Sinnbildung im Rahmen der lebensweltlichen Erfahrung.

Literatur

Aguirre, Antonio. 2010. „Edmund Husserl: Idealisierung und Doxa“. *Phänomenologische Forschungen* 2010, 167-190.

Bernet, Rudolf. 2004. *Conscience et Existence.* Paris: PUF.

Breuer, Irene. 2018. “Kant und Husserl: Ding an sich und Telos des Erfahrungsprozesses“. In *Kant und seine Kritiker – Kant and His Critics*, hrsg. von Antonino Falduto und Heinrich F. Klemme, Zürich/New York: Olms, 275–291.

Cantor, Georg. 1966. „Über unendliche lineare Punktmannigfaltigkeiten.“ In *Abhandlungen, mathematischen* *und philosophischen Inhalts*, hrsg. von E. Zermelo. Hildesheim: Olms, 139-247.

Held, Klaus. 2003. „Krise der Gegenwart und Anfang der Philosophie. Zum Verhältnis von Husserl und Heidegger". In *Kunst und Wahrheit. Festschrift für Walter Biemel*, hrsg. von Madalina Diaconu, 131-144. „Studia Phaenomenologica 3 (Special Issue): 131–144.

Husserl, Edmund. 1950-2012. *Husserliana: Edmund Husserl – Gesammelte Werke*. Bde. 1-41. Haag: Martinus Nijhoff (Bd. 1-26, 1950-1987); Den Haag: Kluwer Academic Publishers (Bd. 27-37, 1988-2004); New York: Springer (Bd. 38-41, 2005-2012). [Abgekürzt: *Hua*].

________. 1973. [Hua III/1]. *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Erstes Buch*. In *Husserliana,* Bd. III.1, hrsg. von Karl Schuhmann. Den Haag: Martinus Nijhoff.

________. 1952. [*Hua* IV]. *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Zweites Buch: Phänomenologische Untersuchungen zur Konstitution*. In *Husserliana,* Bd. IV, hrsg. von Marly Biemel. Den Haag: Martinus Nijhoff.

________. 1953. [*Hua* V]. *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie*, *drittes Buch. Die Phänomenologie und die Fundamente der Wissenschaften*. In *Husserliana,* Bd. V, hrsg. von Marly Biemel. Den Haag: Martinus Nijhoff.

________. 1976. [Hua VI]. *Die Krisis der europäischen Phänomenologie. Eine Einleitung in die phänomenologische Philosophie.* In *Husserliana,* Bd. VI, hrsg. von Walter Biemel. Den Haag: Martinus Nijhoff.

________. 1972. *Erfahrung und Urteil. Untersuchungen zur Genealogie der Logik*, hrsg. von Ludwig Landgrebe. Hamburg: Meiner. [Abgekürzt als EU]

________. 1973. [Hua XV]. *Zur Phänomenologie der Intersubjektivität. Texte aus dem Nachlass*. Dritter Teil. 1929-35. In *Husserliana*, Bd. XV, hrsg. von Iso Kern. Den Haag: Martinus Nijhoff.

_______ . 1973 [Hua XVI]. *Ding und Raum. Vorlesungen 1907.* In *Husserliana*, Bd. XVI, hrsg. von Ulrich Claesges. Den Haag: Martinus Nijhoff.

________. 1984. [*Hua* XIX/2]. *Logische Untersuchungen. Zweiter Teil. Untersuchungen zur Phänomenologie und Theorie der Erkenntnis.* In *Husserliana,* Bd. XIX.2, hrsg. von Ursula Panzer. The Hague: Martinus Nijhoff.

________. 1993. [Hua XXIX] *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie. Ergänzungsband. Texte aus dem Nachlass 1934-1937.* In *Husserliana,* Bd. XXIX, hrsg. von R.N. Smid. Den Haag: Martinus Nijhoff.

Kant, Immanuel. 2016. [AA, A/B]. *Kritik der reinen Vernunft.* Erste Auflage 1781 („A“) und Zweite Auflage 1787 („B“). In *Werke in Sechs Bänden*, Bd. II, hrsg. von Wilhelm Weischedel. Darmstadt: WBG. [Akademie Ausgabe, hrsg. von der Preußischen Akademie der Wissenschaften, Bd. III (1904, 1911) und IV (1911), Hrsg. Benno Erdmann, Berlin: G. Reimer]. [Abgekürzt als AA A/B]. Darmstadt: WBG, Auflage: 6., unveränd. (1. März 2004).

Richir, Marc. 1990. *La crise du sens et la phénoménologie*. Grenoble: Million.

Tengelyi, László. 2007. *Erfahrung und Ausdruck, Phänomenologie im Umbruch bei Husserl und seinen Nachfolgern.* Dordrecht: Kluwer.

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Oskar Becker or the Reconciliation of Mathematics and Existential Philosophy

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Abstract

Oskar Becker's work in the philosophy of mathematics makes an important contribution to the philosophical understanding of the constructivist program. Becker (1889-1964), a student of Edmund Husserl and an associate of Martin Heidegger, initially sought to ground a constructivist view of mathematics in Husserl's transcendental phenomenology; subsequently he adopted a Heideggerian and existential view of mathematics that, he argued, would allow one to rescue large parts of classical mathematics from an intuitionist and constructive perspective. In his later writings he finally turned to a radically historicist interpretation of the constructivist program.

Keywords: Edmund Husserl; Hermann Weyl; L.E.J. Brouwer; Martin Heidegger; Mathematical intuitionism; Constructivism; Historicism

I begin with a piece of autobiography. When I first heard of symbolic logic I was a schoolboy, about fifteen years old. By chance, I had discovered a little book on the topic in some local bookstore and had found its treatment of the propositional and the predicate calculus an endlessly intriguing subject matter. It was a time when I dreamt at night of sex and truth-tables. I don't know any more in what order. Then, as a freshman at the University of Bonn, I discovered a course in symbolic logic taught by one Oskar Becker. He turned out to be a charming, elderly, slightly dotty professor who still taught his regular classes even though he was retired and who was scheduled that semester not only for his logic course but also a seminar with the title "The Principle of Reason." In my innocence I conjectured that this would be more logic and so enrolled in both classes. The seminar, however, turned out to be on the later Heidegger, of whom I had never heard till then, and the text for the course was a small book of lectures entitled "The Principle of Reason," that quickly intrigued me with its darkly poetic prose. Heidegger was asking himself in the work why calculative reason has become so dominant in our culture and was warning of the limits of all such reasoning. While the principle of sufficient reason tells us that everything has its rational ground, its claim to universal validity was itself mysterious; the human condition remained, in fact, unexplained by it. In short, as Heidegger said at the end, "Being: the Abyss."

For Becker, as I saw him in that semester, the conjunction of logic and later Heidegger seemed to come naturally. I learned afterwards that he had been a mathematician at first, then a student of Edmund Husserl's, and finally a personal friend and philosophical associate of Martin Heidegger, that he had written extensively on the history and philosophy of mathematics and had made contributions to modal logic as well as to aesthetics and other parts of philosophy. How interconnected these interests were in Becker's own mind became clear to me only many years later when I studied his book on "the logic and ontology of mathematical phenomena," one of his major pieces of writing, published in 1927 under the title *Mathematische Existenz*. The first thing that struck me about that work was that it had appeared in volume 8 of Husserl's *Yearbook for Philosophy and Phenomenological Research* side by side with Heidegger's *Being and Time*. The two works made up the entire content of that particular yearbook. At first sight, they seemed to have nothing in common. Becker's treatise belonged, after all, to the philosophy of mathematics: it examined the conflict between L.E.J. Brouwer's mathematical intuitionism and David Hilbert's formalism and sought to put mathematics on a new philosophical footing. Heidegger's work, on the other hand, concerned itself with the conditions of human Dasein and with what its author called the temporality and historicality of Being. But it seemed that Husserl, the editor of the *Yearbook,* had considered the works as related and complementary. Part of the explanation I found in Husserl's conviction that Becker and Heidegger represented the two sides of the phenomenological movement in philosophy - the scientific and the humanistic one, for short - and that he hoped his two disciples would bring his own work to further fruition in these two directions.

This was not to be, as we now know. Heidegger's *Being and Time* was, in fact, a radical turn in the road of phenomenology and not a continuation of Husserl's direction of thought. And by 1927, Becker was walking more in the footsteps of his friend Heidegger than those of his teacher Husserl. *Mathematische Existenz* constituted, in fact, an attempt to re-think mathematics with the help of Heidegger's insights. Becker made that explicit in his preface in which he announced that his treatment of the philosophy of mathematics was relying primarily on Heidegger's hermeneutic-phenomenological method of investigation – in addition, he hastened to add, to the methods of Husserl's formal transcendental-constitutive phenomenology. (442) With a further bow to Heidegger, Becker also declared it to be his intention "to put 'mathematical existence' in the context of human Dasein which must be regarded everywhere as the fundamental context of interpretation." (442) In other words, he proposed to treat the problem of the existence of mathematical objects with the tools of existential philosophy; hence, presumably, the title of his work

1. Husserlian Beginnings

Mathematische Existenz was, in fact, Becker's second attempt at a foundational theory for mathematics. In 1923 he had developed an account of the foundations of geometry that had drawn not yet on Heidegger but instead on the philosophical work of his teacher Husserl. Relying on both published and unpublished writings he summarized and commented extensively on Husserl's views in that essay. "In writing this treatise I owe gratitude in the first instance to Edmund Husserl," he declared, "whose research is the foundation on which it arises." At the same time he had spoken of his reliance on the work of the mathematician Hermann Weyl "whose account of the mathematical and physical problems," he added, "provided particularly suitable material for phenomenological analysis because he himself is close to phenomenology." (388) Weyl was a mathematician sympathetic to "the need of a phenomenological perspective on all questions of the clarification of basic concepts." (van Dalen, 3) Husserl had even invited him to submit an article to the phenomenological *Yearbook* and had attached "very high value" to Weyl's proposed contribution, an essay on "The New Foundational Crisis in Mathematics." To Husserl's regret, Weyl, though, finally decided to publish the piece instead in a mathematical journal.

In that essay Weyl had contrasted the classical, atomistic view of the continuum as an ordered set of points (a view he himself had espoused in earlier writings) with Brouwer's conception of "the continuum as medium of free becoming." (Weyl 1998a, 93) Identifying now with Brouwer's views, Weyl had written: "It would have been wonderful had the old dispute led to the conclusion that the atomistic conception as well as the continuous one can be carried through. Instead the latter has triumphed for good over the former. It is Brouwer to whom we owe the solution of the continuum problem." (Weyl 1998a, 99) In a later paper Weyl spoke of the conflict between Hilbert and Brouwer as deeply grounded in fundamental questions of epistemology. "The old opposites of realism and idealism, of the *Being* of Parmenides and the *Becoming* of Heraclitus, are here again dealt with in a most pointed and intensified form." (Weyl 1998b, 141) With Brouwer, he wrote, "mathematics gains the highest intuitive clarity; his doctrine is idealism in mathematics thought to the end." (Weyl 1998b 136) But "full of pain" Weyl also regretted that in Brouwer's reconstruction "the mathematician sees the greatest part of his towering theories dissolve in fog." (Weyl 1998b, 136) This gave him renewed sympathy for Hilbert's attempt to salvage the entire edifice of classical mathematics. He noted that Hilbert was, in reality, not as sharply separated from Brouwer as the polemical tone of their debate made it appear. For Hilbert, too, was "completely convinced that the power of interpreted thought does not reach further than is claimed by Brouwer, that it is incapable of supporting the 'transfinite' modes of inferences of mathematics, and that there is no justification for all the transfinite statements of mathematics qua *interpreted, understandable truths*." (Weyl 1998b, 136) Hilbert had sought to rescue transfinite mathematics, however, by treating its formulas as un-interpreted, contentless combinations of signs whose formal consistency could be established by means of an interpreted finitary metamathematics. For Weyl this proved in the end unsatisfactory. "If Hilbert is not just playing a game of formulae, then he aspires to a theoretical mathematics in contrast to Brouwer's intuitive one. But where is that transcendental world carried by belief, at which its symbols are directed?" (Weyl 1998b, 140) It was these ideas of Weyl's that provided Becker in 1923 with the material for his own thinking about mathematics. Husserl himself summarized Becker's work at the time in a letter to Weyl as coming to the conclusion "that the Brouwer-Weyl theories are the only ones that stand up to the strict, indispensable demands of a constitutive-phenomenological research into foundations." (van Dalen, 7)

Husserl's enthusiasm for Weyl's intuitionism and for Becker's appropriation of Weyl's view calls for explanation. For Husserl is not generally considered to have been inclined towards a constructive view of mathematics. He is, perhaps, best known for his defense of the objectivity of logic and the rejection of all forms of psychologism as spelled out in the first volume of the *Logical Investigations* of 1900. Husserl appears there committed to an uncompromising Platonic realism similar to the one generally ascribed to Frege. In a review of Husserl's total work Becker found it therefore necessary to address this apparent discrepancy between Husserl's objectivism and his apparent inclination towards constructivism. He pointed out that neither Husserl's Philosophy of Arithmetic of 1891 nor the later volumes of the *Logical Investigations* are committed to any form of realism. Becker rejected, moreover, the claims of those interpreters who believe that "Husserl had developed from an extreme representative of psychologism (in *The Philosophy of Arithmetic*) to the most radical anti-psychologist (in volume one of the *Logical Investigations*) and had afterwards (beginning already in volume two of the *Logical*

Investigations and then in further writings) relapsed more or less back into psychologism." (B/H, 120) Instead, he spoke of a continuous development in Husserl's thought which had led phenomenological research step by step to an explicit recognition of the transcendental idealism that was the hallmark of Husserl's later philosophizing. Already in *The Philosophy of Arithmetic*, Becker argued, one could find an endorsement of "the principle of transcendental idealism" which asserts "the universal accessibility in principle to all objects of which philosophy can speak with any sense at all." (B/H, 123) Volume one of the Logical Investigations had to be seen in this context.

> The correctly understood principle of transcendental ("constitutive") idealism is an integral component of phenomenology as such. Accordingly it can be pointed out in different forms in every phase of Husserl's philosophizing. (B/H, 123)

Volume one of the *Logical Investigations* was therefore not to be read as an argument in favor of a Platonic realism, but as an attack on the "empiricism, anthropologism, relativism, and psychologism of the time." (B/H, 124) Husserl's later views on his principle of transcendental idealism should, however, treated as an expression of a "constitutive," and that is constructive view of reality. Husserl had thus been, in essence, a constructivist throughout his career.

2. The Heideggerian Becker

In trying to think of logic and mathematics from the perspective of existential philosophy Becker drew on two propositions fundamental to that tradition, propositions characteristic of Heidegger's thought up to and including *Being and Time* which entirely bypass his later hostility towards logical reasoning. The first of these concerns what Becker called "an 'existentialist' identification of 'reality' with the reality of factual life." (Gr., 61) We can express the idea succinctly in the assertion that (1) the real is the temporal.

Let us call this the anti-Platonic principle of existential thought. It derives, of course, from Nietzsche and is not specific to Heidegger. The principle refuses an interpretation of the temporal world in terms of a supposedly a-temporal realm, be it that of the Platonic ideas, of concepts, numbers, and values conceived as "real" entities, or of transcendent supernatural powers. The proposition is also directed against Kant and the post-Kantians who insist that empirical reality can only be understood by appeal to "transcendental", that is, by appeal to necessary and hence timeless principles of human reason.

This anti-Platonism the existential tradition shares with a number of different movements of nineteenth and twentieth century thought, in particular with the kind of philosophical naturalism and positivism characteristic of some early phases of so-called analytic philosophy. The second proposition on which Becker builds his philosophy of logic and mathematics is, by contrast, more specifically tied to existential thought and even more specifically to the Heideggerian version of it. It maintains that (2) Human existence is through and through historical. We can call this, for lack of a better name, the historicist principle. It implies, in particular, that our understanding of the world, our determination of meanings, that is the whole process of interpreting the world and our symbols (including the linguistic and mathematical ones), is historical in nature. In clarification of the concept of the historical here appealed to Becker agrees with Heidegger that historical time experience has a specific structure which is not captured by the scientific notion of an infinite, linear, neutral time-series. Human time experience is characterized rather by its finitude (hence the central significance of death for interpreting ourselves); it is secondly characterized by our directedness towards the future, "our projective running forwards towards the future," as Heidegger puts it; and it is characterized thirdly by the experience of the uniqueness of the historical event.

I will now try to describe the effect these two propositions have on Becker's thinking about logic and mathematics. Proposition (1), the claim that the real is the temporal, leads him to conclude immediately that logic and mathematics, too, need to be interpreted in temporal terms. He writes:

> Time is not only the form of inner sense, but the fundamental structure of human life altogether... Our existence can be characterized as temporality. Time is not a mere form that surrounds

> us, but permeates our total being and essence. That shows itself also – even though it is often overlooked - in mathematics... We can and must count and calculate only because we are temporal beings. An eternal, infinite being does not need to count. (Gr., 158)

Mathematics, too, must then be interpreted in terms of the notion of time. Becker argues that such an interpretation has already been undertaken by Brouwer. Dutch intuitionism is thus the position in the philosophy of mathematics that corresponds to the existential point of view. Intuitionism can and must be supported by means of considerations drawn from existential philosophy.

Existential philosophy recognizes, first of all, no a-temporal notion of truth. Elaborating on Becker we can point out that Nietzsche already urges us to abandon our Platonistic notion of "objective" truth and to replace it with that of "my truth", that is with the recognition that which propositions we can assert will depend on the occasion and the moment. At every occasion, I will be in a position to assert some propositions and to deny others, but there will also be many propositions about which I am not in a position to make a judgment. Having replaced the notion of absolute truth by that of assertibility, we must conclude that the principle of excluded middle no longer holds. We are not in a position to say of every proposition that either it or its negation are to be asserted. Existential philosophy leads thereby directly into intuitionistic logic. With respect to mathematical objects the existential philosopher and the intuitionist once again agree; they both regard them as temporal constructs.

What changed for Becker in 1927 were then not his views on intuitionistic constructivism but rather his philosophical justification for those views. Where he had previously supported them by appeal to Husserl's principle of transcendental idealism; he now appealed to Heidegger's hermeneutic and historical conception of human understanding. It is not difficult to see that he may have conceived of the latter as a reworking and extension of Husserl's view, as a historicizing of Husserl's transcendental position. In 1927 he writes accordingly in a critical tone about Husserl's position:

> Seen from the conception of a fully "historical" life experience, transcendental idealism (at least in its usual form) appears as an abstract modification of the original historical standpoint. In this transcendental idealism human life manifests itself only "in the faded form of a 'pure consciousness.'" (Becker 1927, 626)

Becker's move from Husserl to Heidegger should, however, prove as more than a change in philosophical foundations. Becker began to argue now that only from the historical-hermeneutic position could intuitionism be fully understood but that such an understanding would at the same time necessitate modifications in the intuitionistic view. Brouwer had tried to construct the numbers in temporal terms. But he had identified only a single characteristic of the experience of time, the fact that every current moment parts in ever repeated form into a past and future. Brouwer had spoken accordingly of the two-oneness of the moment as the fundamental phenomenon for intuitionism. But this aspect of time, Becker argued now, could only justify the construction of rule-governed series of numbers, such as the sequence of natural numbers, that is the construction of series of numbers according to a repetitive, rule-governed principle. Brouwer's thinking about time was, however, insufficient to make sense of free-choice sequences, that is, non-rule-governed, non-repetitive constructions which according to Brouwer's own view were needed to introduce the real numbers. Such sequences could only be conceived in terms of a notion of historical time, Becker argued now. For only historical Dasein could undergo a process of free becoming.

Human thought could, moreover, so Becker, pass in such a process of free becoming through a series of stages of reflections in which the totality of the previous stages becomes the point of a departure for new reflections. This series is potentially infinite. In thought we can, moreover, once again reflect on this series of possible stages of reflection as a whole and can thus initiate an entirely new and higher level form of reflection. To this process there exists, moreover, no inherent upper bound. Becker writes: "In the repeated iterations the uniformity of the concrete sequential stages of iteration become evident. This leads to the idea of envisaging the whole infinite possibility of iterations, in numerical terms, to speak of iterations of stage n." (545) And with this we bring "the finite mechanism to light... which so to say governs the transfinite structures in their peculiar movement and which allows our finite human consciousness to grasp them." (548) He hopes that in this way it might be possible, in contrast to Brouwer's expectations, to salvage large parts of Cantor's theory of transfinite numbers. He allows in any case that at least in the theory of pure transfinite ordinals "one can speak of an ontological foundation of the theory of transfinite numbers." (561)

I have reported these considerations not in order to endorse them, but in order to show how murky the philosophical discussion becomes once one raises the question what is to count as an admissible method of construction. In standard expositions of intuitionistic constructivism these difficulties are usually hidden from view because of insufficient attention to the philosophical details. It is a merit of Becker's account to have sought a systematic exposition and to have thereby exposed problems inherent in the constructivist project.

Becker's attempt to extend this method in order to reach a constructivist mathematics is motivated by his concern with the application of mathematics in scientific theorizing. He realizes that mathematics finds its fulfillment only in its use in the mathematical theories of natural science. But here, it turns out, we need and use parts of mathematics that cannot be justified by intuitive and constructive means. The natural world cannot be completely grasped with the tools of a constructive and interpreted mathematics. But given that we are essentially temporal beings and that our understanding proceeds always in temporal and historical terms, we must grant that in science we make use of parts of mathematics which we can no longer intuitively interpret. This kind of mathematics can only be treated as an un-interpreted formal calculus. We are confronted here, as Becker puts it, with the problem of the alienness and incomprehensibility of nature.

Precisely here mathematics helps us further and it does so precisely in its abstract and formalistic shape which depends no longer on intuition. In quantum mechanics it is the concept of Hilbert space of infinite dimensions which brings clarity into the matter just as the four-dimensional 'space- time-union'

Minkowski's played already a fundamental role in relativity theory. (Gr., 167*ff.*) Neither of these can be explained in terms of an intuitively grounded geometry. Becker finds himself forced back here on a distinction made already by Hilbert between an intuitively interpretable and a purely formal mathematics. It is a distinction which in a modified fashion had also already been appealed to by Husserl in his early Philosophy of Arithmetic.

Where Becker differed from both Hilbert and the early Husserl was in his view of what intuitively interpretable mathematics includes. It includes for him all of intuitionistic mathematics in the extended sense he had delineated. By contrast, interpreted mathematics meant for Hilbert only finitistic mathematics and for the early Husserl even more restrictively only a small fragment of finitary mathematics. In contrast to Hilbert, Becker assumed moreover no longer that the formal mathematics needed in natural science be shown to be consistent and could thus be justified by means of intuitively interpreted mathematics.

From this there arise for Becker philosophical consequences for Heidegger's historical-hermeneutic view. The historical-hermeneutic method claims to be able to understand everything. But the totalizing claim of this mode of thought is shown to fail "wherever nature confronts it." (Gr., 170) In the natural world we find ourselves confronted with phenomena which we can only describe with abstract formulas and which remain therefore hermeneutically impenetrable. It is precisely where hermeneutic thinking fails that the mathematical mode of thought leads further. From this it is evident that human beings are not merely historically "existing" beings, as Heidegger thought.

> Existential analysis is completely justified in its own domain which cannot be circumscribed from outside. But there exist at the same time other powers which are inseparably intertwined with existing Dasein. An 'understanding' of these powers is however impossible; they resist altogether the existential hermeneutic, phenomenological analysis. (DD, 92)

The boundary between these two domains runs through mathematics itself. Within the space of human experience the intuitionistic and constructivist conception of mathematics is undoubtedly phenomenologically correct. But the application of mathematics in natural science manifests a further moment which points to the limits of the constructivist conception. This does not mean that the mythological views of a Platonistic realism are after all justified, but only that in the interplay of history and nature neither the Platonistic nor the constructivist conception can finally triumph.

Looking back from these thoughts to what I learned from Becker's classes at the beginning of my academic career, I begin to see now also why he might have been so fascinated at this time by Heidegger's essay on the principle of reason. Like Heidegger he seems to have ended up with a view that leaves the world no longer entirely comprehensible to us. We can adequately describe it in our formulas but can no longer assign an intuitive meaning to our formulas. This sense of an aporia becomes perhaps most obvious whenever we try to sort out the paradoxes of quantum physics. The formulas fit, but every attempt to interpret them in ordinary words seems to lead to an impasse. Hence, Heidegger's thought finds an unexpected resonance in us. Being is, indeed, the abyss.

One may or may not find these considerations compelling. I have laid them out here in some detail to contrast them to a second line of thought in Becker's work, one particularly noticeable in his later writings, which proceeds more and more from the second of the two principles that Becker lays down as fundamental to existential philosophy. Whereas the line of thought explored so far proceeds most directly from the anti-Platonic assumption that the real is the temporal and which therefore seeks to construct both logic and mathematics in temporal terms, this new line of thought takes its departure from the idea that human understanding is inherently historical in character.

This assumption is already present in Becker's 1927 essay though its consequences are not fully explored till later on. In line with the historicist principle he writes in 1927 that the work of the mathematician itself must become a theme for phenomenological interpretation. We must consciously and philosophically recognize the obvious but often overlooked fact that mathematics is "a human science". He writes at the time:

> The contrast: intuitionism-formalism is rooted in the fundamental philosophical opposition between the anthropological and the "absolute" conception of knowledge (science) and finally of life itself (as the ultimate reality). (Becker 1927, 625)

If we take the anthropological view, we must interpret current work in mathematics as the historical outcome of a prolonged process of mathematical construction. How Becker means to apply this thought, is already apparent in his earliest contribution to the philosophy of mathematics, his 1923 essay on the foundations of geometry. The work contains an extensive critique of Hilbert's formal-axiomatic treatment of geometry. Becker grants that it is, of course, possible to operate with formal axioms without saying anything about what they might mean. But formal geometry presupposes historically and systematically an interpreted geometrical science which refers to our intuitive experience of space. Hilbert's work can only be understood as the endpoint in a process that began in the intuitive geometry of the Greeks, proceeded via the axiomatization of Euclidean geometry, and through the development of non-Euclidean geometries in the nineteenth century to the contemporary formalist view-point.

Hilbert would, of course, not deny that modern geometry has its historical origin in the intuitive geometry of the ancients, but he would consider this fact irrelevant to the determination of what geometry is today. We can call Becker's alternative view a genealogical one, since like Nietzsche's genealogical investigations it assumes that the philosophical understanding of some subject-matter involves a tracing of its historical genealogy.

According to Becker's genealogical story, we discover that though mathematics is initially a demonstrative and intuitive undertaking, formally analytic modes of mathematical thinking can already be found in antiquity. But they become dominant only in modernity and the strictly formalist view is only a product of the late nineteenth century. Becker asks now what kind of care and meaning is hidden in this mathematical formalism. And he concludes that it is the "care for the unlimited continuation of deductions." To put it differently: the business of deduction is to be secured without regard to content and factual problems which are at stake or, at least, might be at stake. (628*ff*.) To say it more neutrally: mathematical formalism is, for Becker, the product of a professionalization and specialization in the field of mathematics in which foundational philosophical problems and the problem of the uses of mathematics are increasingly bracketed out and in which therefore mathematics becomes increasingly more incapable of understanding itself philosophically. The result is that mathematics is now understood as a purely formal operating with un-interpreted symbols.

There is another genealogical line that Becker pursues in his historical reflection. It is the discovery that the modern opposition between a realist and a constructivist conception of mathematics goes back at least to the conflict between Plato and Aristotle. For Plato and the Pythagorean tradition that is linked to him, mathematical reality is autonomous and characterized by its own laws. On Aristotle's view, mathematical objects exist only as products of a process of abstraction. For Plato the infinite is something given, for Aristotle it is only something potential. For Plato the mathematical is above and outside time, the Aristotelian insight that mathematical thinking has to proceed by abstracting and idealizing introduces a human, subjective, and hence temporal element into mathematics. The two views characterize archetypal positions which return over and over again in the history of mathematics. "In contrast to a common view... one must put Plato together with Leibniz and Aristotle with Kant as far the philosophy of mathematics is concerned. Plato and Leibniz start evidently of as mathematical mystics. They are both 'Pythagoreans'; they both end up ascribing to mathematics a decisive role in the construction of the world. They both assume it to represent the metaphysical and ontological structure of the world... Aristotle and Kant are, by contrast, critics, sober opponents of all mythical reminiscences and excesses... They both strip mathematics of its mysterious character. The mathematician remains a finite human; any concept of the infinite that goes beyond the phenomenologically accessible is strictly rejected.” (*747ff.*)

According to Becker, Hilbert's attempt to rescue classical mathematics as a whole remains ultimately committed to the Platonic-Leibnizian form of thought whereas intuitionism is the natural descendent of Aristotelian and Kantian thinking. This is hidden from view only because Hilbert casts his Platonism in the form of a completely formalized mathematics.

What is important here for us is Becker's realization that both the constructivist and the non-constructivist view in mathematics have their own long genealogy within the history of mathematics. The history of mathematics shows us how these views have, over time, been constructed and reconstructed. There emerges thus from these reflections a new concept of construction, that is, the concept of the construction of mathematical theories in historical time. In this history both constructivist and non-constructivist forms of mathematics have been produced; they are both, so to say, possible constructions that have been carried out. In his late work Becker begins therefore to speak of mathematics more generally as a construction of possibilities. “The world of mathematics ties together the possible,” he writes. The realm of mathematics is that of “possible worlds.” (Gr., 66*ff.*) But Becker is in no way a modal realist. He does not believe that these possible worlds are real. They are rather to be thought of in terms of our original constructivist and anthropological conception as results of intuitive human constructions.

From the possibilities that mathematics invents the mathematical physicist selects fitting models for the characterization of the empirical world. We can clarify this idea by reference to non-Euclidean geometry. Mathematicians describe a multiplicity of possible spaces. But it would be absurd to assume that these all coexist in some hyper-space. Geometry is rather the construction of these possibilities. Natural science, in turn, builds with these “all kinds of models *in abstracto* of which it knows, however, that each of them can represent only some features of the observed phenomena.” (Gr. 66)

In this historical and modal conception of mathematics a new attitude towards the conflict between mathematical realism and mathematical constructivism becomes apparent. These two positions can only be considered as constructions of alternative mathematical possibilities. Where Becker had originally assigned pre-eminence to intuitionist constructivism, his new view allows for a neutral distance between the mathematical possibilities. The difference between them, he now allows, may concern the range of their proper applicability. Constructive mathematics may prove to be the appropriate model for the characterization of the space of human experience, whereas classical mathematics may prove indispensable for the scientific description of the world.

This view allows for the redemption of the constructivist view at a different and higher level. For it treats all mathematical structures as constructed, whether they are those espoused by mathematical realists or those put forward by traditional constructivists. But the method of construction to which this new view refers to is not that of piece-by-piece construction of the intuitionist constructivists. The question is now rather what kind of historical events generate mathematical structures and these may be of various sorts. They are certainly not step-by-step, number-by-number, free choice by free choice constructions. The problem with intuitionistic constructivism is now seen to be that it has operated all along with a purely abstract, dogmatic, and formalistic notion of temporal construction.

On Becker's ultimate view, or at least on the view I extract from his words, the nature of mathematical construction can only be understood by attending to the actual genealogy of mathematical thought. The history of mathematics becomes here the key to the philosophy of mathematics, the history of logic the key to the philosophical understanding of logic, and the history of analytic philosophy the key to a philosophical understanding of analytic philosophy. This is a lesson I have learned from Becker's work; one which I have tried to apply over the years in writing about the foundations of mathematics, of logic, and of analytic philosophy. This view does not mean to reduce philosophical inquiry to historical description, but sees an understanding of the historical process as indispensable to the philosophical interpretation of the meaning of the end results of that process.

This is probably not the moment to ask what general characterization we can give of mathematical construction, if we see it as a process in real time. A number of things are clear, though, very quickly. That process is not one that proceeds step by step. The same ground is covered again and again, concepts are clarified and made more precise as we go, distinctions are made which were formerly not seen, assumptions are withdrawn that were previously made. The construction of mathematical possibilities proceeds in some ways not unlike that of other human creations such as, for instance, literary fictions. There we invent figures by specifying in general some properties they are meant to have but leaving other necessary properties of real figures undetermined. We do so by laying down in general who Lady Macbeth is, not by identifying one particular fictional object. The same is true of our mathematical objects. We specify them by certain characteristics and leave others undetermined at least till a later moment. We lay down that the natural numbers are to form a progression but leave unspecified whether they are to be sets and if so, what sets they are. Mathematical objects have no determinate identity, they are rather figures in a design and that design is usually described only in general terms. Frege once wrote:

> The historical approach with its aim of detecting how things begin and of arriving from these origins at a knowledge of their nature, is certainly perfectly legitimate; but it has also its limitations. If everything were in continual flux, and nothing maintained itself for all time, there would no longer be any possibility of getting to know anything about the world and everything would be plunged in confusion... What is known as the history of concepts is really either a history of our knowledge of concepts or of the meaning of words. Often it is only after immense intellectual effort, which may have continued over centuries, that humanity at last succeeds in achieving knowledge of a concept in it pure form, in stripping off the irrelevant accretions which veil it from the eyes of the mind. (FA, VII)

I see it as Becker's merit to have shown how we can avoid such a mythological account without having to abandon large parts of our logical and mathematical thinking, that we can take a constructive view of mathematics without being forced into the procrustean bed of a mathematical constructivism. And this achievement, I believe, has been made possible precisely because throughout his work has held steadfastly on to the existential insights that we are thoroughly temporal and historical beings.

Oskar Becker died shortly after I began my studies and so I moved on to the University of Munich and then to Oxford where I learned that it was quite illegitimate to think about both logic and Heidegger. The gap between so-called analytic and so-called Continental philosophy is, however, not an inevitable one. We should remind ourselves first of all that Husserl initially studied mathematics and remained throughout his life concerned with the foundations of logic and mathematics, that Heidegger, too, had an early interest in logic and the philosophy of mathematics and that he contemplated initially to write his dissertation on the philosophy of mathematics, but then wrote it and his subsequent *Habilitationsschrift* instead on questions in philosophical logic, and that he reviewed the development of modern logic in a series of early essays which show him to be fully familiar with Frege's writings and with Russell and Whitehead's *Principia Mathematica*. It was only much later, after the publication of Being and Time and with his 1929 essay “What is Metaphysics?” that he turned critically against logic. In that essay he announced that the most important and ultimate concern of metaphysics was with nothingness and that of this nothing one could not even say that it exists but only that it nihilates, something that could not be grasped with the means of logic. Shortly afterwards, Rudolf Carnap was to ridicule these remarks as characteristic of metaphysical nonsense and with this the trenches were opened for the war between the “analytic” and the “non-analytic” movement in philosophy and those, like Becker, who refused to take sides in this confrontation were quickly shunted to the sidelines and then forgotten.

Recalling Oskar Becker today may be justified by the very different situation in which we now find ourselves. The confrontation between analytic and Continental philosophy has ceased to be an exciting battle; it has become instead now for the most part a long, dreary, uninformed trench warfare. As a result some philosophers have sought to think across these hardened lines. There is a sense that something is to be gained from such an exchange and it is precisely in this context that Becker becomes once again of interest.

REFERENCES

The abbreviations in the text refers to: Becker 1959 (B/H); Becker 1959 (Gr.); Becker 1963 (DD); Frege 1959 (FA).

Becker, Oskar. 1923. *Beiträge zur phänomenologischen Begründung der Geometrie und physikalischen Anwendung*. In *Jahrbuch für Phänomenologie und phänomenologische Forschung* 6: 385-560. Halle: Max Niemeyer.

Becker, Oskar. 1927. *Mathematische Existenz. Untersuchungen zur Logik und Ontologie mathematischer Phänomene*. In *Jahrbuch für Phänomenologie und phänomenologische Forschung* 8: 439-809. Halle: Max Niemeyer.

Becker, Oskar. 1930. "Die Philosophie Edmund Husserl's." In *Kantstudien* 35: 119-150. [Cited as „B/H".]

Becker, Oskar. 1959. *Größe und Grenze der mathematischen Denkweise*. Freiburg-Munich: Verlag Karl Alber. [Cited as „Gr".]

Becker, Oskar. 1963. *Dasein und Dawesen. Gesammelte philosophische Aufsätze*. Neske: Pfullingen. [Cited as „DD".]

Frege, Gottlob. 1959. *The Foundations of Arithmetic*. Translated by J. L. Austin. Oxford: Basil Blackwell. [Cited as „FA".]

van Dalen, Dirk. 1984. "Four letters from Edmund Husserl to Hermann Weyl." *Husserl Studies* 1: 1-12.

Weyl, Hermann. (1998a). "On the New Foundational Crisis in Mathematics". In *From Brouwer to Hilbert: The Debate on the Foundations of Mathematics in the 1920s*. Edited by Paolo Mancosu, 86-118. Oxford: Oxford University Press.

Weyl, Hermann. (1998b). "The Current Epistemological Crisis in Mathematics." In *From Brouwer to Hilbert: The Debate on* *the Foundations of Mathematics in the 1920s*. Edited by Paolo Mancosu, 123-142. Oxford: Oxford University Press.

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Weyl's Phenomenological Constructivism

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Abstract

Several scholars support the view that Weyl's investigations have undergone many changes along his life. Among them, there is no common agreement, but many authors set an early phase connected with Weyl's adherence to intuitionism and a later phase usually referred as Weyl's symbolic constructivism. The paper aims to show that previous interpretations are reasonable though they miss the phenomenological framework in which they can be better understood. I will focus on some Husserlian issues that I think were overlooked in the literature. Husserl's distinction between descriptive and exact concepts delineates the difference between a descriptive analysis of a field of inquiry and its exact determination. Clarifying how they are related is not easy. Nonetheless, the proposed difference between descriptive and exact sciences does not exclude the fact that they might coexist as two correlated investigations in the same field of inquiry once we were able to establish a connection by means of some idealizing procedure intuitively ascertained. A uniform interpretation of Weyl's investigations is proposed within Husserl's phenomenological framework, at least in the period 1917-1927.

Keywords: Weyl, Husserl, phenomenology, constructivism, descriptive, exact, limiting ideas, space, continuum

Hermann Weyl (1881-1955) was a leading mathematician at the beginning of the twentieth century. His major contributions concerned several fields of research, both in pure mathematics and theoretical physics, and, most importantly, his pioneering work was carried out in the light of his unique philosophical view. As only few mathematicians of his time, Weyl dealt with both scientific and philosophical issues with great skill, becoming a very unique figure among scientists and mathematicians of his time.[1] Although Weyl was a well-known mathematician, philosophers of mathematics have started getting interested in his work only recently, and, even though several authors have tried to uncover the philosophical framework that underlies Weyl's studies, many of them did not identify a coherent perspective in his philosophical view, arguing that his foundational research changed over the years.

Both Sieroka (2009) and Mancosu (2010) recognize at least two main tendencies in Weyl's work between 1917 and 1927: A first phase mainly characterized by his criticisms against set theory and classical analysis, and by his rejection of Hilbert's formalism and adherence to the intuitionistic-oriented account of Husserl and then Brouwer; and a second phase characterized by his tendency toward a sort of symbolic constructivism and his reconciliation with Hilbert's formalism. A similar interpretation is also supported by Da Silva (1997), Bell (2004), and Folina (2008), who identify a changeable perspective over the period 1917-1927, which went from an intuitionistic-oriented approach to a constructivist account.[2]

Not everyone agrees with the interpretation that supports a perspective changing over time. For instance, Scholz gives a more uniform interpretation of Weyl's research, and, in Scholz (2000), he suggests a constructive reading of Weyl's work since the publication of *Das Kontinuum*, arguing that Weyl was strongly influenced by Fichte's constructive philosophy. A similar constructive interpretation is also defended by Tieszen (2000), although he takes into account also the influence of Husserl's philosophy by suggesting that the philosophical framework of Weyl's mathematical constructivism should be understood in the light of transcendental idealism, which finds its roots in Kant, Fichte, and Husserl. Thus, Tieszen (2000) proposes a strong constructivist reading of Husserl's philosophy.

This paper aims to support a more uniform interpretation of Weyl's research in the period 1917-1927.[3] We will focus on three main works: *Das Kontinuum* (1918), *Raum-Zeit-Materie* (1921), and *Philosophy of Mathematics and Natural Science* (1949).[4] In certain respects, my interpretation will be close to Tieszen's reading, although it will also highlight some important Husserlian issues that in my opinion were previously overlooked.

1. The Mathematical Form of the Euclidean Space

Weyl's research on the nature of intuitive space constitutes an important body of work. The space of intuition pertains to our experience of spatiality and it should not be confused with any of its conceptualization. We "have to differentiate carefully between phenomenal knowledge or insight", and "theoretical construction" (Weyl 1949, 61): The first is conveyed by statements like "this leaf (given to me in a present act of perception) has this green color (given to me in a present act of perception)" (Weyl 1949, 61); on the other hand, the second is characterized by rational principles and it allows us to "'jump over its own shadow', to leave behind the stuff of the given, to represent the *transcendent*" (Weyl 1949, 66). Mathematics and physics allow us to achieve this sort of theoretical construction, and Weyl's mathematical formulation of *affine geometry* is an attempt in this direction. Indeed, he aims to develop a mathematical account of our intuitive space that is not "demanding the reduction of all truth to the intuitively given" (Weyl 1949, 65).

For any intuitively given field of inquiry, we should be able to first identify the *basic categories of objects* (*Grundkategorien*) and the *primitive relations* among these objects (*ursprünglichen Relationen*) that pertain to it.[5] A *primitive judgment scheme* (*ursprüngliche Urteilsschema*) is associated with each primitive relation, which "yields a meaningful proposition" only when each blank of the relation is filled by an object of its corresponding category (Weyl 1994, 41). In the first part of *Das Kontinuum* Weyl, deals with this subject matter and gives some examples. The proposition "this leaf is green", whose judgment scheme is "G(x): x is green", is meaningful (*sinnvoll*) because the blank x is affiliated with the category "visible thing" and it is filled by the object "leaf", which is indeed a visible thing (Weyl 1994, 5).[6] Weyl aims to avoid any mathematical account that makes use of judgment schemes that yield meaningless propositions. He remarks that "anyone who forgets that a proposition with such a structure can be meaningless is in danger of becoming trapped in absurdity" (Weyl 1994, 6).[7] For this reason, Weyl takes into consideration only "well-structured" primitive judgment schemes, from which further judgment schemes can be derived by applying certain principles of logical construction, without bringing again intuition into play. Weyl refers to these judgement schemes as *complex judgment schemes* and calls *derived relations* the associated relations. What sort of new judgment schemes "will unfold before our intuition in the development of the life of the mind can certainly not be anticipated *a priori*" (Weyl 1994, 113).[8] Despite this, the principles of logical construction "can be set down once and for all (just like the elementary forms of logical inference)" (Weyl 1994, 113).[9] Among these principles, Weyl identifies the judgments that express a state of affairs regarding the given field of inquiry: They are called *pertinent judgements* and they allow us to acquire a "complete knowledge of the objects of the basic categories as far as they are connected by the basic relations" (Weyl 1949, 7). Therefore, a meaningful mathematical analysis of an intuitively given field of inquiry starts with the identification of its basic categories and primitive relations. A mathematical theory can then be logically built on them, without bringing again intuition into play.

On this basis, Weyl develops the affine geometry and identifies two "fundamental categories of objects", namely *spatial-point category* and *translation category* (Weyl 1952, 18). Weyl also refers to them as the category of *points* and the category of *vectors*, respectively. Few primitive relations are found among these objects, i.e. the *axioms* concerning the operations of *addition* and *multiplication*, and the relationships between points and vectors. Weyl then points out that all concepts that may be defined, only by using logical reasoning from the basic notions of vector and point and their primitive relations "belong to affine geometry" (Weyl 1952, 18). For instance, it is possible to define the concept of a *straight line* and a *plane*:

- given a point O and a vector $\vec{e}$, the end-points of all vectors $\overrightarrow{OP}$ which have the form $\lambda\vec{e}$ constitute a *straight line*;

- given a point O, a vector $\overrightarrow{e_1}$, and a vector $\overrightarrow{e_2}$ which is not of the form $\lambda\overrightarrow{e_1}$, then the end-points of all vectors $\overrightarrow{OP}$ that have the form $\lambda_1\overrightarrow{e_1} + \lambda_2\overrightarrow{e_2}$ constitute a *plane*.

It is then possible to derive the totality of all possible formations concerning that field of inquiry from a few basic notions and relations. Moreover, all theorems that can be logically deduced within this framework constitute "the doctrine of affine geometry" (*Lehrgebäude der affinen Geometrie*) (Weyl 1952, 18). In this sense, geometry turns out to be a "*theory of space*" (Weyl 1949, 18).[10]

Furthermore, Weyl introduces the notion of *n-dimensional linear vector-manifold* (*n-dimensionale lineare Vektor-Mannigfaltigkeit*), which consists of all vectors of the form $\lambda_1\overrightarrow{e_1} + \cdots + \lambda_n\overrightarrow{e_n}$ (where $\overrightarrow{e_1}, \dots, \overrightarrow{e_n}$ are n linearly independent vectors, i.e. their linear combination only vanishes when all the coefficients vanish).[11] Affine geometry is obtained when n=3. He then formulates the last axiom of affine geometry, the *dimensional axiom*, which states that in affine geometry (3-dimensional linear manifold) there are three linearly independent vectors, but every 4 vectors, the vectors become linearly dependent on one another.[12]

However, this mathematical conceptualization is not unique: Any field of inquiry allows us to identify only certain categories of objects or primitive relations but their choice can be "arbitrary to a considerable extent" since they are not uniquely determined by the field of inquiry (Weyl 1949, 20). The difference between "essentially originary and essentially derived notions lies beyond the competence of the mathematician" (Weyl 1949, 20). The classical concept of space that concerns *Euclidean geometry* provides another possible conceptualization of the space of intuition. Specifically, Euclidean geometry is able to account for its homogeneity. In this case, we deal with three categories of objects, *spatial-point*, *line*, and *plane*, that are not defined but rather "assumed to be intuitively given" (Weyl 1949, 3). Few primitive relations are associated with these categories: *incidence*, *betweenness*, and *congruence*. Weyl also remarks that the category of points "reflects the intuitive homogeneity of space" (Weyl 1949, 8).

Indeed, any judgment scheme "P(x)" with blank x relating to this category and derived from the primitive judgement schemes without any reference to individual spatial-points, lines or plane "is always true either of *each* or of *no"* point (Weyl 1994, 16). For instance, the property "P(x): there exists a line such that the point x lies on it" is always true for any given point. On the other hand, the property "P(x): there exist three points y_1, y_2, y_3 lying on a line (y_2 being between y_1 and y_3) such that x is between y_1 and y_2 and it is also between y_2 and y_3" is always false. For this reason, Weyl refers to this category as a *homogeneous category*. Therefore, this mathematical conceptualization allows us to account for the intuitive homogeneity of space.

Although Weyl acknowledges the possibility of different conceptualizations of the space of intuition, the choice of which conceptualization to opt for is somehow limited; indeed, in some cases we should prefer one conceptualization over another. For instance, the axiomatic construction of affine geometry seems to be a better conceptualization of the space of intuition as it consists of "a system that, also in logical respect, is of a much more transparent and homogeneous structure than the purely geometrical axioms of Euclid or Hilbert" (Weyl 1949, 69). This theoretical construction reveals "a wonderful harmony between the given on one hand and reason on the other" (Weyl 1949, 69). Moreover, the derived concepts of straight line and plane "correspond to those which suggest themselves most naturally from the logical standpoint" (Weyl 1949, 69). For these reasons, Weyl claims that affine geometry best conceptualizes what is intuitively given.

To conclude, we will sum up the main features that characterize Weyl's studies. His research implies a distinction between two kinds of knowledge: The first concerns our sense perception, and Weyl refers to it as a phenomenal knowledge; the second seems to pertain to a domain of mathematical concepts, and Weyl refers to it as a sort of theoretical construction. Although being two different kinds of knowledge, Weyl seems to believe in the possibility of establishing a connection between them. He attempts to formulate a mathematical conceptualization of the space of intuition starting from few basic notions and relations that are intuitively grasped. However, our mathematical knowledge of the real world is not limited to this intuitive source of knowledge, but it is logically built on the basic notions and relations without bringing again intuition into play. This is how the mathematical knowledge of real world can represent the transcendent. Finally, Weyl suggests that different mathematical conceptualizations are possible, but deciding which approach to adopt is not a matter of choice, and one conceptualization might be preferred to another. This arises the problem of finding which mathematical conceptualization best suits what is intuitively given.

2. The Continuum

In *Raum-Zeit-Materie* Weyl remarks that his axiomatic formulation of affine geometry is still far from being satisfactory since it lacks a proper understanding of continuity. In *Das Kontinuum* Weyl does not deduce the notion of multiplication and the related laws from the principles of addition because the axioms of multiplication "cannot be derived in the general form from the axioms of addition by logical reasoning alone" (Weyl 1952, 17). The continuum "is so difficult to fix precisely, from the logical structure of geometry" (Weyl 1952, 17).[13] For this reason, Weyl deals with the nature of continuum in several works aiming to better understand the issue. In *Das Kontinuum*, for instance, Weyl explores the extent to which our mathematical theories of space and time reflect the intuitive content that we experience. Since we experience them as two continuous entities, our mathematical theories should reflect their continuous nature. Hence, understanding the nature of continuum turns out to be especially important for understanding the real world. It contributes "to critical epistemology's investigation into the relations between what is immediately (intuitively) given and the formal (mathematical) concepts through which we seek to construct the given in geometry and physics" (Weyl 1994, 2).

We shall now focus on the mathematical formulation of these continua as it is developed in *Das Kontinuum*. Weyl clarifies that the object of his investigation is the *phenomenal continuum*, be it spatial or temporal. By temporal continuum he means the constant form of our experiences of consciousness by virtue of which they appear to us to flow by successively. He further explains that by experience he does not mean "real psychical or even physical processes" which occur in an individual, "belong to a real world and, perhaps, correspond to the direct experiences". He means what we experience, exactly how we experience it (Weyl 1994, 88). Thus, the phenomenal time should be understood as a pure experience, it refers to the direct perception that we have of it, and it should not be confused with the time of physics or with any other notion of time derived from a certain view of the world.[14] Weyl aims to develop a mathematical theory of the phenomenal continuum. In order to do this, we need first to identify which kinds of basic categories and primitive relations belong to this field of inquiry. However, this is not easy and Weyl needs to postulate the possibility that a "now" is intuitively given in order to have "some hope of connecting phenomenal time with the world of mathematical concepts" (Weyl 1994, 88). By making this assumption, we are able to dissolve the phenomenal time into isolated time-points, rigidly punctual "now", and then, by identifying this sequence of time-points, we can grasp this species of time in an exact way. The time-points belong to a basic category – the *time-point category* – and the following primitive relations can be associated with them:

- the binary relation E_{arlier}(A,B): A is earlier than B;
- the quaternary relation E_{qual}(A,B,A',B'): A is earlier than B, A' is earlier than B', and AB is equal to A'B'.[15]

A mathematical theory of time could be logically built on the above-mentioned basic category and primitive relations, but first some issues must be solved. Indeed, these relations are not sufficient to conceptually differentiate every time-point in the given continuum. The phenomenal time is homogeneous and, as in the case of the homogeneity of the space of intuition, it can be shown that any judgment scheme (whose blank x is associated with the time-point category) that is derived from the primitive judgement schemes without any reference to individual time-points is always true either of each point or for none. Therefore, a single time-point "can only be given by being specified individually", i.e. by a direct intuition (Weyl 1952, 8). There is no intrinsic property that we can assign to a specific time-point in order to differentiate it from all the others.

According to Weyl, the issue could be solved by establishing an *isomorphism* between the domain of time-points and the domain of real numbers (as they are constructed in *Das Kontinuum*).[16] Each time-point will then be associated with a definite real number and vice versa. Specifically, we first need to fix two time-points, O and E, by means of a direct intuition such that $E_{arlier}(O,E)$ holds true. Then we can "fix conceptually further time-points P by referring them to the unit-distance OE" (i.e. the time span OE taken as unit) (Weyl 1952, 8). This is done by establishing a connection between a time-point P and the relation $R_t(P,O,E)$ that can be expressed in the form OP=t*OE. Our mathematical theory of time will have the same structure of real numbers, if this relation, logically derived from the primitive relations, reflects Weyl's construction of a real number. In this case, we could establish an isomorphism between the two domains, and we could associate a real number t with each time-point P. Moreover, Weyl speaks of *co-ordinate system* centered at O (with OE being the unit of length), where t represents the *abscissa* with respect to this co-ordinate system.[17] Weyl further points out that this conceptualization of phenomenal time relies on the individual exhibition of the time-point O. Only through this intuitive act we are able to differentiate time-points in the temporal continuum. Weyl claims that this fact is due to "the unavoidable residue of the eradication of the ego" in that theoretical construction of the world whose existence can only be given "as the intentional content of the processes of consciousness of a pure, sense-giving ego" (Weyl 1994, 94).[18]

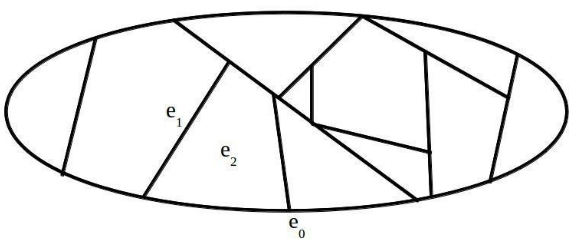
If we can indeed establish an isomorphism, we should be able to confirm it by direct inspection of phenomenal time. That is to say, our intuition should confirm "whether this correspondence between time-points and real numbers holds or not" (Weyl 1994, 90). However, our "intuition of time provides no answer" (Weyl 1994, 90). We face this situation because such interrogation is meaningless: Our mathematical theory of time, indeed, fails to satisfy a fundamental criterion of any theoretical construction, i.e. the time-point category "lacks the required support in intuition", no judgment scheme involving this category can be filled by time-points given by an individual intuition (Weyl 1994, 90). What is given in consciousness presents itself "not simply as a being" but "as an enduring and changing being-now" (Weyl 1994, 91). This being-now is "in its essence, something which, with its temporal position, slips away" (Weyl 1994, 92). For this reason, a mathematical theory of time that dissolves the phenomenal time into time-points turns out to be inadequate. This is due to the continuous nature of phenomenal time: A time-point "exists only as a 'point of transition' […] always only an *approximate*, never an *exact* determination is possible" (Weyl 1994, 92).[19]

Similar observations are also put forward in regard to the spatial continuum. In *Das Kontinuum*, Weyl deals with the phenomenal continuum of spatial extension and, by following his previous work *Die Idee der Riemannschen Fläche*, he attempts to conceptualize the continuous connectedness of the points on a two-dimensional surface. Since he needed to postulate that a "now" is intuitively given, he now needs to assume that an exact "here" can be fixed. However, the continuum does not consist of isolated individual points, and a fixed spatial-point "cannot be exhibited in any way", meaning that an exact determination is never possible (Weyl 1994, 92). Moreover, Weyl acknowledges that additional problems arise even if we accept this postulate. Indeed, we can regard a spatial surface as a "two-dimensional manifold" of surface-points, whose continuous connectedness can be grasped by means of the notion of *neighborhood* (Weyl 2009, 16). Given two surface-points P and Q, and a relation N that satisfies certain conditions, we say that Q lies in the n-neighborhood of P, if the relation N(P,Q;n) holds. This relation aims to represent the structural properties involved in the common notion of neighbourhood $|x - x_0| < n$, so that all ideas of continuity in a two-dimensional surface can be developed within this abstract scheme, free from intuitive knowledge. Although this approach offers many advantages, reducing the continuous connectedness to the concept of neighbourhood is not satisfactory. When a relation N(P;Q;n) establishes the n-neighbourhood of P, "much more occurs than is given by the continuous connectedness itself" (Weyl 1994, 106). In the case of the plane, for instance, "we could choose the interior of the circle of radius 1/n about a point as the nth neighbourhood of that point, but the circle of radius $1/2^n$ would serve just as well" (Weyl 1994, 107). We could also employ several other shapes in place of the circular ones (elliptical, square, etc.). No clear-cut answer "is yet at hand to the question of how we shall establish the link between the given and the mathematical in a perspicuous manner" (Weyl 1994, 107). Thus, dissolving the phenomenal continuum into isolated spatial-points turns out to be deeply unsatisfactory.[20]

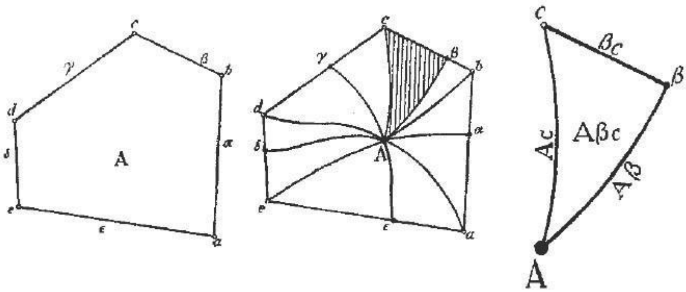
Weyl's studies on the nature of space and time are not pointless, on the contrary, they are of great importance for our understanding of the real world. The abstract schemata of our mathematical theories "must underlie the exact science of domains of objects in which continua play a role" (Weyl 1994, 108). Weyl indeed believes that a sort of "Logos" dwells within reality and we can try to reveal it as much as possible. Our mathematical theories are not a matter of choice just like "our inability to connect up the continuous with the schema of the whole numbers is not just a matter of personal preference" (Weyl 1994, 93, note 11). In this sense he claims that his construction of analysis "contains a *theory of the continuum* which must establish its own reasonableness (beyond its mere logical consistency) in the same way as a physical theory" (Weyl 1994, 93).

After the publication of *Das Kontinuum*, Weyl revises his mathematical approach to the continuum. His first approaches rely on the assumption that it is possible to exhibit a time-point or spatial-point in an individual intuition. However, this assumption violates the essence of continuum, which, by its very nature, cannot be shattered into a multitude of individual elements. The relation between parts and the whole, and not the relation between each element and the set of elements, should underlie the analysis of the continuum.[21] The continuum "falls under the notion of the 'extensive whole', which Husserl describes as what 'permits a dismemberment of such a kind that the pieces are by their nature of the same lowest species as is determined by the undivided whole'" (Weyl 1949, 52).[22] Weyl first tries to improve his approach to the continuum in *Über die neue Grundlagenkrise der Mathematik* published in 1921 (transl. Weyl 1998). In this paper, Weyl emphasizes the "the inner groundlessness of the foundations" upon which the current mathematics rests (Weyl 1998, 86). By following Brouwer's ideas, he then attempts a different approach to the concept of real number and continuum. At that time, Weyl was deeply impressed by the work of Brouwer and his foundational viewpoint: he states: "[…] Brouwer – that is the revolution!" (Weyl 1998, 99).[23] In the last pages of the *Über die neue Grundlagenkrise der Mathematik*, Weyl stresses the need for a different mathematical approach to the continuum of a two-dimensional manifold.

He first formulates the schema S concerning the *topological structure* of the manifold. It consists of finitely many *corners* e_0 (elements of level 0), *edges* e_1 (elements of level 1) and *surface pieces* e_2 (elements of level 2).



Few basic properties can be established: Each surface is limited by certain edges and each edge by certain corners. These properties represent the content of the schema S, i.e. the *topological framework* of the manifold. This schema "has to satisfy certain requirements, which can easily be stated" (Weyl 1998, 115). Weyl then outlines a *process of division* by dividing each edge into two edges by means of one of their points. Analogously, each surface piece is divided into triangles by using lines that start from a center, arbitrarily chosen within the surface piece, and that are connected to the corners of the surface piece.[24]



The figure is an example of how a surface piece, in this case a pentagon, is divided, and it shows the first step of the process of division from S to S'. We can easily identify the elements resulting from the process of division. For instance, the edge β is divided by setting an arbitrary point that leads to the generation of two new edges, namely βc and βb. In addition, an arbitrary point set within the surface piece A is used to divide the surface piece into triangles, obtaining the new surface piece Aβc. All other elements can be identified in a similar way and then properly named. Weyl then points out that we can identify a general pattern: Given the initial schema S, any symbol $e_2e_1e_0$ represents a surface piece e'_2 of the subdivided scheme S': through the iteration of this symbolic process, we obtain a sequence of derived schemes S,S',S'',S''', etc. so that what "we have done is nothing else than devise a systematic cataloguing of the parts created by consecutive subdivisions" (Weyl 2012, 76). The *sequence* ee'e'' and so forth pinpoints a point in the continuum; the sequence starts with a surface piece e of S and provides that the surface piece $e^{(n)}$ of the scheme $S^{(n)}$ is followed by a surface piece $e^{(n+1)}$ of $S^{(n+1)}$, leading to the further division of $e^{(n)}$. From the surface pieces of the initial topological framework, i.e. the schema S, we then reach the points of the manifold by iterating the process of division infinitely many times. This mathematical conceptualization is able to account for the essential feature of the continuum, which relies on the relation between part and whole, where "*every part of it can be further divided without limitation*" (Weyl 1998, 115). A point in a manifold must be seen as a *limiting idea* (*Grenzidee*): the concept of a point is indeed "the idea of the *limit* of a division extending *in infinitum*" (Weyl 1998, 115). Hence, Weyl believes that everyone feels "how truly the new analysis conforms to the intuitive character of the continuum" (Weyl 1998, 117).[25]

To conclude, Weyl's most recent studies on the nature of the continuum seem to be similar to his previous investigations. Indeed, they both underlie a distinction between two kinds of knowledge, one related to sense perception and one concerning the domain of mathematical concepts. Also, what is intuitively given seems to be the starting point of both approaches. Our mathematical understanding of the continuum should rely on intuitive insight, and a theoretical construction should be developed on the basis of basic notions and relations that are intuitively given. Moreover, our mathematical conceptualizations are not a matter of choice: Weyl seems to suggest that a sort of "Logos" dwells into reality and that these studies allow us to grasp the abstract schemata that underlie what is immediately given. However, the analysis of the continuum turns out to be more complicated as the assumption that it is possible to exhibit a time-point or a spatial-point in an individual intuition arises several problems. Hence, Weyl aims to improve his analysis of the continuum in later research, and, specifically his work in topology addresses the issue by regarding a point in a continuum as a limiting idea, i.e. the idea of the limit of a division extending *in infinitum*. According to Weyl, this approach is a more faithful analysis of the continuum. Thus, Weyl's research seems to be characterized by a constant search for the mathematical conceptualization that best suits what is intuitively given.

3. A Phenomenological Framework

Weyl's studies can be better understood within the philosophical framework of Husserl's phenomenology. Edmund Husserl (1859-1938) came to Göttingen as *extraordinarius* professor of philosophy in 1901. In 1904, Weyl moved to Göttingen to study mathematics and physics, and he received his doctorate in 1908 under Hilbert's supervision. Therefore, in the years 1904-1913 Husserl and Weyl worked at the same university. Historical records show that they knew each other, but Weyl's interest in phenomenology was actually sparked by his future wife Helene Joseph (1893-1948). Helen moved to Göttingen to become a student of Husserl in 1911, and, since then, her philosophical thinking was deeply influenced by phenomenology. In the years following the period in Göttingen, Weyl and his wife became friends with Husserl and his family and, when Husserl's youngest son fled Germany during Nazism, he was hosted for some time by the Weyls in Princeton.[26] Weyl sent a copy of *Das Kontinuum* and *Raum-Zeit-Materie* to Husserl, who in turn sent a copy of the second edition (revision of the sixth logical investigation) of *Logische Untersuchungen*. Four letters from Husserl to Weyl have been preserved, and they clearly provide evidence of the Weyl's close affiliation with phenomenology during the years 1917-1927.[27]

Weyl's studies that we have shown strongly suggest a Husserlian influence, since several Husserlian issues underlie Weyl's investigations, such as Weyl's distinction between phenomenal and conceptual knowledge and his theory of meaning. Moreover, Weyl makes explicit reference to Husserl's writing several times. As mentioned in the introduction, several works in recent literature have shown Husserl's influence on Weyl's scientific investigations. I will now focus on some Husserlian issues that I think were overlooked in the literature, and I will put forward a more uniform interpretation of Weyl's studies in the period 1917-1927.

In *Ideen I*, Husserl emphasizes a distinction between *descriptive sciences* and *exact sciences* and argues that, although they are both eidetic sciences, they are essentially different. Geometry is a good example of exact science as it is an *axiomatic science* that operates with *exact concepts*, which express *ideal essences*. Geometry derives every ideally possible spatial form starting with few basic concepts and by using few primitive axioms. However, all these "derived essences" are not usually intuited, which means that geometry does not grasp each essence directly but it derives them by mediate reasoning. For this reason, Husserl refers to exact science also as *explicative sciences*. Moreover, geometry "can be completely certain of dominating actually by its method all the possibilities and of determining them exactly" (Husserl 1982, 163). Husserl refers to this "fundamental logical property" in terms of *definite manifold* (Husserl 1982, 163).[28] A field of inquiry is articulated as a definite manifold if it is possible to derive all the possible formations concerning that field by starting from a few basic concepts and a given set of axioms. On the other hand, a descriptive science is purely descriptive and it operates with *inexact concepts*, which express *morphological essences*. A descriptive science investigates its field of inquiry though a direct seeing of essences. In this sense, we can refer to phenomenology as a descriptive science as its phenomenological descriptions are based on a direct seeing of essences.[29] Nonetheless, the proposed difference between descriptive and exact sciences does not exclude the fact that they might coexist as two correlated investigations in the same field of inquiry. A field of inquiry, for instance, might be articulated as a definite manifold. However, this fact is not a matter of choice and it "must be demonstrable in immediate intuition" (Husserl 1982, 165). One of the necessary conditions, for instance, has to be "the exactness in '*concept-formation*', which is by no means a matter of free choice and logical technique" (Husserl 1982, 165). The exactness of the basic concepts has to be grounded on the descriptive analysis of the field of inquiry itself, so that *their meaning is completely clarified* within this phenomenal domain. There must be some idealizing procedure, intuitively ascertained, that replace morphological essences with ideal essences. Husserl further points out that these ideal essences, grasped by such an idealization, have to be considered as a sort of "limit", that is *limiting ideas* (*Grenzideen*) in the Kantian sense. In this way, it might be possible to regard a field of inquiry as a definite manifold.[30] An important case is represented by the relationship between intuitive space and geometry. The former is extensively described by Husserl's eidetic investigations on our spatial experience: these phenomenological descriptions constitutes a *descriptive material eidetic science of space*. On the other hand, the latter is an eidetic science dealing with all possible *spatial forms* by means of exact concepts, that is an *exact material eidetic science of space*. Clarifying all connections between these two sciences is not an easy task. In *Ideen I*, Husserl acknowledges that further investigations are needed for "a clarification of the so-little understood relationship between 'descriptive' and 'explanatory' science" (Husserl 1982, 165). This field of phenomenological research belongs to a more general issue concerning the complex relationship between phenomenology and ontology. Although shedding light upon Husserl's complex view of this issue goes beyond the scope of the present paper[31], we would like to point out that a connection between a descriptive analysis of a field of inquiry and its exact determination can be established via an idealizing procedure intuitively ascertained. Such a connection is important if we want to provide an exact determination of that very field of inquiry, or we can say, of that *regional ontology*. We should interpret Weyl's investigations within Husserl's phenomenological framework.

Weyl's research on the nature of intuitive space aims to uncover the structure of space that underlies the domains of objects immediately given in our experience of space. Whereas "in examining a real object we have to rely continually on our sense perception in order to bring to light ever new features, capable of *description in concepts of vague extent only*", the structure of space "can be exhaustively characterized with the help of a few *exact concepts* and in a few statements, the *axioms*, in such a manner that all geometrical concepts can be defined in terms of those basic concepts and every true geometrical statement follows as a logical consequence from the axioms" (Weyl 1949, 3, my emphasis). Once intuition has "furnished us with the necessary basis", we shall "enter into *the region of deductive mathematics*" (Weyl 1952, 16, my emphasis). In this sense geometry turns out to be a "*theory of space*" (Weyl 1949, 18, my emphasis). Moreover, "the scientific theory in question is said to be *definite* (*definit*) according to Husserl" (Weyl 1949, 18).[32] Weyl's preference for the axiomatic construction of affine geometry over Euclid and Hilbert's approach can be also better understood within Husserl's phenomenological framework. Indeed, only the former theoretical construction takes into account the idealizing procedure involved in the constitution of the ideal essences of line and plane.[33] Moreover, the meanings of the exact concepts that express these ideal essences are better clarified within the phenomenal domain of intuitive space. Affine geometry, therefore, reveals "a wonderful harmony between the given on one hand and reason on the other" because it reflects the descriptive analysis of this field of inquiry more accurately (Weyl 1949, 69).

Similarly, Weyl's mathematical conceptualizations of the continuum find their roots in Husserl's phenomenological framework. In *Das Kontinuum*, Weyl tries to establish a connection between something given in the "*morphological description* of what presents itself in intuition" and "something constructed in a logical conceptual way" (Weyl 1994, 49, my emphasis). Nevertheless, any idealizing procedure can be intuitively ascertained as regards the constitution of the category of point. His research in topology improves this approach by developing a theoretical construction that takes into account the idealizing procedure involved in the constitution of the ideal essence of a point. This ideal essence is then expressed by an exact concept, whose meaning can be clarified within the phenomenal domain of intuitive continuum. Weyl further claims that, in order to improve this approach, the process of division itself should not be regarded as given in an exact way. In fact, we should assume that the divisions are given only vaguely and are not accurately done since an exact division would contradict the essence of the continuum. However, as the division progresses, the accuracy will increase indefinitely.[34] Topological studies allow us to exactly address these problems "even though the continua to which they are addressed may not be given exactly but only vaguely, as is always the case in reality" (Weyl 1949, 90). These studies provide an intermediate level of analysis since a rational analysis of continua "proceeds in three steps: (1) *morphology*, which operates with *vaguely circumscribed types of forms*; (2) *topology*, which, *guided by conspicuous singularities* or even in free construction, places into the manifold a vaguely localized but combinatorially exactly determined skeleton; and (3) *geometry* proper, whose *ideal structures* could only be carried with exactness into a real continuum after this has been spun over with a subdivision net of a fineness increasing *ad* *infinitum*" (Weyl 1949, 91, my emphasis). Husserl's influence on these topological studies is highlighted by Weyl's reference to O. Becker (1889-1964), who wrote a *Habilitationsschrift* in 1922 on the phenomenological foundations of geometry and relativity theory under Husserl's direction. For a "more careful phenomenological analysis of the *contrast between vagueness and exactness and of the limit concept*, the reader may be referred to the work by O. Becker", his *Beiträge zur phänommenologischen Begründung der Geometrie und ihrer physikalischen Anwendungen* (Weyl 1949, 91, my emphasis; cf. Becker 1923). Becker indeed further develops this analysis by improving especially the foundational aspects involved in the connection between a descriptive analysis of a field of inquiry and its exact determination.[35]

4. Conclusion

Weyl's research turns out to be an attempt to establish a connection between a descriptive analysis of phenomena and their exact determination within Husserl's phenomenological framework we have outlined. He tries to untangle this connection in different ways, and, for this reason, we should not come to the conclusion that Weyl keeps changing perspective in his studies between 1917 and 1927. Instead, his theories should be read as different attempts to attain a theoretical construction that is as much phenomenologically grounded as possible. In this sense, we can refer to Weyl's *phenomenological constructivism*. I would like to specify that I am not arguing that Weyl's studies can be defined exactly as phenomenological research. Weyl himself acknowledges that he broaches only "lightly on the philosophical implications" since he is not "in a position to give such answers to the epistemological questions involved" as his conscience would allow him to uphold (Weyl 1952, 2). In *Das Kontinuum*, for instance, he remarks that his research on the continuum is "only a slightly illuminating surrogate for a genuine philosophy of the continuum" since his task "is mathematical rather than epistemological" (Weyl 1994, 97). He also admits that for him it is "very difficult to give a precise analysis of the relevant mental acts" (Weyl 1995, 454).

Therefore, my interpretation suggests that Weyl's studies are developed by taking into account Husserl's phenomenological framework though they should be further clarified through an in-depth phenomenological analysis.

These considerations could be extended beyond the few examples we have shown. In particular, Weyl's development of *infinitesimal geometry* should be understood within this framework. In *Raum-Zeit-Materie*, Weyl addresses the rising theory of general relativity and aims to develop a theoretical construction of the real world whose meaning is phenomenologically clarified within the domain of our experience.[36] To conclude, here are the philosophical reasons that underlie Weyl's famous remark in *Raum-Zeit-Materie*:

> The investigations about space that have been conducted in chapter II appear to me to offer a good example of the essential analysis (*Wesenanalyse*) striven for by phenomenological philosophy (Husserl), an example that is typical for such cases where a non-immanent essence is dealt with (Weyl 1921a, 133, my translation).

NOTES

[1] For a general introduction to Weyl's scientific and philosophical work see Scholz (2001) and Bell and Korté (2016).

[2] These studies, of course, shed light on many further details. For instance, in Da Silva (1997), Weyl's predictivism is clarified by reference to Husserl's theory of meaning proposed in *Logische Untersuchungen*.

[3] In Weyl's obituary, appeared in the *Biographical Memoirs of Fellows of the Royal Society* in 1957, the years between 1917 and 1927 are described as the period when Weyl "was at the height of his powers" (Newman 1957, 306). It was a rich and stimulating period for Weyl's mathematical and philosophical production and a substantial body of work was published at that time. For this reason, the decade 1917-1927 is an important period to focus on.

[4] *Philosophy of Mathematics and Natural Science* (1949) is the revised English version of the first German edition published in 1927 (Weyl 1927). The text was translated by O. Helmer with the help of J. Weyl, and it was reviewed by Hermann Weyl himself. There are no significant changes from the first edition, except for six essays that Weyl added. For these reasons, in most cases I will refer to the English edition, while I will quote directly from the German edition when needed.

[5] We are considering properties among relations as a special case.

[6] On the contrary, a proposition is meaningless (*sinnlos*) when this condition is not satisfied. For instance, the judgment scheme "H(x): x is honest" does not yield a meaningful proposition if x is filled by the object "leaf". Weyl's theory of meaning is thoroughly described in Tieszen (2000).

[7] Actually Weyl seems to believe that a proper intuitive analysis of the given field of inquiry would prevent us from being trapped in such absurdities. He says: "Perhaps meaningless propositions can appear only in thought about language, never in thought about things" (Weyl 1994, 5).
[8] This quotation is taken from an article published in 1919 (Weyl 1919). It has been translated in English and added as an appendix in Weyl (1994).
[9] Weyl's principles of logical construction belong to a more comprehensive "logical critique of language" (Weyl 1949, 7). Specifically, he speaks of *pure grammar* when referring to Husserl's *Logische Untersuchungen*. See Weyl (1994, 113, note 2). He makes reference to Husserl's philosophy of logic also in the preface of *Das Kontinuum* when he says: "Concerning the epistemological side of logic, I agree with the conceptions which underlie Husserl's *Logische Untersuchungen*. The reader should also consult the deepened presentation in Husserl's *Ideen* which places the logical within the framework of a comprehensive philosophy" (Weyl 1994, 2).
[10] A similar remark can be found in Weyl's *infinitesimal geometry*. This more general approach improves our analysis of space to such an extent that Weyl refers to it as "the climax of a wonderful sequence of logically-connected ideas, and in which the result of these ideas has found its ultimate shape, is a true *geometry*, a doctrine of *space itself*" (Weyl 1952, 102).
[11] For a detailed account of the notion of manifold (*Mannigfaltigkeit*) from the mid-nineteenth century to the mid-twentieth century, see Scholz (1999). The historical development of this concept is complex and it is not always easy to recognize the meaning assigned by each author. For a better understanding of Weyl's notion of manifold, however, we can notice what he says with regard to the notion of *surface* in *Die Idee der Riemannschen Fläche*: "[…] the concept 'two-dimensional manifold' or 'surface' will not be associated with points in three-dimensional space; rather it will be a much more general abstract idea. If any set of objects (which will play the role of points) is given and a continuous coherence between them, similar to that in the plane, is defined, then we shall speak of a two-dimensional manifold" (Weyl 2009, 16). Therefore, Weyl's notion of manifold can be broadly understood as the "abstract form" of a given field of inquiry.
[12] He further adds that a point O and three linearly independent vectors constitute a *coordinate system*. This system allows us to identify a point by its *coordinates* $\lambda_1, \lambda_2, \lambda_3$ by means of the relation $\overrightarrow{OP} = \lambda_1\overrightarrow{e_1} + \lambda_2\overrightarrow{e_2} + \lambda_3\overrightarrow{e_3}$.
[13] In *Das Kontinuuum* Weyl remarks that: "[…] the *continuity* given to us immediately by intuition (in the flow of time and in motion) has yet to be grasped mathematically as a totality of discrete "stages" in accordance with that part of its content which can be conceptualized in an "exact" way. More or less arbitrarily axiomatized systems (be they ever so "elegant" and "fruitful") cannot further help us here" (Weyl 1994, 24).
[14] Weyl follows implicitly Husserl's approach. Husserl argues that a preliminary act is needed in the analysis of experience. He refers to it as *epoché* and it is conceived as the suspension of judgment about the natural world, setting aside all objective theses and focusing on the phenomenon as it presents itself. See Husserl (1913b).

[15] This equality refers to the equality of experiential content of the two *time spans* AB and A'B', into which falls every time-point that is later than A(A'), but earlier than B(B'). Weyl actually remarks that such an equality might be very controversial, however, he chooses to not delve into it. As we will see, the previous postulate will turn out to be an even bigger issue.

[16] Weyl's construction of real numbers in *Das Kontinuum* is logically built on the basic category of natural numbers and the primitive relation "S(n',n): n' is the successor of n". He develops this construction in detail and introduces many other notions, such as the notions of *set* and *function*. For further details, see Mancosu (2010). For an axiomatic interpretation, see Feferman (1988).

[17] The idea of isomorphism then turns out to be "of fundamental importance for epistemology" (Weyl 1949, 25). Weyl also refers to *transfer principle* (*Übertragungsprinzips*). By adopting an isomorphic mapping between two domains "is possible to transfer any insights gained in one field to the isomorphic field" (Weyl 1949, 26). Similar considerations are also supported in the case of a mathematical theory of space. With regard to space Weyl states: "[…] for example, Descartes' construction of coordinates maps the space isomorphically into the operational domain of linear algebra" (Weyl 1949, 25). Weyl further claims that a mathematical theory of time or space cannot be pursued as an independent axiomatic science but it should rely on this transfer principle. We should transfer any result pertaining to analysis into the domain of time-points by means of "a transfer principle based on the introduction of a coordinate system" (Weyl 1994, 96). Weyl finally remarks that the notion of isomorphism "induce us to conceive of an axiom system as a *logical mold* (*Leerform*) of possible sciences" (Weyl 1949, 25). A concrete interpretation is given "when designata have been exhibited for the names of the basic concepts, on the basis of which the axioms become true propositions" (Weyl 1949, 25). "Pure mathematics, in the modern view, amounts to a general hypothetico-deductive theory of relations; it develops the theory of logical 'mold' without binding itself to one or the other among the possible concrete interpretations" (Weyl 1949, 27). In line with Husserl, Weyl points out that the notion of *formalization* reflects a point of view "without which an understanding of mathematical methods is out of the question" and suggests the reader to "compare Husserl, *Logische Untersuchungen*, I, Section 67-72" (Weyl 1949, 27).

[18] Weyl inherits this conception of the real world from Husserl. He explicitly makes reference to Husserl's *Ideen* when he claims: "[…] the real world, and every one of its constituents with their accompanying characteristics, are, and can only be given as, intentional objects of acts of consciousness" (Weyl 1952, 4).

[19] Weyl recommends reading Husserl's phenomenological description of time (Husserl 1913b, § 81,82) for further details. He makes also reference to Bergson's philosophy (Bergson 1907).

[20] Weyl's studies are often characterized by a continuous tension between a temporary solution and a call for a better solution. For this reason, these considerations are not in conflict with previous mathematical conceptualizations of space. In this case, Weyl is showing us the underlying

problems concerning a finer analysis of a mathematical theory of time or space.

[21] "[…] sie dadurch gegen das Wesen des Kontinuums verstößt, als welches seiner Natur nach gar nicht in eine Menge einzelner Elemente zerschlagen werden kann. Nicht das Verhältnis von Element zur Menge, sondern dasjenige des Teiles zum Ganzen sollte der Analyse des Kontinuums zugrunde gelegt warden" (Weyl 1988, 5).

[22] Weyl is referring to Husserl's *Logische Untersuchungen*. See Husserl (1973, vol II, 29).

[23] Note that he did not always agree with Brouwer and that he actually put forward his own foundational account. For a comparison between them, see van Atten *et al.* (2002).

[24] The picture and the following remarks are taken from another paper published in 1940 and titled *The Mathematical Way of Thinking* (Weyl 1940). In that paper, this account is better explained.

[25] Weyl outlines how we can develop a mathematical analysis of this manifold in accordance with his previous foundational remarks. However, he was aware that several issues should have been addressed, and his research on *combinatorial topology* aims to further develop this approach. He published two important contributions in that direction in 1923 and 1924. See Weyl (1923, 1924) and Scholz (2000).

[26] See Weyl (1948, 381). For further details about the personal contacts between Weyl and Husserl, see Ryckman (2005, § 5).

[27] The correspondence is published in Schuhmann (1996) and in van Dalen (1984). Few excerpts are translated and discussed in Ryckman (2005, § 5). See also Tonietti (1988). For a French translation that also includes a noteworthy letter from Weyl to Husserl, see Lobo (2009).

[28] Husserl's notion of *definiteness* (*Definitheit*) has been a matter of debate, especially in relation to the modern notion of *completeness*. A number of different interpretations of this notion have been proposed in the literature. See, for instance, Ortiz Hill (1995), Majer (1997), Da Silva (2000) and Centrone (2010). For a detailed account of the various notions of completeness that were theorized in connection with the development of the axiomatic method in the late nineteenth and early twentieth century mathematics, see Awodey and Reck (2002).

[29] Husserl points out, however, that phenomenology is not an inadequate science because it is not an exact science. Our prejudices on the well-known exact sciences, such as geometry, should not make us fail to recognize that "transcendental phenomenology, as a descriptive science of essence, belongs however to a *fundamental class of eidetic sciences totally different* from the one to which the mathematical sciences belong" (Husserl 1982, 169).

[30] Husserl says: "In the eidetic province of reduced phenomena (either as a whole or in some partial province) […] the pressing question of whether, besides the descriptive procedure, one might not follow - as a counterpart to descriptive phenomenology - an idealizing procedure which substitutes pure and strict ideals for intuited data and might even serve as the fundamental means for a mathesis of mental processes" (Husserl 1982, 169).

[31] For further details, see Husserl (1982, § 72-75) and Husserl (1980, § 13-17). In later years, Husserl revises his analysis of idealization as part of his historical reflection on the origins of philosophical and scientific thought. Important remarks concerning the origin of geometry can be found in his *Krisis*. See Husserl (1970, 353).

[32] For a better understanding of Weyl's notion of definiteness note that Weyl distinguishes it from the notion of *completeness* (*Vollständigkeit*). He claims that, in a complete system of axioms, for every pertinent general proposition a the question 'does a or ~a hold?' could be answered by using logical inference on the basis of the axioms, however, "mathematics would thereby be trivialized" (Weyl 1949, 24). Intuition and "the life of the scientific mind pose the problem, and these cannot be solved by mechanical rules like computing exercise" (Weyl 1949, 24). Cf. Centrone (2010, § 3.6.2) for a comparison between Husserl's notion of *Definitheit* and Hilbert's notion of *Vollständigkeit*.

[33] Note that affine geometry does not take into account any idealizing procedure regarding the basic categories of objects. Line and plane are indeed "derived exact concepts". However, the connection between a descriptive analysis of a field of inquiry and its exact determination should be established, wherever appropriate.

[34] "In der Wirklichkeit muß man sich vorstellen, daß die Teilung auf der 0^{ten} Stufe Σ_0 [on S] nur vage, mit einer beschränkten Genauigkeit gegeben ist; denn eine exakte Teilung widerspricht dem Wesen des Kontinuums. Aber bei fortschreitender Teilung soll sich auch die Genauigkeit, mit der die anfänglichen Ecken und Seiten und die auf den vorhergehenden Stufen neu eingeführten festgelegt sind, unbegrenzt steigern" (Weyl 1988, 8).

[35] In a letter to Weyl dated April 9, 1922, Husserl wrote: "Dr Becker also found it necessary in the first part of his work to enter into the general fundamental questions concerning the theorization of vague experiential data, with its vague continuity, and to sketch a constitutive theory of the continuum" (Mancosu and Ryckman 2010a, 282). For further details, consult the correspondence between Weyl and Becker, which is discussed in Mancosu and Ryckman (2010a, 2010b). See also Lobo (2009).

[36] In a letter to Weyl dated April 12, 1923, Becker remarks that Weyl's work on general relativity has for the first time "made possible a complete phenomenological foundation for geometry (in the sense of 'world geometry')". He further adds that "the same idealistic conception" underlies both Weyl's theory of continuum and his infinitesimal geometry. See Mancosu and Ryckman (2010b, 309).

REFERENCES

Awodey, Steve and Erich H. Reck. 2002. "Completeness and Categoricity. Part. I: Nineteenth-Century Axiomatics to Twentieth-Century Metalogic." *Hist Philos Logic* 23: 1-30.

Becker, Oskar. 1923. "Beiträge zur Phänomenologischen Begründung der Geometrie und Ihrer Physikalischen Anwendung." *Jahrbuch für Philosophie und Phänomenologische Forschung* 6: 385-560.

Bell, John L. and Herbert Kortè. 2016. "Hermann Weyl." In *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2016/entries/weyl/

Bell, John L. 2004. "Hermann Weyl's Later Philosophical Views: his Divergence from Husserl." In *Husserl and the Sciences*, edited by R. Feist, 173-185. Ottawa: Ottawa Univesity Press.

Bergson, Henri-Louis. 1907. *L'Évolution Créatrice.* Paris: Alcan.

Centrone, Stefania. 2010. *Logic and Philosophy of Mathematics in the Early Husserl.* Dordrecht: Springer.

Da Silva, Jairo Josè. 1997. "Husserl's Phenomenology and Weyl's Predicativism." *Synthese* 110: 277-296.

________. 2000. "Husserl's Two Notions of Completeness. Husserl and Hilbert on Completeness and Imaginary Elements in Mathematics." *Synthese* 125: 417-438.

Feferman, Solomon. 1988. "Weyl Vindicated: 'Das Kontinuum' 70 Years Later." In *Atti del Congresso, Temi e Prospettive della Logica e della Filosofia della Scienza Contemporanee, Cesena, 7–10 gennaio 1987*, vol I, 59-93. Bologna: CLUEB.

Folina, Janet. 2008. "Intuition between the Analytic-Continental Divide: Hermann Weyl's Philosophy of the Continuum." *Philosophia Mathematica* 16: 25-55.

Husserl, Edmund. 1982. *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy: First Book.* Translated by F. Kersten. Dordrecht: Kluwer.

________. 1913a. *Logische Untersuchungen.* 2nd. Halle: Neimeyer.

________. 1913b. "Ideen zu Einer Reinen Phänomenologie und Phänomenologischen Philosophie." *Jahrbuch für Philosophie und Phänomenologische Forschung* 1: 1-323.

________. 1973. *Logical Investigations*. Translated by J.N. Findlay (2nd ed.). Vol. I and II. London: Routledge and Kegan Paul.

________. 1970. *The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy*. Translated by D. Carr. Evanston, Ill: Northwestern University Press.

Lobo, Carlos. 2009. "Mathématicien Philosophe ou Philosophe Mathématicien." *Annales de Phénoménologie* 8: 205-252.

Majer, U. 1997. "Husserl and Hilbert on Completeness: a Neglected Chapter in Early Twentieth Century Foundation of Mathematics." *Synthese* 110, Issue 1: 35-56.

Mancosu, Paolo. 1998. *From Brouwer to Hilbert : the Debate on the Foundation of Mathematics in the 1920s*. New York: Oxford University Press.

________. 2010. "Hermann Weyl: Predicativity and an Intuitionics Excursion." In *The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic, 1900-1940*, edited by P. Mancosu, 259-276. New York: Oxford University Press.

Mancosu, Paolo, and Thomas Ryckman. 2010a. "Mathematics and Phenomenology: The Correspondence Between O. Becker and H. Weyl." In *The Adventure of Reason : Interplay between Philosophy of Mathematics and Mathematical Logic, 1900-1940*, edited by P. Mancosu, 277-307. New York: Oxford University Press.

Mancosu, Paolo, and Thomas Ryckman. 2010b. "Geometry, Physics, and Phenomenology: Four Letters of O. Becker to H. Weyl." In *The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic, 1900-1940*, edited by P. Mancosu, 308-345. New York: Oxford.

Newman, Maxwell H. A. 1957. "Hermann Weyl , 1885-1955." *Biographical memoirs of Fellow of The Royal Society* 3: 305-328.

Ortix Hill, Claire. 1995. "Husserl and Hilbert on Compleneteness." In *From Dedekind to Gödel. Essay in the Development of the Foundations of Mathematics*, edited by J. Hintikka, 143-163. Dordrecht: Kluwer.

Ryckman, Thomas. 2005. *The Reign of Relativity: Philosophy in Physics 1915-1925*. Oxford: Oxford University Press.

Scholz, Erhard. 1999. "The Concept of Manifold, 1850-1950." In *History of Topology*, edited by I.M. James, 25-65. Amsterdam: Elsevier.

Scholz, Erhard. 2000. "Hermann Weyl on the Concept of Continuum." In *Proof Theory: History and Philosophical Significance*, edited by V. Hendricks et al., 195-217. Dordrecht: Kluwer.

Scholz, Erhard (ed.). 2001. *Hermann Weyl's Raum-Zeit-Materie and a General Introduction to His Scientific Work*. Basel: Birkhauser.

Schuhmann, Karl & E. Schuhmann (eds.). 1996. *Edmund Husserl Briefwechsel, Bd. 7*. In *Husserliana: Edmund Husserl Dokumente*, Bd. III/1-10. Den Haag: Kluwer Academic.

Sieroka, Norman. 2009. "Husserlian and Fichtean Leanings: Weyl on Logicism, Intuitionism, and Formalism." *Philosophia Scientiae* 13: 85-96.

Tieszen, Richard. 2000. "The Philosophical Background of Weyl's Mathematical Constructivism." *Philosophia Mathematica* 8: 274-301.

Tonietti, Tito. 1988. "Four Letters of E. Husserl to Hermann Weyl and their Context." In *Exact Sciences and Their Philosophical Foundations*, edited by W. Deppert, 343-384. Frankfurt am Main: Lang.

Van Dalen, Dirk. 1984. "Four Letters from Edmund Husserl to Hermann Weyl." *Husserl Studies* 1: 1-12.

Weyl, Hermann. 1913. *Die Idee der Riemannschen Fläche*. Leipzig: Teubner.

________. 1918. *Das Kontinuum*. Leipzig: Veit & Co.

________. 1919. "Der Circulus Vitiosus in der Heutigen Begründung der Analysis." *Jahresbericht der Deutschen Mathematikervereinigung* 28: 85-92. [English translation in Weyl (1994)].

________. 1921a. *Raum-Zeit-Materie*. 4th ed. Berlin: Springer.

________. 1921b. "Über die Neue Grundlagenkrise der Mathematik." *Mathematische Zeitschrift* 10: 37-79.

________. 1923. "Análisis Situs Combinatorio." *Revista Matematica Hispano-Americana* 5: 43-68.

________. 1924. "Análisis Situs Combinatorio (continuación)." *Revista Matematica Hispano-Americana* 6: 1-9 and 33-41.

________. 1927. *Philosophie der Mathematik und Naturwissenschaft*. Munchen: Oldenbourg.

________. 1940. "The Mathematical Way of Thinking." *Science* 92: 437-446. [Garbled text, improved version reprinted in Weyl (2012, 67).]

________. 1949. *Philosophy of Mathematics and Natural Science*. Translated by O. Helmer. Princeton: Princeton University Press.

________. 1952. *Space-Time-Matter*. Translated by H. L. Brose 4th ed. Mineola, NY: Dover.

________. 1988. *Riemanns Geometrische Ideen, ihre Auswirkungen und ihre Verknüpfung mit der Gruppentheorie*. Berlin: Springer-Verlag.

.

________. 1994. *The Continuum*. Translated by S. Pollard and T. Bole. New York: Dover.

________. 1995. "Topology and Abstract Algebra as Two Roads of Mathematical Comprehension." Translated by A. Shenitzer. *American Mathematical Monthly* 102: 453-460 and 646-651.

________. 1996. "In memorian Helene Weyl (1948)." In *Arnold Zweig, Beatrice Zweig, Helene Weyl: Komm her, Wir lieben Dich. Briefe einer ungewohnlichen Freundschaft*, edited by I. Lange. Berlin: Aufhan-Verlag.

________. 1998. "On the New Foundational Crisis of Mathematics (1921)." In *From Brouwer to Hilbert: the Debate on the Foundation of Mathematics in the 1920s*, edited by P. Mancosu, translated by B. Müller, 86-118. New York: Oxford University Press.

________. 2009. *The Concept of a Riemann Surface*. Translated by G. E. Maclane (3rd ed.). Mineola, NY: Dover.

________. 2012. *Levels of Infinity*. Translated by P. Pesic. Mineola, NY: Dover.

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Husserl and Bourbaki on Mathematics: Similarities, Possible Influence and some Dissimilarities

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Abstract

Husserl's mature philosophy of mathematics has, on the logic side the influence of Leibniz, Bolzano, Lotze and Hume, and on the mathematics side the influence of Leibniz and Riemann. What is not clear are the influences Husserl's views exerted on those of later researchers. There is, however a remarkable similarity between Husserl's conception of mathematics as a theory of structures and the views of the school of Bourbaki. Was there some direct or indirect influence of Husserl on the Bourbakians?

Keywords: Bourbaki, Husserl, logic, mathematics, theory of structures

1. Introduction

The evolution of Husserl's views on logic and mathematics from his youth work, *Philosophie der Arithmetik* (Hua XII), to his mature views of the first volume of his *opus magnum Logische Untersuchungen* (Hua XVIII and XIX), expounded also much later in the first part of *Formale und transzendentale Logik* (Hua XVII), as well as in his posthumous *Einleitung in die Logik und Erkenntnistheorie* (Hua XXIV) and *Logik und allgemeine Wissenschaftstheorie* (Hua XXX) has been the matter of some discussion, including distortions and superficial renderings by scholars (and Fregean fans)[1] working in the so-called analytic tradition, and some of which never studied Husserl seriously.[2]

The fact of the matter, as has been pointed out many times by the present author and by other Husserlian scholars, is that *Philosophie der Arithmetik* is basically a work stemming from Husserl's professorship's thesis of 1887, *Über den Begriff der Zahl*[3], and corresponds to Husserl's views at most up to 1890, being that the main reason why the second planned volume of that work was never published and it seems that never written. In fact, if one examines the writings included in Husserl's posthumous book *Studien zur Arithmetik und Geometrie* (Hua XXI), one can very well trace the evolution of some of Husserl's views on those mathematical disciplines from 1886 to 1894. On the other hand, if one reads Husserl's posthumous paper 'Zur Logik der Zeichen (Semiotik)'[4], written in 1890 and his critical review of the first volume of Ernst Schröder's *Vorlesungen über die Algebra der Logik*[5], which was almost surely written in 1890, since it was already in press in January of 1891 when Frege's 'Funktion und Begriff'[6] was published, there is absolutely no doubt that Husserl discovered the distinction between, in Frege's terminology, 'sense and reference (better: referent)' with complete independence of Frege and probably at the same time of his a decade older rival. In fact, that distinction was clearly anticipated by Bolzano, as pointed out by the present author in a recent paper.[7]

In fact, as Husserl pointed out in the first volume of *Logische Untersuchungen* (Hua XVIII, Ch. X, §§ 60-61, and Appendix; see also Hua XIX, 35-38) and elsewhere, Leibniz, Bolzano, Lotze and Hume were the philosophers who played a decisive role in making Husserl abandon the mild Brentanian psychologism of his *Philosophie der Arithmetik*, not the 1894 late review of that book by Frege[8]. By the way, Husserl's mature conception of logic, mathematics and their relationship dates precisely from 1894 and is clearly different from Frege's. Husserl was never a logicist, and not even a reductionist.

2. The Influence on Husserl of both Leibniz and Riemann

Husserl's conception of the relation between logic and mathematics is certainly different from Frege's, though not unrelated, In fact, both are heirs, as is also David Hilbert, of the seminal contributions to philosophy of the great German mathematician and philosopher Gottfried Leibniz. From the time of its early systematization by Aristotle –and even earlier- in Ancient Greece, logic was conceived as a philosophical discipline with little relationship to mathematics. Moreover, whereas mathematics grew gradually from its early origins in Greece, India and the Middle East, until the revolutions made by Descartes (analytic geometry) and Newton and the same Leibniz (differential and integral calculus) immensely accelerated that process, logic remained basically the same from Aristotle's systematization to the nineteenth century. However, precisely Leibniz had already somehow anticipated the modern view of logic and mathematics by bringing them together as fundamentally intertwined in his conception of a *mathesis universalis* (see Leibniz 1982).[9] That conception of the essential connection between logic and mathematics was taken by the great philosopher and mathematician Bernard Bolzano[10], and later developed in more concrete and diverse fashions by the three illustrious intellectual grandsons of the great Leibniz –and, thus, intellectual cousins – Gottlob Frege, Edmund Husserl and David Hilbert. The three intellectual cousins were originally mathematicians, who turned to philosophy in different degrees. Hilbert was certainly the only one who remained essentially a mathematician and, by the way, probably the greatest mathematician of the first half of the twentieth century. Frege remained a mathematics professor all his life, but his research was essentially in logic and philosophy, being certainly one of the greatest logicians ever, as well as one of the best and most influential contemporary philosophers. Husserl, on the other hand, made the turn from mathematician to philosopher more completely than the other two, being a philosophy professor all his life and, by the way, being one of the greatest philosophers ever.

Hilbert tried to develop logic and arithmetic at the same time, as parts of a common discipline, without clearly articulating their relationship (see Hilbert 1964 and 2013). Frege articulated the relation between logic and mathematics in a much clearer fashion. Non-geometrical mathematics can be obtained analytically, by definitions and derivations, from logic. The latter is the mother discipline, while non-geometrical mathematics is the daughter discipline.[11] That conception has been baptized "logicism", and since Frege – and also since Richard Dedekind – has played an important role in the discussions on the philosophy of mathematics. And since Frege was not only a logicist, but also a Platonist in the philosophy of mathematics, he was forced to introduce so-called "logical objects", and to conceive the truth-values – the true and the false – as logical objects *par excellence*.

As Leibniz, Bolzano, Frege and Hilbert, Husserl also conceived logic and mathematics as strongly related. But his conception was more articulated than those of his predecessors and his two contemporaries, and that was partially due to the fact that Husserl had another strong intellectual influence from another source, namely, from one of the greatest mathematicians of the nineteenth century, and interestingly not from his teacher Karl Weierstraß, but from Bernhard Riemann.

Already in a letter to his teacher Brentano of the 29th of December of 1892 Husserl informed him that he had accepted Riemann's twofold conception of the nature of geometry, namely, (i) that from a mathematical point of view all geometrical structures, be it of three, four or n dimensions, be it of zero, negative or positive curvature stand at the same level, and geometry in the mathematical sense is the study of all those different sorts of geometrical manifolds (or structures); and (ii) that with respect to physical space one cannot decide *a priori*, but only empirically whether it has zero, negative or positive curvature, as well as three, four or whatever dimensions. In later letters of the 29th of March of 1897 and the 7th of September of 1901 to Paul Natorp[12] – thus, clearly before the advent in 1905 of Einstein's special relativity, Minkowski's 1908 refinement and Einstein's and Hilbert's 1915 general relativity – Husserl reasserted such convictions. On this point Husserl and his since 1901 near friend Hilbert were far ahead of their stubborn older intellectual cousin Frege, who in a paper written between 1902 and 1906, but published only posthumously, compared non-Euclidean geometries to alchemy and astrology.[13]

3. Husserl's Conception of Logic: a Brief Survey

Although our interest here is mainly on Husserl's conception of mathematics and that of the Bourbaki group, a few words have to be said about Husserl's conception of logic, in order to explain Husserl's understanding of the *mathesis universalis* and contrast it to that of Frege.[14]

First of all, Husserl was neither a logicist and, thus, did not need to try to derive mathematics from logic, nor was he a logical Platonist, thus, had no necessity to postulate the existence of any so-called logical objects. To say it briefly, for Husserl there were no logical objects. Logic was for Husserl a syntactic-semantic discipline, based on what he called "meaning categories", which are on the basis of the formation of all sorts of sentences. Besides the formation of elementary (or atomic, in contemporary parlance) sentences, the most important aspect of the formation of the first and fundamental stage in the edification of the logical syntactical-semantic building is what Husserl calls somewhat negatively "the laws that protect against nonsense". Those laws allow us to form complex sentences from more elementary sentences with the help of what are now called logical connectives. Thus, beginning with elementary sentences, by means of the reiterated application of the logical connectives, one could form complex sentences of any finite level of complexity. It should be perfectly clear for anyone with a minimum of knowledge of logic and of contemporary analytic philosophy that this first level of the logical building is that of what Carnap, without citing Husserl or even including *Logische Untersuchungen* in the bibliography, called "formation rules" in his *Logische Syntax der Sprache*.[15]

The second level of the logical building was for Husserl that – once more negatively expressed- of the laws that protect against formal countersense, that is, against contradiction, and more positively expressed, guide derivations. These are what Carnap in *Logische Syntax* called, once more without any reference to Husserl, "transformation rules", and which, as the formation rules, are now part of the standard rigorous presentations of logic in textbooks. Husserl called this part of logic "apophantic logic", that is, the theory of the proposition (or of the sentence), and in more modern parlance could have been called syntax or theory of deduction.

In *Logische Untersuchungen* Husserl had still not neatly distinguished between syntax and semantics. This distinction was clearly made in *Formale und transzendentale Logik* when he added above the level of apophantic logic a level of the logic of truth. One obtained this level from the previous one by introducing the notion of truth and related notions of a semantic flavour. On this point Husserl also anticipated a little what occurred a few years later at the hands of the great Tarski.

But though logic was a syntactical-semantic discipline and mathematics was not derivable from logic, it does not mean that logic and mathematics were separated from each other. For this intellectual grandson of Leibniz, logic and mathematics were very related, but not as mother and daughter as in Frege, but as sister disciplines. Mathematics, geometry included, was also a formal discipline, though not a syntactical-semantic discipline, but an ontological one. Mathematics was a sort of ontological counterpart of logic, the ontologically fat sister discipline of logic, which Husserl used to call "formal ontology".

4. Husserl's Views on Mathematics as a Theory of Structures

Husserl considered mathematics a formal ontology. From an etymological standpoint that means a domain of purely formal objects, in contrast to the regional (material) ontologies that are the objects of study –or prospective objects of study- of the material sciences. But what was meant by 'formal ontology' was a plurality of formal structures. Husserl had generalized Riemann's conception of geometry as the study of geometrical manifolds or structures to the whole of mathematics. For Husserl there was a plurality of fundamental formal-ontological categories, which served as the building blocks of the most basic and fundamental mathematical disciplines. This point should be stressed, since Husserl never envisaged the reduction of all mathematical concepts to a single one: he was certainly not a reductionist. The lists of formal-ontological categories – as he called them- fluctuated a little from exposition to exposition, but it usually included the notions of set, relation, whole and part, and of number (presumably cardinal number) and of ordinal number.

On this point, mathematicians, logicians and philosophers schooled in the set-theorist tradition would certainly point out that the notions of cardinal and ordinal number, as well as the notion of relation can be defined in terms of that of set. Leaving aside whether those definitions are natural or somewhat forced, it should, firstly, be pointed out that the notions of set and of relation, and with them the fundamental mathematical notion of function are really interderivable. In set-theoretical mathematics a function of n arguments can be defined as a relation of n+1 arguments univocally determined in its last argument. On the other hand, Frege defined the notion of relation in terms of the notion of function: a relation of n arguments being a function of the same number n of arguments, whose value is a truth-value.[16] But the notion of set can also be defined, as pointed out by Saunders Mac Lane[17], in terms of that of relation. In fact, the notion of set can also be defined directly in terms of that of function.[18] And as is very well known, the notion of set can be defined in terms of the notion of category, as shown in any textbook on category theory.

The most interesting and less considered of the formal-ontological categories is that of whole, or if you prefer, of whole and part. Probably most mathematicians do not consider the notion of whole a mathematical notion, probably because it is too loosely characterized. Nonetheless, firstly it should be pointed out that the great Polish logician Stanislaw Lesniewski developed a theory of parts and wholes, a mereology, as a fundamental part of his alternative logical building and presumably a nominalist replacement of set theory. On the other hand, in *Logische Untersuchungen* Husserl considered a somewhat particular case of the notion of whole, namely, the notion of extensive whole, which certainly admits a formal mathematical treatment, and even Whitehead considered a rigorous treatment of a theory of parts and wholes in his theory of the extensive continuum (see Whitehead (1979, 294-301). One could ask whether the notion of (extensive) whole is definable in terms of that of set or (and) the other way around.[19]

Continuing with Husserl, the formal-ontological categories give rise to the fundamental mathematical disciplines, for example, to set theory, to a mereology, to a theory of relations, to cardinal number theory. The remaining mathematical disciplines are obtained, according to Husserl by one of two procedures or by a combination of the two procedures. Firstly, we obtain new areas of mathematics by specialization. Secondly, we obtain new areas of mathematics by bringing together two or more of the fundamental mathematical disciplines. And thirdly, we obtain new areas of mathematics by combining the two procedures of specialization and of bringing together structures to form more complicated structures.[20]

5. Bourbaki's Views on Mathematics as a Theory of Structures

The collective French mathematician Nicholas Bourbaki is certainly one of the most distinctive components of twentieth century mathematics. In the early 1930s a group of young but already distinguished French mathematicians, among them Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Dieudonné, André Weil, Charles Ehresmann and René de Possel, organized the Bourbaki group, whose project was to rewrite the whole of mathematics under strict foundations.[21] The group was constantly renewed, both because new members were added and others opted to abandon the group, as well as because the organizing members established an age limit of 50 for all (original and future) members of the group. Due to the development of set theory as a founding discipline during the first decades of the twentieth century at the hands of Ernst Zermelo, Abraham Fraenkel, John von Neumann and others, the notion of set was taken as the most basic mathematical concept. Nonetheless, there was no attempt at reducing mathematics to set theory, but instead the notion simply served as the language used for the introduction of the basic mathematical structures, what the Bourbakians called the "mother structures".[22]

According to Bourbaki, the mother structures were threesome, namely, algebraic, topological and order structures. These were the ground structures. All other mathematical structures were obtained from them by three processes with which the reader should already be familiar, namely, the specialization of a structure, the connecting of two (or more) structures and the combination of those two procedures. Thus, Hausdorff spaces are specializations of topological spaces, being uniform spaces and metric spaces further specializations in that order, whereas topological groups are structures obtained by connecting topological and algebraic structures. Banach spaces and the structure of the real numbers bring together specializations of different sorts of mathematical structures.

It is easy to see that the conception of mathematics as a theory of structures of the Bourbaki group is very similar to that of Husserl even in fundamental details.[23] In both cases there are what the Bourbaki group called "mother structures" –three in the case of the Bourbaki group, a not definitely determined, but probably a little larger number in the case of Husserl. The rest is basically identical, namely, one can obtain other structures either (i) by specialization of the mother structures –that is: incorporating additional structure-, (ii) by connecting two or more mother structures to form a complex one, or (iii) by combining the two procedures of specialization and connection of structures at any level, for example, bringing together specializations of mother structures to form complex less abstract structures, or obtaining specializations of complex structures resulting from the connection of mother structures, etc.

6. Small but Important Divergences

From a purely theoretical –not necessarily historical-standpoint, the Bourbaki group's conception of mathematics as a theory of structures can be seen as an elaboration or refinement of Husserl's views. It seems as if the Bourbaki mathematicians had made Husserl's views more precise. Whether a historical link could also be traced is another matter, one that we will touch briefly below.

There are however, some small differences of detail, on which we want to dwell now. The most obvious difference between the two theories of structures is that the Bourbaki group, probably influenced by the development of set theory, takes the notion of set as the basic notion that even the mother structures should take for granted. The three mother structures

–topological, algebraic and order structures- are, thus, not based on a fundamental mathematical notion, but on a notion that can be expressed in set-theoretical language. On the other hand, in Husserl's theory the notion of set is only one of the fundamental notions of mathematics and each fundamental notion originates a mother structure. Set theory is just one of maybe five or six mother structures, though such a number could be reduced in view of the definability of some of the candidates for formal ontological categories –for example cardinal number and ordinal number- in terms of the notion of set and the interdefinability of others – namely, the notions of relation and function- with the notion of set.

A second seemingly small but very important difference is the inclusion by Husserl of the notions of whole and part in the list of formal ontological categories, and the urge to develop a mathematical theory of wholes and parts. Tough as stated above, in some mathematical contexts the notions of set and of whole seem not to be clearly distinguished –is the spatial continuum the set of which a point is a member or the whole of which the point is a part?-, the notions are not only different but not definable in terms of the other in a non-artificial way. Of course, in Bourbaki's views, as in those of all of current mathematics, the notions of whole and part are not mathematical. However, the fact that Lesniewski could develop a formal theory of wholes and parts should be a reminder for mathematicians that such a mathematical theory is feasible and not an unfounded speculation of Husserl.

Things get more interesting if we bring to the fore the most important structuralist rival of Bourbaki's views, namely, category theory, developed a decade after the surge of Bourbakian mathematics by some collaborators of the Bourbakians, especially Saunders Mac Lane and Samuel Eilenberg. In category theory the notion of set does not play any decisive role. It is one of many mathematical notions that can be dealt with without difficulty in the context of category theory. A category consists of two components, namely, objects and morphisms (or arrows) between the objects. Somewhat more precisely, a category **K** consists of a collection of objects Obj(**K**), together with, for each pair of objects A and B in the collection of objects **K**, a possibly empty collection of morphisms $f{:}A\to B$ such that:

(i) For any three, not necessarily distinct objects A, B, C in Obj(**K**) and any $f{:}A\to B$ and $g{:}B\to C$, there is an operation "$*$", called the composition of f and g such that $g*f{:}A\to C$, which is associative, that is, $h*(g*f)=(h*g)*f$.

(ii) For every object A of **K**, the collection **K**(A,A) contains an identity morphism id_A, that is, one such that if f is a morphism in **K**(A,B), respectively, in **K**(B,A), we have $id_A*f=f$, respectively, $f*id_A=f$.[24]

Category theorists, in their textbook expositions sometimes use the word "set" instead of the more neutral word "collection", but we have preferred to avoid it, in order not to create the suspicion that when one later in such textbooks introduces the notion of set in terms of categories we are operating in a circle. Nonetheless, the fact that categories can also be introduced with help of the notion of set debilitates the claim that category theory can serve as the foundation of the whole of mathematics. In the best of cases, it would seem to be on equal stand with set theory

A very different objection could come if we take Husserl seriously and consider the theory of (at least extensive) wholes and parts as a mathematical theory. Probably the attempt to define wholes in terms of categories would be at least as difficult or artificial as to define them in terms of sets. Hence, in reality there are not two but three rival foundations of structural mathematics, namely: (i) set-theoretic foundations, (ii) categorical foundations and (iii) none of the above, but a foundation on a plurality of fundamental (formal ontological categories), none of which seem more fundamental than the others and some that could only be, in the best of cases, artificially defined in terms of some other.

7. Did Husserl influence the Bourbaki group?

This question admits presently no definitive answer, and we will only pave the way for future investigations –most probably by younger authors. Nonetheless, there are some interesting factors that point to a possible somewhat indirect influence of Husserl on the Bourbaki group. First of all, there are two general circumstances present in Germany and France in the first decades of the twentieth century that should not be ignored. The first one is very simple and general, namely, that contrary to what one could think today, especially if you see the history of contemporary philosophy through the muddy lenses of North American empiricism[25] and of the multiply-broken ones of its magician cousin nominalism, in Germany, France, Poland and other European countries the spectre of empiricism did not blind the spirits of mathematicians and philosophers, forbidding them to read non-empiricist philosophers.

The second more specific one is that in some of the most important centres of mathematical research both in Germany and in France in the first decades of the twentieth century mathematicians and philosophers were in near intellectual contact. In Gottingen, for example, which had been one of the most important centres of mathematical research since Gauß, before and after 1900 mathematicians and philosophers were not only administratively linked, but in some cases also intellectually and personally strongly related. That was the case from 1901 to 1916 between Felix Klein and, especially David Hilbert with precisely the mathematician turned philosopher Edmund Husserl. In fact, Husserl initiated his tenure as philosophy professor in Göttingen with a double conference at meetings of their mathematical society, and that just a year after the publication of the first volume of *Logische Untersuchungen*, in whose last chapter he had presented for the first time his mature conception of logic and mathematics. In particular, Husserl and Hilbert developed a friendship that lasted for their whole lives. Moreover, many of Hilbert's collaborators, like Ernst Zermelo and Constantin Carathéodory were also in friendly terms with Husserl, and many students of Hilbert, as Hermann Weyl and Max Born, were also students of Husserl, and these two also developed a lifelong friendship with the great philosopher.

The situation was very similar to that in Göttingen two to three decades later in Paris at the École Normale, which usually assembled the best French mathematicians, both as students and as faculty. Mathematicians, like Henri Cartan, André Weil and others of the founders of the Bourbaki group had a strong intellectual and personal relation with philosophers also schooled in mathematics, as Jean Cavaillès and Albert Lautman. Cavaillès was an excellent Husserlian scholar intensively working both on Husserl's views on mathematics and logic and on the philosophy of mathematics, in general.[26] Henri Cartan had not only been one of Cavaillès' teachers, but he was also in the doctoral committee of Jean Cavaillès' dissertation and, moreover, wrote the Preface to the second edition of one of Cavaillès' two doctoral theses, *Méthode Axiomatique et Formalisme*. But Cavaillès seemed to have been near also to many other members of the Bourbaki group, among them André Weil, René de Possel, Paul Dubreil and, especially, Claude Chevalley,[27] though his strongest relation with any member of the Bourbaki group was that with another of its founders, Charles Ehresmann, who was one of Cavaillès best friends, and after Cavaillès death in 1944 was one of the editors of Cavaillès' posthumous *opus magnum*, *Sur la Logique et la Théorie de la Science*, and co-author of its Preface. Thus, Cavaillès belonged to the periphery of near friends of the Nicholas Bourbaki group, and received regularly their manuscripts before their printing. Most surely, besides Ehresmann, Chevalley and Cartan[28], also other members of the group took seriously any constructive criticism of his and respected his well-founded philosophical views on mathematics. Cavaillès' views on mathematics, however, had essentially two fundamental sources, namely, Husserl's conception of mathematics as a formal ontology, that is, as a theory of structures as we described above, and the development of set theory from Cantor to its axiomatization beginning with Zermelo and up to at least 1930. We can suppose that the "working mathematicians" of the Bourbaki group were at least partially acquainted with the development of set theory, to which French mathematicians of the prior generation, like Émile Borel, had contributed. It is certainly not excluded that one or two of them had read some Husserl, since Husserl was highly esteemed in those days in French philosophical circles, and precisely in 1928 he had lectured in Paris at La Sorbonne on the foundations of our knowledge, lectures that were the basis for his book of the same year under the title

Cartesianische Meditationen (Hua I). But even in the case that none of the founders of the Bourbaki group had ever acquainted himself directly with Husserl's views on mathematics, they were most surely informed of them by their highly respected Cavaillès. It is by no means preposterous to sustain that the Bourbakians were somewhat at least indirectly influenced by Husserl's views on mathematics as a theory of structures -or at least Husserl's views strengthened theirs-, with its mother structures and all the ways in which one can obtain new structures from already existing structures, and the fundamental similarities between both conceptions were not a purely lucky coincidence.[29] Nonetheless, as we stressed above, there were also two major discrepancies, namely, (i) the presupposition by the Bourbakians of the set-theoretic language and of set theory as a sort of basis of even the mother structures, and (ii) the inclusion by Husserl of a mereology – in Lesniewski's parlance – as a fundamental mathematical structure, that is, as a mother structure. Concerning the first discrepancy, due to the fact that sets can be defined in terms of other mathematical concepts, it should be clear that Husserl was right and the Bourbakians wrong: the notion of set is not the fundamental mathematical notion, not even in a linguistic-pragmatist sense. There does not seem to exist a unique most fundamental mathematical notion. Concerning the second discrepancy, both the Bourbakians and the category theorists would have to show that everything that can be obtained in a mereology can be obtained in their respective conceptions, making the introduction of the notions of whole and part in mathematics superfluous. If that were not the case, then Husserl would have been justified in introducing the notions of part and whole as fundamental mathematical notions. Only the future development of mathematics can decide this question.[30]

NOTES

[1] Dagfinn Føllesdal (1958, 1969) has been very influential in the propagation of this incorrect view, as was also Evert W. Beth (1959) and much earlier than both Alonzo Church in his review of Marvin Farber's book *The Foundation of Phenomenology*. For the three authors, see the references.

[2] Some scholars cannot even distinguish well between Husserl and Kant, and have argued that Kant was the main influence on Carnap's *Logische Aufbau der Welt*, whereas in reality it was Husserl who exerted a not acknowledged fundamental influence on that work, to the point that we should seriously speak about plagiarism. See on this subject endnote 14 below as well as writings of Verena Mayer and the present author in the references.
[3] Published for the first time as Appendix A to the *Husserliana* edition of *Philosophie der Arithmetik* (Hua XII; Husserl 1970b).
[4] 'Zur Logik der Zeichen (Semiotik)', was written in 1890 but published for the first time only as Appendix B.(I) to the Husserliana edition of *Philosophie der Arithmetik* (Hua XII; Husserl 1970c).
[5] 'Besprechung von E. Schröders *Vorlesungen über die Algebra der Logik I*' 1891, reprinted in (Hua XXII; Husserl 1979b).
[6] Frege's 'Funktion und Begriff' was most probably also written in 1890. It was reprinted in his *Kleine Schriften* (Frege 1967/1990).
[7] See Rosado Haddock (2018, 199-219).
[8] 'Rezension von E. G. Husserl, *Philosophie der Arithmetik I*' 1894, reprinted in *Kleine Scriften* (Frege 1967/1990, 179-192).
[9] For an excellent treatment of Leibniz and the *mathesis universalis*, see the recent paper by Centrone and Da Silva (2017, 1-23). For Husserl's assessment of Leibniz' influence, see also Hua XVIII, §§ 60-61.
[10] See Hua XVIII (Appendix to Chapter 10), for Husserl's assessment of Bolzano's work and influence and, very especially, the section 26d of his *Formale und transzendentale Logik* (Hua XVII). See also Centrone and Da Silva's (2017), as well as Casari (2017, 75-91), and the references therein. For a thorough treatment of both Leibniz and Bolzano's views on mathematics, see Danek (1975).
[11] On Frege's logicism and Platonism, see his philosophical masterpiece *Die Grundlagen der Arithmetik* (Frege 1986), as well as the Introduction to his *Grundgesetze der ArithmetikI (*Frege 1962), a certainly failed but nonetheless impressive attempt to derive arithmetic and mathematical analysis from logic.
[12] Since we have quoted extensively from those three letters in two older papers, namely, in the already mentioned 'Husserl and Riemann' and in 'Husserl's Conception of Physical Theories and Physical Geometry in the Time of the Prolegomena: a Comparison with Duhem and Poincaré', we refer the reader to those papers (Rosado Haddock 2012 and 2017).
[13] See Frege's posthumous 'Über Euklidische Geometrie', in Frege (1983, 182-184).
[14] We will follow basically *Logische Untersuchungen*, though *Formale und transzendentale Logik*, *Einleitung in die Logik und Erkenntnistheorie* and *Logik und allgemeine Wissenschaftstheorie* could very well had been used. There is only one point, emphazised in *Formale und transzendentale Logik*, in which we will refer especially to this last work.
[15] This is a clear case of dishonesty by Carnap without any possible excuse. In former books of his, namely, in his dissertation, *Der Raum*, and in *Der logische Aufbau der Welt*, Carnap included *Logische Untersuchungen* in the bibliography, but not in his 1934 book nor in 'Die Überwindung der Metaphysik durch logische Analyse der Sprache', in both of which he

appropriated material from Husserl's *Logische Untersuchungen*. Of course, as Verena Mayer and the present author have shown in various writings (see Mayer 2016), in *Der logische Aufbau der Welt* he appropriated many more ideas from Husserl, this time precisely from the latter's "Ideas", namely, from *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie I* (Hua III) and from other then unpublished manuscripts of Husserl, most probably from the then still unpublished second volume of that work.

[16] Frege really mentioned explicitly only the case in which n=2, but the generalization to any finite n is trivial.

[17] See, for example, Mac Lane (1986, 359, 407).

[18] Already in 1925 John von Neumann defined the notion of set in terms of that of function, as pointed out to me many years ago by Philippe de Rouilhan.

[19] By the way, it is interesting that when studying general topology and considering notions like that of neighbourhood I have always wondered whether in that fundamental area of mathematics –and maybe in parts of mathematical analysis- one could very well replace the notion of set with the notion of extensive whole, thus, obtaining another sort of "non-standard" analysis.

[20] See on this whole last point, e.g. Hua XVIII, §§ 69-70.

[21] For a general exposition of the views of the Bourbaki group see Corry (2004, chapter 7, especially, pp. 292-293).

[22] For more detailed expositions of the views of the Bourbaki group, see Bourbaki (1949 and 1950).

[23] It should be pointed out, however, that Bourbaki's conception, though fundamentally similar to Husserl's, is far more elaborated than Husserl's sketches. For example, in order to combine two structures, some law of compatibility is usually necessary. Thus, in the case of topological groups, which combine topological and algebraic structures, a law of compatibility requires that homomorphisms between groups be continuous. However, though Husserl did not make explicit such a requirement, there is little doubt that he, as a well trained mathematician, would have accepted it. There are other components in the more sophisticated presentation by the Bourbaki group, though they are not essential for the conception itself, but for the particular presentation. See on this issue Bourbaki (1966, chapters 1 and 4).

[24] We have followed here, with small modifications, the definition of category in Michael A. Arbib's and Ernest G. Manes book *Arrows, Structures, and Functions: The Categorical Imperative*, though we could very well had used Saunders Mac Lane's classic book *Categories for the Working Mathematician* or any other textbook on category theory.

[25] A friend of mine who made her undergraduate studies in philosophy in one of the most renowned North American universities told me that one of her philosophy professors – around 1970 – told her that after 1905 all good philosophy was written in English and, thus, it was not necessary to learn other languages, especially German. *Lang lebe die Unwissenheit und Ihre Schwester die Doofheit!*

[26] For the life and work of this philosopher and hero of the French resistance against the Nazis during the second-world war, we refer to the very valuable biography written by his sister Gabrielle Ferrières (1982).

[27] A possible example of this nearness is the following: In a footnote on p. 78 of his 1937 *Méthode Axiomatique et Formalisme* Cavaillès mentions that the most general definition of integration had been given just recently simultaneously and independently of each other by Hans Hahn and René de Possel. In the references there is a paper of Hahn (1933) – to which Cavaillès most surely refers – but no writing of de Possel is included. That points either to having obtained that information directly from de Possel or from another member of the Bourbaki group, in any case to the information being obtained from the inner circle of the group.
[28] See Cartan's assessment of Cavaillès in the already mentioned preface to *Méthode Axiomatique et Formalisme* (Cavaillès 1981).
[29] Besides Cavaillès two books already mentioned, his biography written by his sister, Gabrielle Ferrières, is also very informative with respect to his relation to members of the Bourbaki group. Thus, for example, on pp. 106-107 Ferrières quotes a letter from her brother, in which Cavaillès not only shows his great esteem for Chevalley, but also mentions that the latter is working on one of the monographs for the Bourbaki group and that he – Cavaillès – will be taking part in the discussion of the monograph. Moreover, and also as an example, Ferrières (1982, 124) refers to Cavaillès' great friendship with Ehresmann; Cavaillès (125) mentions that the Bourbaki group continues to send him parts of their projected treatise on analysis; and Ferrières (211) mentions that in a book on algebra published after Cavaillès' death Paul Dubreil recommends the reader to read Cavaillès, Thus, it is very difficult to argue that the Bourbakians were not informed about Husserl's views on mathematics.
[30] For an exposition of both the Bourbakian conception of mathematics and of that of the category theorists, as well as a comparison between them that favours category theorists, see Corry (2004, Chapters 7-9). Of course, nothing is said about Husserl or the possibility of conceiving a theory of wholes and parts as a mathematical discipline.

REFERENCES

Arbib, Michael A. & Ernest G. Manes. 1975. *Arrows, Structures, and Functions: The Categorical Imperative*. New York: Academic Press.

Beth, Evert Willem. 1959. *The Foundations of Mathematics.* Amsterdam: North Holland

Bolzano, Bernard. 1963. *Grundlegung der Logik*. Selected sections from volumes 1 and 2 of *Wissenschaftslehre*. Hamburg: Felix Meiner.

Bourbaki, Nicholas. 1949. "Foundations of Mathematics for the Working Mathematician." *Journal of Symbolic Logic* 14(4): 1-8

Bourbaki, Nicholas. 1950. "The Architecture of Mathematics." *American Mathematical Monthly* 57: 221-232

Bourbaki, Nicholas. 1966. *Éléments de Mathématique: Théorie des Ensembles*. Paris : Hermann.

Casari, Ettore. 2017. "Husserl and Bolzano". Centrone 2017, 75-91.

Cavaillès, Jean. 1981. *Méthode Axiomatique et Formalisme*. Paris: Hermann. (First published 1937).

Cavaillès, Jean. 1960. *Sur la Logique et la Théorie de la Science*. Paris: Presses Universitaires de France. (First published 1947).

Centrone, Stefania (ed.). 2017. *Essays on Husserl's Logic and Philosophy of Mathematics*. Dordrecht: Springer.

Centrone, Stefania & da Silva, Jairo. 2017. "Husserl and Leibniz: Notes on the *Mathesis Universalis*'. In Centrone 2017, 1-23.

Church, Alonzo. 1944. "Review of Marvin Farber, *The Foundations of Phenomenology*." *Journal of Symbolic Logic* 9: 63-65.

Corry, Leo. 2004. *Modern Algebra and the Rise of Mathematical Structures*. Revised edition. Basel: Birkhäuser.

Danek, Jaromir. 1975. *Les Projets de Leibniz et de Bolzano: deux Sources de la Logique Contemporaine*. Québec : Les Presses de l"Université Laval.

Ferrières, Gabrielle. 1982. *Jean Cavaillès: un Philosophe dans la Guerre 1903-1944*. Paris : Éditions du Seuil.

Føllesdal, Dagfinn. 1958. *Husserl und Frege, ein Beitrag zur Beleuchtung der phänomenologischen Philosophie*. Oslo: H. Aschehoug.

Føllesdal, Dagfinn. 1969. "Husserl's Concept of Noema." *Journal of Philosophy* 66 (20): 680-697.

Frege, Gottlob. 1962. *Grundgesetze der Arithmetik (1893, 1903)*. Reprinted in a single volume.

________. 1967. "Funktion und Begriff' (1891). In *Kleine Schriften*, edited by Ignacio Angelleli. Darmstadt: Hildesheim: Georg Olms.

________. 1974. *Wissenschaftlicher Briefwechsel.* Hamburg: Felix Meiner.

________. 1986. *Die Grundlagen der Arithmetik (1884).* Centenary Edition. Edited and with an Introduction by Christian Thiel. Hamburg: Felix Meiner.

________. 1990. *Kleine Schriften*. Revised edition. Edited by Ignacio Angelelli. Hildesheim: Georg Olms.

________. 1983a. *Nachgelassene Schriften*. Revised edition. Edited by Hans Hermes. Hamburg: Felix Meiner.

________. 1983b "Über Euklidische Geometrie." In *Frege* 1983a, 182-184.

Hilbert, David. 1964. *Hilbertiana: Fünf Aufsätze.* Darmstadt: Wissenschaftliche Buchgesellschaft.

________. 2013. *Lectures on the Foundations of Arithmetic and Logic: 1917-1933.* Edited by William Ewald & Wilfried Sieg. Berlin: Springer.

Husserl, Edmund. 1950-2012. *Husserliana: Edmund Husserl – Gesammelte Werke*. Bde. 1-41. Haag: Martinus Nijhoff (Bd. 1-26, 1950-1987); Haag: Kluwer Academic Publishers (Bd. 27-37, 1988-2004); New York: Springer (Bd. 38-41, 2005-2012). [Abbreviated *Hua*].

________. 1950. [*Hua* III]. *Ideen zu einer reinen Phänomenologie und Phänomenologischen Philosophie, Erstes Buch, Allgemeine Einführung in die Reine Phänomenologie*. In *Husserliana,* Bd. III, hrsg. von Walter Biemel. Den Haag: Martinus Nijhoff.

________. 1952. [*Hua* IV]. *Ideen zur einer reinen Phänomenologie und phänomenologischen Philosophie. Zweites Buch: Phänomenologische Untersuchungen zur Konstitution*. In *Husserliana,* Bd. IV, hrsg. von Marly Biemel. The Hague: Martinus Nijhoff.

________. 1970a. [Hua XII]. *Philosophie der Arithmetik*. In *Husserliana*, Bd. XII, hrsg. von Lothar Eley. Den Haag : Martinus Nijhoff.

________. 1970b. "Über den Begriff der Zahl (1887)." [Appendix A]. In *Husserliana*, Bd. XII, 289-339. Den Haag: Martinus Nijhoff.

________. 1970c. "Zur Logik der Zeichen (Semiotik) (1890)." [Appendix B.(1)]. In *Husserliana* XII, 340-373.

________. 1973. [*Hua* I] *Cartesianische Meditationen und Pariser Vorträge*. In *Husserliana*, Bd. I, hrsg. und eingeleitet von Stephan Strasser. Haag: Martinus Nijhoff.

________. 1974. [Hua XVII]. *Formale und transzendentale Logik. Versuch einer Kritik der logischen Vernunft*. In *Husserliana*, Bd. XVII, hrsg. von Paul Janssen. Haag: Martinus Nijhoff.

________. 1975. [*Hua* XVIII]. *Logische Untersuchungen. Erster Teil. Prolegomena zur reinen Logik*. In *Husserliana,* Bd. XVIII, hrsg. von Elmar Holenstein. Haag: Martinus Nijhoff.

________. 1979a. [Hua XXII]. *Aufsätze und Rezensionen (1890-1910).* In *Husserliana,* Bd. XXII, hrsg. von B. Rang. The Hague: Martinus Nijhoff.

________. 1979b. "Besprechung von E. Schröders *Vorlesungen über die Algebra der Logik I* (1891)". In *Husserliana*, Bd. XXII: *Aufsätze und Rezensionen 1890-1910*, 3-43. Den Haag: Martinus Nijhoff.

________. 1983. [Hua XXI]. *Studien zur Arithmetik und Geometrie. Texte aus dem Nachlass*. In *Husserliana,* Bd. XXI, hrsg. von I. Strohmeyer. Den Haag: Martinus Nijhoff.

________. 1984. [*Hua* XIX]. *Logische Untersuchungen. Zweiter Teil. Untersuchungen zur Phänomenologie und Theorie der Erkenntnis*. In *Husserliana,* Bd. XIX, hrsg. von Ursula Panzer. The Hague: Martinus Nijhoff. [Text der 1. und der 2. Auflage, Halle, 1901, 1922].

________. 1985. [Hua XXIV] *Einleitung in die Logik und Erkenntnistheorie. Vorlesungen 1906/07*. In *Husserliana*, Bd. XXIV, hrsg. von Ullrich Melle. Den Haag: Martinus Nijhoff.

________. 1995. [Hua XXX]. *Logik und allgemeine Wissenschaftstheorie. Vorlesungen 1917/18. Mit ergänzenden*

Texten aus der ersten Fassung 1910/11. In *Husserliana*, Bd. XXX, hrsg. von Ursula Panzer. Den Haag: Kluwer Academic.

Kant, Immanuel. 1990. *Kritik der reinen Vernunft* (1781/1787). Reprint of both editions. Third edition. Hamburg: Felix Meiner. (First published in 1930).

Leibniz, Gottfried W. 1982. *Allgemeine Untersuchungen über die Analyse der Begriffe und Wahrheiten*. Bilingual edition Latin-German. Edited by Franz Schupp. Hamburg: Felix Meiner.

Mac Lane, Saunders. 1971. *Categories for the Working Mathematician*. New York: Springer

________. 1986. *Mathematics, Form and Function*. New York: Springer.

Mayer, Verena 1991 „Die Konstruktion der Erfahrungswelt: Carnap und Husserl" In *Erkenntnis* Orientated, edited by W. Spohn 287-303. Dordrecht: Kluwer

________ 1992 „Carnap und Husserl" In *Wissenschaft und Subjektivität*, edited by D. Bell and W. Vossenkuhl, 181-205. Berlin: Akademie Verlag.

________ 2016. "Der Logische Aufbau als Plagiat." In *Husserl and Analytic Philosophy*, edited by Guillermo E. Rosado Haddock, 175-260. Berlin: Walter de Gruyter.

Rosado Haddock, Guillermo E. 2008. *The Young Carnap's Unknown Master*. Aldershot: Ashgate.

________. 2012a. "Husserl's Conception of Physical Theories and Physical Geometry in the Time of the *Prolegomena*: A Comparison with Duhem's and Poincaré's Views". *Axiomathes* 22 (1): 173-191.

________. 2012b. *Against the Current*. Frankfurt am Main: Ontos Verlag.

________. 2012c. "On the Interpretation of the Young Carnap's Philosophy." In Rosado Haddock 2012b, 261-284.

________ (ed.). 2016a. *Husserl and Analytic Philosophy*. Berlin: Walter de Gruyter.

________, 2016b. "The Old Husserl and the Young Carnap." In Rosado Haddock 2016a, 167-198.

________. 2017. "Husserl and Riemann." In Centrone (ed.) 2017, 229-243.

________. 2018a. *Unorthodox Analytic Philosophy*. London: College Publications.

________. 2018b. "Husserl and Kant: voilà la difference." In Rosado Haddock 2018a, 115-141.

Whitehead, Alfred North. 1979. *Process and Reality*. London: Macmillan. (First published 1929).

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Symbol and Number

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Abstract

This paper explores the view that numbers are symbolically constituted, that numbers just are meaningful symbols. Such a view is what results if we take the conception of number spelled out by Husserl in the second part of his *Philosophy of Arithmetic* to be self-standing rather than supported by the conception of numbers as abstracted from sets. It will be argued that this latter conception is problematic in itself and, moreover, that it cannot be regarded as providing a foundation for the former.

Keywords: philosophy of arithmetic, Husserl, meaningful formalism, calculation, mathematical ontology

Each of the two parts of Husserl's *Philosophy of Arithmetic* (Husserl 1891) presents a separate conception of numbers. According to the first conception, numbers are abstractions from sets; according to the second conception, numbers are mirror images of symbols and cannot be thought of independently of a system of symbols. Husserl regarded the two conceptions as connected. In particular, he regarded the first conception as a foundation for the second: numbers as understood by the second conception form a superstructure built on top of numbers as understood by the first. Owing to our limitations in forming real, or authentic, representations of sets, we are for the most part relegated to this superstructure. In fact, arithmetic as we know it moves entirely within this superstructure.

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After giving a brief sketch of these two conceptions of number as spelled out in the *Philosophy of Arithmetic*, I shall question whether they in fact are connected in the way Husserl thought. A negative verdict leads me to explore the second conception of number as self-standing. Thus I shall explore the thesis that numbers are symbolically constituted objects, that numbers in effect just are meaningful symbols. The qualifier "meaningful" is important here: the philosophy of arithmetic to be explored is not formalism, namely the thesis that the real objects of mathematics are all finitary, such as strokes and rows of strokes; rather, the non-finitary notion of meaning will be invoked. Whether the resulting philosophy of mathematics has any affinity with Husserl's thought on mathematics, I shall not discuss in any detail; but it does seem to me to be an eminently phenomenological philosophy of mathematics. There are also interesting connections to the type theory of Per Martin-Löf, some of which will be noted.

1. According to the conception of number spelled out in the first part of the *Philosophy of Arithmetic* numbers are abstractions from sets (15-16.).[1] For instance, from the set {redness, the Moon, Napoleon} the number 3 is abstracted, and from the set of the Apostles the number 12 is abstracted. Husserl provides a detailed psychological description of set conception, or set constitution. What makes a representation into a representation of a set is a certain relation, or connection, that obtains between certain parts of the representation that are clearly distinguished from each other (what will become the elements of the set) (20). Much effort is spent on characterizing this relation, which Husserl calls the collective connection (*kollektive Verbindung*). It is, for instance, not the relation of compresence in one consciousness; nor the relation of temporal succession; nor that of sameness or of difference (64). Rather, it is a peculiar relation that is established by an intention directed towards clearly distinguished parts of the representation and that, as it were, holds these parts together. What are to become the elements of the set are all parts of the given representation; but, simply by being parts of a representation, they are still not represented as elements of a set; rather, it takes a second-order intention directed towards these elements to form the set representation (74).

Given such a set representation one can (according to Husserl's abstraction theory) disregard the particular nature of each of its elements and concentrate entirely on the collective connection itself (79). The result is a representation of what Husserl calls a plurality form (*Vielheitsform*), which we may symbolize in language as "something and something and something…" (80). Each occurrence here of the word "something" indicates an arbitrary object, and the ellipsis indicates indeterminacy, namely that the same pattern may continue arbitrarily long (81). Corresponding to the plurality form is the concept of plurality (Husserl is not clear about how to understand the relation between the form and the concept). The concept inherits the indeterminacy of the form (81). Any particular determination of the concept of plurality is a particular number, for instance 3 or 12. Borrowing a famous piece of terminology—though perhaps not the associated doctrine—from (Johnson 1921), we may call the concept of plurality a determinable whose determinations are particular numbers. Particular numbers are in turn described as species of the general concept of number, which concept we obtain by noting the similarity of particular numbers with each other (82). The relation between the general concept of number and particular numbers is therefore one of specification, whereas the relation between the concept of plurality and particular numbers is one of determination.

2. According to Husserl's other conception of number, numbers are the mirror images of numerical expressions. Thus, in the second part of the *Philosophy of Arithmetic* Husserl repeatedly speaks about a parallelism that obtains between number concepts and number signs (e.g. 228, 234, 237-241): the development of a system of number signs is at the same time a development of number concepts. The build-up of the signs mirrors the build-up of the concepts; in particular, the successive construction of number signs mirrors the succession of the number concepts. It is clear that the resulting conception of number is entirely different from the first conception. Here there is no reliance on sets, nor on abstraction. Rather, numbers are here conceived as elements of an ordered sequence, in which each element presupposes its predecessors. A number as abstracted from a set does, by contrast, not thus presuppose its predecessors, but only a set to abstract on. Whereas the first conception is thus a conception of cardinal number, the second conception is a conception of ordinal number.

The system of signs through which numbers are introduced provides a unique expression for each number (260). Numbers represented by these introductory expressions are called *normal* (261). For instance, in the standard decimal system the normal numbers are 0, 1, 2, 3,…, 10, 11, 12,…, 100, 101, 102,… Numbers not given in normal form are called *problematic*; examples are 2 + 3 and 7 × 5. Each problematic number presents us with a task, namely that of reduction to normal form (261). When the decimal numbers are taken to be the normal numbers, then the reduction of 2 + 3, for instance, yields 5, and the reduction of 7 × 5 yields 35. Reduction is thus just what we usually call calculation (258). A basic task of arithmetic is to delineate the various ways of forming problematic numbers and describe the methods of reducing numbers thus formed to normal numbers, that is, to describe methods of calculation (262). Given the parallelism between number signs and number concepts, calculation may be carried out on signs alone without regard to their content, since rules regarding signs will directly translate into rules regarding their content. The rules of manipulating signs are thus sound with respect to their intended content. Husserl considers in detail rules of calculation associated with the usual arithmetical operations—addition, multiplication, subtraction, division—and argues that these rules are indeed sound (264-272).[2]

3. A central thesis in the *Philosophy of Arithmetic* is that the two conceptions of number just described are in fact connected. The first conception describes numbers as they are given to us "authentically" (e.g. 15-16). In an authentic representation, a number is given to us just as it is, we see the "number in itself". A finite mind is, however, limited in how large numbers it can represent authentically. Humans, in particular, may not be able to conceive authentically numbers greater than, say, ten or twelve. Since arithmetic presupposes arbitrarily large numbers, it therefore has to be based on some other conception of number, namely one according to which numbers are not authentically, but rather "symbolically" given (190-92). In a symbolic, or "inauthentic" representation, a number is given to us only indirectly, through a sign (193-94). The second conception of number is just such a symbolic conception, and it, according to Husserl, is the conception of number assumed in arithmetic.

Although arithmetic thus assumes numbers to be symbolically, and not authentically, represented, it would seem odd to say that symbolic, or inauthentic, representations lie at the foundations of arithmetic; for this would be to say that the science of number is based on representations in which numbers are in fact not properly given to us. Rather, one would expect that authentic representations of numbers in some way provide a foundation for symbolic representations of numbers and, thereby, also for arithmetic.

It is, however, difficult to see precisely how Husserl's account of symbolically given numbers is to be grounded in his account of authentically given numbers. The impossibility for us to form authentic representations of numbers larger than, say, twelve stems from the impossibility for us to form authentic representations of sets with more than that number of elements. For larger sets we are confined to symbolic representations. For instance, the set representation that we form when we enter a lecture hall and see a large audience is, in the typical case, only symbolic. Husserl provides a detailed account of such symbolic set representations (195-218). This account does, however, not play any role in Husserl's account of symbolically given numbers (contrary to what Husserl seems to suggest at 222-23). In particular, a symbolic representation of a number is not, according to Husserl's account, obtained by abstraction from a symbolically conceived set. Rather, Husserl's introduction of symbolic numbers follows an entirely different plan, with no reliance on symbolic sets or abstraction.

It is true that, at the initial stages of the introduction of symbolic numbers authentic number representations are involved. Symbolic numbers are introduced through a (potentially infinite) system of signs. Husserl prefers a system in base X that employs addition, multiplication, and exponentiation for forming numbers larger than X (228-33). Examples of numbers in this system are therefore X, $(2 \times X) + 3$ and $(3 \times X^2) + 4$ (assuming that X is greater than 4). When X is 10, these symbolic numbers are just decimal numbers written out completely. Authentic number representations are involved in the description of these symbolic numbers through the requirement that the base number X be authentically given. That is, there is some freedom in choosing X, but it has to be a number of which we can form an authentic representation.

Once in place, this decimal-like system yields, according to Husserl, a unique representative for each number in itself (*Zahl an sich*). That is, we get a unique representative for each number as it is given in an authentic representation, or rather: as it would be given in an authentic representation, if we could form such a representation (260). But it is unclear how the existence of such a one-to-one correspondence can be justified for numbers that we cannot authentically represent to ourselves. The thought is perhaps that since there is a one-to-one correspondence for each of the numbers up to X, there is one as well for any number built up from these numbers, just as the numbers in the described system are. But this is a *non sequitur*. If, for instance, we can have no authentic representation of numbers greater than twelve, then how can we possibly know that, say, $(3 \times 10^2) + (7 \times 10) + 1$ is the unique representative of a number in itself, the true three-hundred-and-seventy-one? Although the symbolic number in question is built up from numbers—3, 10, 2, 7, 10, 1—each of which may be taken to correspond uniquely to a number in itself, we have no guarantee that the same can be said of a number formed through addition, multiplication, and exponentiation from these. These operations may well take us out of the range of authentically representable numbers; and for any number outside that range, we have no way of ascertaining whether it has a unique representative in the system of symbolic numbers.

Since sufficiently large numbers are inaccessible to us, we thus seem to have no way of assessing claims regarding their relation to symbolic numbers. A similar problem affects Husserl's whole conception of numbers as abstractions from sets. If we are in principle denied knowledge of numbers sufficiently large, then what right do have to claim, for instance, that there are infinitely many numbers? For all we know, the sequence of numbers in themselves might terminate at some point outside the range of authentically representable numbers. And how can we be certain that the numbers are linearly ordered? Perhaps at some point far out in the number sequence there is a number that has two immediate successors. If only an initial segment of the number sequence is accessible to us, then we have no right to exclude the possibility of such a situation. That no such situation arises in the part of the number sequence that we do have access to is no proof that it cannot arise in some part that we do not, and indeed cannot, have access to.

There is anyway something strange in developing an account of number and operations on numbers and then go on to say, as Husserl does (190-92), that arithmetic is in fact grounded on an entirely different account of number. Indeed, Husserl thinks that if we could conceive of any number in the way the first account describes, then there would be no arithmetic, since then all relations among numbers would be immediately evident to us (191).[3] The existence of arithmetic thus shows that the first account cannot be our only account of number. The difficulty in seeing how the first account might serve as a foundation for the second account suggests, to my mind, that we might as well forget about it altogether as an account of number. What is presented in the first part of *Philosophy of Arithmetic* should be regarded as an account of finite sets, not as an account of number. We are there given a detailed account of the conception of finite sets; but the abstraction theory of number that is based on it is idle, since it cannot serve as a foundation of arithmetic.

I should emphasize that my criticism here does not concern the distinction between authentic and symbolic representations as such. This is clearly an important distinction, as witnessed by its refined reappearance in Husserl's epistemology from the *Logical Investigations* onwards as the distinction between an intention and its fulfilment. The problem in the *Philosophy of Arithmetic* is not the invocation of this distinction as such, but rather the use Husserl makes of it: he posits objects of which authentic representations are in principle excluded. He says that there are such and such objects, although we shall never be able to see them with evidence, we shall never have proper knowledge of them. The positing of such knowledge-transcendent objects is, to my mind, quite foreign to phenomenology. The phenomenological point of view is a first-person point of view, so a phenomenologist's positing of objects should always be accompanied by a description of how such objects can be given to us. Husserl does quite the opposite when he posits certain objects and, at the same time, denies that they can ever be given to us.[4]

4. We are thus led to explore the prospects of Husserl's second account of number as self-standing, without the spurious support from the account of numbers as abstractions from sets. The main tenet of the resulting philosophy of arithmetic—however it is worked out in detail—is that numbers are symbolically constituted objects. Numbers are given by a system of meaningful symbols; not, however, in the sense that the "numbers in themselves" are mirror images of these symbols—as Husserl seems to have held—but in the sense that the numbers are these very symbols. There is no number in itself apart from the meaningful symbol that you see on the page in front of you. Apart from a short remark at the beginning of section 6 below, I shall not discuss here whether this can be taken to be Husserl's own philosophy of arithmetic at any point of its development. It does, in any event, seem to me to be an eminently phenomenological philosophy of arithmetic. Working out the details will require considerable effort. Here I wish only to note some possible sources of inspiration from the second part of the *Philosophy of Arithmetic*.

Symbolically constituted numbers are, as already noted, introduced through a system of signs. Husserl considers several alternative sign systems: apart from the base-*X* system, which he prefers, also a unary system (228); a "non-systematic" system (*sic*), where numbers are given only provisional definitions in terms of auxiliary number signs and addition (224-5); and a system where the numbers are generated in the natural order, but where each number is named quite independently of the names of smaller numbers, unlike what is the case in the base-*X* and the unary system (226-7).

Since we regard symbolically constituted numbers as self-standing, we cannot follow Husserl in his preference for the base-*X* system. This system, namely, relies on the arithmetical operations of addition, multiplication, and exponentiation; and we cannot take an understanding of these for granted when we first introduce the numbers. Husserl could perhaps do so, since he took symbolic numbers to correspond to numbers as abstractions from sets, and for these he had given an account of the basic arithmetical operations (182-90). We cannot do the same, however, since we have rejected this account of number and wish to consider symbolic numbers as self-standing. (The positional system in base *X*, in which, for instance, $(3 \times X^2) + (7 \times X) + 1$ is written "371" does not avoid the reliance on addition, multiplication and exponentiation either, since it is merely an abbreviation of the more long-winded base-*X* system that Husserl employs.[5])

From the foundational point of view, a unary system must be preferred. It should, however, not be formulated as Husserl formulates it (228), in terms of successive additions of 1. The numbers would then be introduced as 1, 1+1, 1+1+1, …, so we should again be relying on addition. It is true that we here invoke only a special case of addition, namely where the second argument is 1. But addition as such is a binary function defined on all pairs of numbers, so we need to see (1+1)+1, say, as an instance of the general form $m + n$. Since it is not by this general form that the numbers are generated, we cannot regard addition as being defined simultaneously with the introduction of the numbers. Rather, the definition of addition has to wait until we have explained how the numbers are generated. For the generation of the numbers in the first place, we should rely on a successor function and the basic number 0. (These primitive notions may be regarded as being explained simultaneously with the introduction of the numbers.) The numbers are thus generated as 0, **s**(0), **s**(**s**(0)), **s**(**s**(**s**(0)))… Addition, multiplication, and exponentiation can then later be defined by well-known recursion equations.[6]

5. We thus take numbers to be introduced as 0, **s**(0), **s**(**s**(0)), **s**(**s**(**s**(0)))… It is, however, obvious that not all numbers are given in this unary form. The number 371 in the decimal positional system, for instance, is not so given, nor is, say, 7 × 5. Clearly, we do not want to be forced to say that these are not numbers. They do not look like numbers as introduced by the unary system, but they are numbers nevertheless. An important idea in the second part of the *Philosophy of Arithmetic* is the distinction between what Husserl calls normal and problematic numbers (261). Normal numbers are numbers in introductory form, whereas problematic numbers are numbers in non-introductory form. Employing this terminology we can say that 371 and 7 × 5 are indeed numbers, but problematic numbers. They are not called problematic because their status as numbers is somehow problematic. Rather, they are called problematic because each poses—or, better, *is*—a problem, or a task (*Aufgabe*), namely that of reduction to normal form. In particular, each of the numbers 371 and 7 × 5 is a task of reduction to a number in the form of 0 followed by some number of iterations of the successor function.

Husserl's immediate aim in introducing the distinction between normal and problematic numbers is to be able to say when two symbolically given numbers are identical. It is natural to stipulate that not only any problematic number, but also any normal number is a task: a normal number is the trivial task that is solved by itself. In terms of reduction to normal form, we thus stipulate that a normal number reduces to itself. As a consequence of this stipulation, it will make sense to speak, for any number, of its reduction to normal form. Two symbolically constituted numbers can then be said to be identical if they reduce to the same normal number. For instance, the numbers 7 × 5, 27 + 8, 35 and $\mathbf{s}^{35}(0)$ are all identical to each other, since each reduces to $\mathbf{s}^{35}(0)$ that is, 0 followed by thirty-five iterations of the successor function.

Contained in these stipulations are both a criterion of application and a criterion of identity for numbers. Recall that a criterion of application associated with a concept *C* determines what it is for an object to fall under *C*; and that a criterion of identity associated with *C* determines what it is for objects falling under *C* to be identical.[7] We take numbers to be introduced as 0, **s**(0), **s**(**s**(0)), **s**(**s**(**s**(0)))… These are the normal numbers. By the introduction of numbers we thus know what a normal number is. A number quite generally can then be said to be a task of reduction to a normal number. This is the criterion of application for numbers. The criterion of identity says that numbers are identical if they reduce to one and the same normal number.

These criteria of application and identity for numbers agree with those given by Per Martin-Löf as part of the so-called meaning explanations for his constructive type theory (Martin-Löf 1984). The numbers—or, more precisely, the natural numbers—are there an instance of the more general concept of a type of individuals. A type *A* of individuals is defined by laying down how the elements of normal form of that type are constructed. Thus, one defines a type of individuals by displaying a mode of generation of its normal-form elements, in a manner similar to how numbers were introduced above as 0, **s**(0), **s**(**s**(0)), **s**(**s**(**s**(0)))… In this context elements of normal form are usually called "canonical elements". From the definition of *A* we thus know what the canonical elements of *A* are. The criterion of application for *A* is then formulated as follows:

> an element *a* of a set *A* is a method (or program) which, when executed, yields a canonical element of *A* as result (Martin-Löf 1984, 9)

A canonical element of a set is a method that yields itself as result when executed, hence any canonical element of *A* is also an element of *A* according to this criterion. The criterion of identity for *A* is formulated as follows:

> two arbitrary elements *a*, *b* of a set *A* are equal if, when executed, *a* and *b* yield equal canonical elements of *A* as results (ibid.).

The similarity between these explanations and those we extracted above from the *Philosophy of Arithmetic* should be obvious. We took a number to be a problem, or task, of reduction to normal form, and we may think of such a problem as a programme which, when executed, yields a normal-form number; and we took numbers to be identical if they reduce to the same normal-form number, that is, the same canonical number. This similarity may not be a coincidence: the thesis that mathematical objects quite generally (not only numbers) are symbolically constituted is a philosophy of mathematics that is quite congenial to the spirit of constructive type theory.

The task presented by a number is solved by calculation. Thus the reduction of 7×5 to $\mathbf{s}^{35}(0)$ is just the calculation of 7×5, a calculation whose result is $\mathbf{s}^{35}(0)$. Calculation itself may be regarded as the unravelling of definitions. For instance, 7 is defined as $\mathbf{s}(6)$, 6 is defined as $\mathbf{s}(5)$, 5 is defined as $\mathbf{s}(4)$, etc. Moreover, we have definitions of functions such as addition, multiplication, and exponentiation. Employing these definitions we reduce a number by continued substitution of *definiens* for *definiendum*.[8] Thus, from the defining equations of multiplication we find that $7 \times \mathbf{s}(4)$ reduces to $(7 \times 4) +7$, and from the defining equations of addition that $(7 \times 4) + \mathbf{s}(6)$ reduces to $\mathbf{s}((7 \times 4) + 6$. Likewise we find that $(7 \times 4) + 6$ reduces to $\mathbf{s}((7 \times 4) + 5)$. Continuing this procedure we shall eventually reach $\mathbf{s}^{35}(0)$. The conception of calculation as the unravelling of definitions, later made precise by Kleene, Curry, Martin-Löf and others,[9] is not quite what one finds in the *Philosophy of Arithmetic*; but it can be found in *Logical Investigations* VI §18 (Husserl 1901), where Husserl describes in some detail the reduction of the number $(5^3)^4$ to unary form. (At this point, therefore, Husserl seems to prefer the unary form as the normal form; indeed, in the cited section he in effect says that a decimal number is a task of reduction to unary form.)

6. The section of the *Logical Investigations* just cited is part of a discussion of the notions of intention and fulfilment. Husserl suggests that we may regard the substitution of *definiens* for *definiendum* as a step of partial fulfilment;

complete fulfilment is then reached with what Curry calls the ultimate *definiens*, an expression that can no longer be reduced. Given the understanding of such a process of substitution as a calculation, this section of the *Investigations* thus suggests that we think of the relation between a problematic and a normal number as the relation between an intention and its complete fulfilment. In the theory of fulfilment in the *Investigations* complete fulfilment is achieved by the presence of the intended object itself. And indeed, in the cited section Husserl speaks of the end result of the process of substitution as the "number itself". Here, therefore, the number itself is just the meaningful expression that is the number in its unary form, quite in line with the doctrine currently being explored. Normal numbers have thus taken over the role of numbers in themselves. There is, for instance, nothing beyond the meaningful symbol **s**(**s**(**s**(0))) that is the number three itself. Rock bottom has been reached already with this normal form.

It was already noted that the pair of notions of intention and fulfilment has a precursor in the pair of notions of inauthentic, or symbolic, and authentic representations. That we may think of the relation between a problematic number and the normal number to which it reduces in terms of the former pair of notions suggests that we may also think of it in terms of the latter pair of notions. For Husserl, of course, a symbolically constituted number cannot be authentically represented: for him, only numbers as abstractions from (authentically represented) sets can be so represented. But, for us, since we here take normal-form numbers to play the role of numbers in themselves, it is natural to say that such numbers are authentically given, whereas numbers in problematic form are inauthentically given. That is, it is natural to say, for instance, that the number **s**(**s**(**s**(0))) is here authentically given, whereas 2 + 1 is the same number inauthentically given.

We then recognize a phenomenon that was central in Husserl's original employment of this terminology: of sufficiently large numbers it is physically impossible for us to achieve an authentic representation, though we shall always be able to construct an inauthentic representation. Take, for instance, the number $10^{10^{10}}$. We have no choice but to present this number inauthentically, since an authentic representation of it, consisting of a sequence of **s**'s appended to 0, is out of reach for us.[10] (This point does not depend on the use of the unary system: when numbers get sufficiently large, we cannot write them out in the base-X positional system either.) That an authentic representation of $10^{10^{10}}$ is out of reach for us is, however, not to say that we need to withhold judgement as to whether this number in fact exists. The number $10^{10^{10}}$ is a well-defined problem, we know precisely what it means to calculate it. And given our criterion of application for numbers, this is all that is required for us to have the right to say that this is indeed a number, that this number exists. The contention that sufficiently large numbers can only be inauthentically represented therefore does not, for us, lead to any doubts regarding the existence of large numbers, as it does in Husserl's original theory.

7. A final pair of notions that we shall invoke in elucidating the relation between problematic and normal numbers is the pair of sense and reference. It has been noticed by several readers of (Frege 1892) that the relation between the sense of an expression and its reference may, at least in some cases, be understood as the relation between a programme, or task, and the result of its execution.[11] Frege himself suggests this idea in a *Nachlass* piece:

> "4", "2^2", "$(-2)^2$" are only different signs for the same, whose difference merely indicates the different ways along which one may reach the same thing. (Frege 1983, 95)

Thus, different signs for the same thing indicate different ways leading to that thing. But a way leading to a certain goal is just what a programme or method is.[12] In light of the sense/reference-distinction, the quoted passage can be read as saying that "4", "2^2", "$(-2)^2$" express different senses, whose difference indicates different ways of reaching the same reference. Indeed, the root of the words "*Sinn*" and "sense" has to do with locomotion and can mean way or route.[13] A remnant of this root meaning can be seen when these words are used today to mean direction. Taking advantage of this etymology, we might say that the sense of an expression charts out a route to its reference. In a similar way, a problematic number, being a task, or a programme, can be said to chart out a route to a normal number, namely the route that we follow when solving the task, or executing the programme. It is therefore natural to say that a problematic number stands to the normal number to which it reduces as sense stands to reference. The fact that syntactically different problematic numbers may evaluate to the same normal-form number then reflects the fact that different senses may determine the same reference.

The reduction of a problematic number to its normal form can be represented by a sequence of numbers in which each element is obtained from the previous one by reduction on the basis of definitions. For instance, the reduction of 2 + 1 to $\mathbf{s}^3(0)$ may be represented as the sequence

$$(2 + 1) \rightarrow (2 + \mathbf{s}(0)) \rightarrow \mathbf{s}(2 + 0) \rightarrow \mathbf{s}(2) \rightarrow \mathbf{s}(\mathbf{s}(1)) \rightarrow \mathbf{s}(\mathbf{s}(\mathbf{s}(0)))$$

Although all elements of this sequence are identical, they are also all of them syntactically different from each other. In the terminology of sense and reference we may say that we here have different senses determining one and the same reference. Frege would certainly say that this reference is an object—the number three—that exists apart from this sequence of senses. For us this is not so. For us, the object in question is the final element of the sequence, viz. $\mathbf{s}(\mathbf{s}(\mathbf{s}(0)))$. The reference in this case thus resides at the same level as the senses determining it. Our adoption of the sense/reference-terminology was suggested by the understanding of the sense of an expression as a programme the result of executing which is the corresponding reference. According to our criterion of application for numbers, any number is a programme. A normal-form number, in particular, is a programme the result of executing which is itself. It must therefore be regarded as a sense that determines itself as reference. In a normal-form number we thus have a collapse of sense and reference.

8. It should be clear by now that the philosophy of arithmetic that has been explored here is not the doctrine that the objects of arithmetic are meaningless signs, a view Husserl himself criticizes in the *Philosophy of Arithmetic* (e.g. 170-178). The doctrine explored is rather that the objects of arithmetic are meaningful symbols. Unlike objects such as trees, books, and people, numbers are symbolically constituted: a number is nothing apart from the meaningful expression that presents it; the number is there in the expression. We can of course consider an expression as a formal entity devoid of sense, but in that case, what we see is not a number, but a certain "formal object" (to use another piece of terminology from Curry). In the usual object-directed attitude, by contrast, the expression is regarded as meaningful, and then we see, for instance, the number 7 × 5. What the meaning of this expression is we in effect specified when giving the criterion of application for numbers: the meaning is specified by saying that 7 × 5 is a programme for obtaining a number in normal form. That this is indeed a notion of meaning was brought out by our invoking Frege's sense/reference-distinction to illuminate the relation between a programme and the normal number that it yields upon calculation.

A natural question now is whether the view explored here can be extended to all of mathematics: can we say that not only the objects of arithmetic, but in fact all mathematical objects are symbolically constituted? What we cannot say, it seems, is that all mathematical objects are senses as explicated here, viz. programmes of reduction to normal form; for it should not be expected that all mathematical objects can be regarded as such programmes. In particular, a doubt may arise concerning functions. A programme as understood here is something that can be calculated, or reduced, to normal form. We do, however, not calculate a function such as the multiplication function, ×, in isolation. What is calculated is rather 7 × 5 or 3 × 10 or any other result of supplying the multiplication function with two numbers as arguments.

Although a function is thus not a programme in the specified sense, that is, an object of calculation, and therefore not a sense as explicated here, we may nevertheless regard it as a meaningful symbol. For instance, we can take the meaning of × to consist in the fact that when supplied with two numbers as arguments we get a programme that can be calculated to normal form, say by relying on the definition of expressions of the form $m \times n$. Assuming that we have explained what it is to be a function quite generally in some such way,[14] then we should have made important steps towards the justification of viewing functions as symbolically constituted objects. That in turn would be an important step towards extending the work Husserl began in the second part of the *Philosophy of Arithmetic* to a general philosophy of mathematics.

NOTES

[1] All page references given without any further specification are to the *Husserliana* edition of the *Philosophy of Arithmetic* (Husserl 1891).
[2] For a more detailed overview of the contents of the *Philosophy of Arithmetic*, see (Centrone 2010, ch. 1).
[3] This claim is stronger than the claim that there is no arithmetic for an infinite intellect; for it is here presupposed only that *any* number is authentically conceivable, not that they are all authentically conceivable at once.
[4] A conclusion similar to that reached here is also reached in (Miller 1982, 77): “Large numbers would seem to have no being whatsoever, if the act which is their unity cannot be performed.”
[5] Husserl’s preference for the more long-winded system stems, probably, not only from the fact that it is more transparent, but also from the fact that the positional system presupposes 1 and 0: from Husserl’s point of view these are not unproblematically called numbers (129-34).
[6] The need for using a separate successor function rather than the special case of addition, $m + 1$, was seen by Dedekind. In (Dedekind 1888), where the generation of the numbers by means of the successor function is made precise (albeit by the use of impredicative methods), one also finds recursive definitions of addition, multiplication, and exponentiation. Husserl had read this work, was impressed by its rigour, but found that “in its strange artificiality, it strays far from the truth” (125).
[7] A concept with which both a criterion of application and a criterion of identity are associated is usually called a *sortal* concept. The notion of a criterion of identity is often traced back to (Frege 1884, §62). The term “criterion of application” stems from (Dummett 1973, 74).
[8] If the definition of a function is given in terms of variables, then besides substitution we also a need to rely on instantiation. For instance, from the definitional equation $x + \mathbf{s}(y) = \mathbf{s}(x + y)$ we get $7 + \mathbf{s}(4) = \mathbf{s}(7 + 4)$ by instantiation.
[9] See (Kleene 1952, esp. §54) and (Curry and Feys 1958, esp. ch. 2E).
[10] Consider the problem of printing this many **s**’s. A quick calculation shows that if we print, say, 1000 **s**’s per second, then a conservative estimate of the number of years required is the number written in decimal notation as 1

followed by a milliard zeros. In comparison, the number of years written in decimal notation as 1 followed by 11 zeros is already larger than the estimated age of the universe.

[11] See, for instance, (Dummett 1991, 123), stemming from lectures given in 1976, or (Tichý 1988). An unpublished lecture by Martin-Löf given in 2001 should also be mentioned, since it has been important for the current presentation.

[12] The Greek word *methodos* is a combination of *meta* and *hodos*, and the primary meaning of this latter word is way or route.

[13] See Pokorny's *Indogermanisches etymologisches Wörterbuch* s.v. "sent-"

[14] A part of the general explanation would be: *f* is a function if *f*(*n*) is a number whenever *n* is a number. This explanation presupposes that all functions are total, namely that *f*(*n*) is always a programme that in fact yields a normal number upon evaluation, whichever number *n* might be. It would therefore not work for Husserl's conception of functions (or operations, in his terminology) if it is right, as Centrone (2010) has stressed, that Husserl allows partial functions.

REFERENCES

Centrone, Stefania. 2010. *Logic and Philosophy of Mathematics in the Early Husserl*. Dordrecht: Springer.

Curry, Haskell and Robert Feys. 1958. *Combinatory Logic*. Volume 1. Amsterdam: North-Holland.

Dedekind, Richard. 1888. *Was sind und was sollen die Zahlen?* Braunschweig: Vieweg.

Dummett, Michael. 1973. *Frege: Philosophy of Language*. London: Duckworth.

________. 1991. *The Logical Basis of Metaphysics*. London: Duckworth.

Frege, Gottlob. 1884. *Grundlagen der Arithmetik*. Breslau: Wilhelm Koebner.

________. 1892. "Sinn und Bedeutung." *Zeitschrift für Philosophie und philosophische Kritik* 100: 25-50.

________. 1983. *Nachgelassene Schriften*. Hamburg: Felix Meiner.

Husserl, Edmund. 1891. *Philosophie der Arithmetik*. Halle: Pfeffer. Page references are to Husserliana volume XII, edited by Lothar Eley. The Hague: Martinus Nijhoff.

________. 1901. *Logische Untersuchungen. Zweiter Teil*. Halle: Niemeyer.

Johnson, W. E. 1921. *Logic. Part I.* Cambridge: Cambridge University Press.

Kleene, Stephen Cole. 1952. *Introduction to Metamathematics*. New York: Van Nostrand.

Miller, J. Philip. 1982. *Numbers in Presence and Absence: A Study of Husserl's Philosophy of Mathematics*. The Hague: Martinus Nijhoff.

Martin-Löf, Per. 1984. *Intuitionistic Type Theory*. Naples: Bibliopolis.

Tichý, Pavel. 1988. *The Foundations of Frege's Logic*. Berlin: Walter de Gruyter.

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Le schématisme transcendantal dans l'arithmétique : la lecture richirienne de Frege

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Abstract

Transcendental Schematism in Arithmetic: On Richir's Reading of Frege

This article shows how the contemporary phenomenologist Marc Richir developed his reflection on the foundation of arithmetic. Despite Frege's criticism of the Kantian thesis of arithmetic, Richir discovers, in his reading of Frege's logicist foundation for arithmetic, a key to rediscovering the Kantian conception of the number as a transcendental schema of quantity (*quantitas*). We begin this presentation by considering the background of the problem through examining the Kantian and Husserlian notions of the number. At the same time, we show the fundamental difference between Husserl's and Richir's phenomenologies by referring to the problem of the intuition of the infinite. Secondly, we show the key points of Richir's reading of Frege's work, *The Foundation of Arithmetic* (1884), which are developed in Richir's article 'Heredity and Numbers' (1983). The impossibility of the intuition of the infinite, which is, for Frege, one of the examples that attest to the impossibility of founding the number's existence on intuition, proves, for Richir, the impossibility of the thoroughgoing determination of the elements of an infinite set. Departing from this premise, Richir discovers, between the lines of Frege, an undeclared phenomenological foundation of arithmetic.

Keywords: thoroughgoing determination, the transcendental ideal, transcendental schema, zero, the infinite

La phénoménologie est étroitement liée à la considération du fondement de l'arithmétique depuis sa naissance. Dans cet article, nous montrerons comment Marc Richir, un phénoménologue francophone contemporain, prend

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en considération la fondation phénoménologique de l'arithmétique. Il reprend dans son travail le concept kantien du nombre comme schème transcendantal de la quantité (*quantitas*[1]). Bien que cette conception ait été l'objet de la critique émise par Frege, Richir dégage, dans sa lecture de la tentative fregéenne de fonder logiquement l'arithmétique, une clé pour redécouvrir la fondation kantienne de l'arithmétique.

Notre démonstration sera menée en deux étapes. Premièrement, nous reviendrons sur l'arrière-plan de notre problématique en présentant la notion de nombre chez Kant et chez Husserl respectivement. Nous démontrerons également la différence entre la phénoménologie de Husserl et celle de Richir en prenant comme points de repère la problématique de l'intuition de l'infini et celle de la détermination complète (*durchgängige Bestimmung*) des éléments de l'ensemble infini. Deuxièmement, après avoir démontré ces points, nous présenterons les points clés de la lecture richirienne de l'œuvre de Frege, *Die Grundlagen der Arithmetik* (Frege 1884), déployée dans son article « L'Hérédité et les nombres » (Richir 1983).

L'impossibilité de l'intuition de l'infini est, chez Frege, l'un des exemples qui attestent de l'impossibilité de fonder l'existence des nombres à partir de l'intuition. Néanmoins, chez Richir, cette même impossibilité va de pair avec celle de la détermination complète des éléments de l'ensemble infini. En partant de cette prémisse différente de celle de Frege, Richir dégage d'entre les mots de Frege une fondation phénoménologique inavouée.

1. L'arrière-plan : le nombre comme schème transcendantal de la quantité

L'une des révolutions qu'introduit Kant dans sa philosophie transcendantale consiste à considérer la proposition arithmétique comme jugement synthétique *a priori*, écartant à la fois la psychologie empirique qui l'expliquerait comme jugement synthétique *a posteriori*, et la logique formelle qui la tiendrait pour jugement analytique *a priori*. Ces deux extrêmes mis à l'écart ne peuvent pas donner une explication suffisante pour la fondation de l'arithmétique : la première (la psychologie empirique) n'arrivant pas à expliquer l'universalité de l'arithmétique, la deuxième (la logique formelle) ne réussissant pas à expliquer l'élargissement de la connaissance ayant lieu dans le jugement arithmétique.

Afin de montrer comment la proposition arithmétique s'établit comme jugement synthétique *a priori*, Kant a recours à son schématisme transcendantal. Néanmoins, le chapitre du schématisme transcendantal dans la *Critique de la raison pure* n'a pas été facilement accepté par ses lecteurs de l'époque ultérieure : Mill propose de retourner à la psychologie empirique, d'une part, et Frege soutient qu'il faut retourner à la logique formelle, d'autre part. L'une des raisons de ces deux sortes de retour consisterait en le fait que le schématisme transcendantal que développe Kant reste obscur à leurs yeux.

Nous commencerons donc par démontrer en quoi consiste le schème transcendantal de la quantité. Le schème, un produit de l'imagination, est l'une des conditions de possibilité de la connaissance, qui permet à la fois « de rendre sensibles ses concepts » et « de se rendre intelligibles ses intuitions » (Kant 2006, 144 ; Kant 1911, 1904/1911, A51/B75) et ainsi applique les concepts aux intuitions. Dans le cas du nombre, celui-ci permet d'appliquer les concepts de la quantité (*quantitas*) à une forme de l'intuition qui est le temps (Kant 2006, 227 ; Kant 1911, 1904/1911, A142-143/B182 ; Kant 1911, *Prol*. 283). Le nombre joue le rôle d'intermédiaire entre les catégories de la quantité et le temps.

Que ce schème soit « transcendantal » signifie qu'il est lié aux concepts purs de l'entendement. Le schématisme transcendantal est la fonction de médiatiser les catégories et les phénomènes (Kant 2006, 224 ; Kant 1911, 1904/1911, A138/B177). Les catégories de la quantité – l'unité, la pluralité et la totalité – ne sont que les cadres vides de la pensée, elles ne peuvent en tant que tels être appliquées aux phénomènes sans intervention du schématisme transcendantal qui les rend sensibles (Kant 2006, 228-229 ; Kant 1911, 1904/1911, A145/B184).

Kant présente le schématisme transcendantal de la quantité comme une « série du temps » (*Zeitreihe*). L'unité, la pluralité et la totalité ne deviennent sensibles que si elles sont liées à une forme de l'intuition qu'est le temps, à travers la suite du temps. L'acte de compter est donc l'acte de produire le temps comme une suite du temps.

Néanmoins, cette description du schématisme transcendantal par Kant n'était pas suffisamment claire pour de nombreux héritiers de sa philosophie. Tout en donnant son consentement à la thèse kantienne selon laquelle la proposition de la géométrie est un jugement synthétique *a priori*, Frege refuse d'accepter une thèse parallèle selon laquelle la proposition arithmétique l'est aussi. Le logiciste critique non seulement le psychologisme, mais aussi le schématisme kantien qui tente de surmonter l'empirisme, en argumentant qu'il est incompréhensible sans impliquer l'aspect empirique (Frege 1969, 129 ; Frege 1884, 6, §5).

Dans cette critique du psychologisme empirique, Frege dénonce trois apories qu'il attribue à l'argument psychologiste du fondement de l'arithmétique selon lequel le nombre naît de l'abstraction psychique des objets intuitionnés.

La première aporie consiste en l'incompatibilité de la différence (*Verschiedenheit*) et de l'identité (*Gleichheit*). D'une part, une simple accumulation d'objets différents ne donne pas un nombre. L'acte de compter suppose que chaque objet compté apparaisse comme identique. D'autre part, une accumulation de choses identiques ne donne pas non plus un nombre. S'il y a trois monnaies et si elles sont identiques – à savoir, si elles ont les mêmes propriétés et occupent un temps et un espace identiques –, nous les comptons comme 1 et non 3. C'est un non-sens de compter ceux qui sont identiques.

La deuxième aporie est celle de zéro et de 1. Zéro et 1 ne sont pas la pluralité. Si l'on fait abstraction de la lune, on pourrait en tirer « satellite de la Terre » ou « satellite d'une planète », mais pas le nombre 1. Dans le cas de zéro, il n'y a même pas de quoi extraire une abstraction.

La troisième aporie est celle des grands nombres ou de l'infini. Nous pourrions avoir l'intuition des trois chaises, mais il nous serait impossible d'avoir l'intuition de 50 chaises. S'il y avait des chaises de nombre infini, il est évident que cela dépasserait notre capacité d'intuition sensible. Frege considère que ces trois apories peuvent être balayées par la position logiciste. Néanmoins, en intervenant dans cette polémique philosophique autour du fondement de l'arithmétique, Husserl critique dans la *Philosophie de l'arithmétique* (Husserl 1970) à la fois Kant, Mill et Frege. Selon notre interprétation pourtant, la thèse que soutient Husserl hérite d'une manière essentielle de celle de Kant en ce qu'elle considère comme deux écueils le psychologisme (de Mill) et le logicisme (de Frege). Nous l'expliquerons.

Husserl critique Kant qui lie le nombre au temps, de même qu'il critique Lange qui l'associe à l'espace (Lange 1894, 138 *et seq.*). Selon Husserl, si le nombre était quelque chose de fondé sur le temps ou sur l'espace, l'ordre temporel ou spatial exercerait son influence sur la constitution du nombre. Néanmoins, ce qui est trois est toujours trois, que l'on compte de gauche à droite, de droite à gauche, du temps précédent au temps ultérieur ou du temps ultérieur à temps précédent. Cela attesterait de l'indépendance du nombre, du temps et de l'espace (Husserl 1970, 31 *et seq.*, 36 *et seq.).*

Tout en ayant ainsi critiqué la thèse kantienne selon laquelle le nombre est la « série du temps », Husserl hérite, selon notre interprétation, de l'enseignement essentiel de Kant selon lequel les propositions arithmétiques sont des jugements synthétiques *a priori*. Cela est perceptible lorsque Husserl définit l'acte de compter comme l'acte de lier quelque chose (*Etwas*) à quelque chose (*Etwas*) : comme une liaison collective (*kollektive Verbindung*). Le nombre est constitué par cette liaison par « et » (*und*), sans que cette liaison ne dépende du contenu des « quelque choses » (Husserl 1970, 79 ; Derrida 1990, 65, n. 26). Dans cette conception husserlienne, l'aporie première que Frege met en lumière ne pose pas de problème. L'acte de la pensée de lier les termes constitue un nombre indifféremment de si ceux-ci sont identiques ou différents au niveau de leurs contenus.

Husserl donne également la solution à la deuxième aporie de Frege (l'aporie de zéro et de 1) ainsi qu'à la troisième (l'aporie des nombres grands et de l'infini), en introduisant la distinction entre les nombres propres (*eigentliche Zahl*) et symboliques ou celle des nombres fondés sur l'acte de penser et ceux qui en sont déduits ultérieurement (Husserl 1970, 133 *et seq.*).

De là, bien que Husserl finisse *La philosophie de l'arithmétique* en un seul tome, sans y ajouter la publication du deuxième tome qu'il avait préparé, nous pouvons y découvrir une esquisse de la fondation phénoménologique de l'arithmétique. Il s'agit du changement de l'intérêt qui s'est orienté aux contenus des objets comptés, vers l'intérêt à « et » que l'acte de penser attribue en liant les objets. Nous pouvons considérer – bien que Husserl ne le précise pas ainsi – que c'est une fonction de l'imagination qui entrevoit derrière des objets comptés un « quelque chose » absolument formel et eidétique.

Ainsi, Husserl semble nous annoncer la notion ultérieure de variation imaginative qui lui permettra de développer la notion de l'intuition eidétique. Ce « quelque chose », quoique Husserl n'emprunte pas ce terme, n'est autre qu'une sorte de schème transcendantal, produit de l'imagination apriorique, sans que ce schème ne soit lié au temps.

Néanmoins, après la critique de Frege émise dans sa recension de la *Philosophie de l'arithmétique*, Husserl n'a laissé aucune œuvre ultérieure concernant la fondation de l'arithmétique, du moins sous forme d'œuvres publiées. D'où surgit la question : comment la phénoménologie peut-elle répondre, après Husserl, à la tentative logiciste de Frege ? C'est afin de répondre à cette question que nous nous dirigeons vers Richir. N'oublions pas cependant que Richir n'est pas un héritier soi-disant orthodoxe de Husserl. Tout en estimant très hautement Husserl, Richir considère que la phénoménologie de ce dernier commet l'erreur de tomber dans l'apparence transcendantale prévenue par Kant dans sa *Dialectique transcendantale* (Richir 2004). Comme c'est les cas d'Aristote et de Kant, Richir considère que l'intuition de l'infini actuel est impossible.

Nous l'expliquons. Dans le chapitre de « L'idéal transcendantal » dans la *Dialectique transcendantale*, Kant traite de la problématique de l'apparence transcendantale de la connaissance de l'infini, en rappelant le principe de « *omnimoda determinatio* » chez Leibniz (2002, 18-21). Il s'agit du principe selon lequel « *toute chose existante est intégralement déterminée* (alles Existierende ist durchgängig bestimmt) » (Kant 2006, 518 ; Kant 1911, 1904/1911, A573/B601).

Kant évoque ce principe comme un indice qui permet de distinguer la détermination logique d'un concept et la détermination ontologique d'une chose. Pour qu'un concept soit possible, il suffit qu'il ne contienne pas de contradiction. Pourtant, pour qu'une chose existe, non seulement il est nécessaire qu'elle ne contienne pas de contradiction, mais aussi qu'elle soit soumise au principe de la détermination intégrale (Kant 2006, 518 ; Kant 1911, 1904/1911, A571/B599). Qu'une chose existe implique donc que tous ses attributs et leurs degrés sont intégralement déterminés.

Ce qui fonctionne comme mesure de cette détermination n'est autre que l'idéal transcendantal, *Urwesen* (archi-essence) qui contient en soi le degré parfait de tous les attributs possibles dans la simplicité, car sans cette mesure qui concrétise la perfection, il serait impossible de déterminer les attributs ou les degrés d'une chose. Kant dit : « Tout comme l'*idée* fournit la règle, l'*idéal* sert […] de prototype pour la détermination complète de la copie » (Kant 2006, 517 ; Kant 1911, 1904/1911, A569/B597). En d'autres termes, l'idéal sert de « mesure indispensable » « pour pouvoir apprécier et mesurer d'après lui le degré et le défaut de ce qui est imparfait » (ibid.). Ainsi, Kant montre que l'idéal transcendantal n'est autre que la présupposition transcendantale de la détermination intégrale d'une chose. Il est donc pensable mais non intuitionnable. La raison humaine ne peut donc pas affirmer qu'il existe.

L'impossibilité de l'intuition de l'infini actuel rend en même temps impossible, pour Richir, la détermination complète des éléments de l'ensemble infini. C'est une différence essentielle à l'égard de Husserl qui reconnaît l'intuition de l'infini actuel, bien qu'il s'agisse de l'intuition catégoriale distinguée de l'intuition sensible. Ainsi avons-nous confirmé l'arrière-plan de notre problématique.

2. La lecture richirienne de la tentative fregéenne de fondement de l'arithmétique

Après avoir parcouru cet arrière-plan, nous voudrions dès à présent monter comment Richir lit Frege et quelles conséquences il en tire. Pour commencer, il nous faudrait prêter encore une fois attention au fait que Richir et Frege partagent la même prémisse selon laquelle l'intuition de l'infini est impossible (la troisième aporie indiquée ci-dessus). Néanmoins, à partir de cette même prémisse, les deux philosophes se dissocient pour s'opposer l'un à l'autre : Frege en conclut la thèse selon laquelle le nombre existe sans s'appuyer sur aucune intuition. Contrairement à Richir qui conçoit que l'impossibilité de l'intuition de l'infini est inséparable de l'impossibilité de la détermination complète des éléments de l'ensemble infini, Frege soutient l'impossibilité de l'intuition de l'infini et *en même temps* la possibilité de la détermination complète des éléments de l'ensemble infini. C'est ce point qui devient problématique dans la lecture richirienne de Frege.[2] Nous suivrons donc l'article « L'Hérédité et les nombres » dans lequel Richir examine la tentative fregéenne de fondement logiciste du nombre déployée dans *Les fondements de l'arithmétique*.

i. La détermination complète des éléments de l'ensemble infini présupposée dans la tentative logiciste fregéenne de fonder l'arithmétique

Afin de montrer que le nombre existe indépendamment de l'intuition et que la proposition de l'arithmétique se justifie sans être fondée sur la connaissance de la subjectivité, Frege aborde une considération hautement innovatrice du nombre : l'extension du concept.

Ordinairement, nous avons tendance à penser qu'il faut d'abord définir ce qu'est le nombre afin de comprendre ce que signifie le fait que deux nombres soient égaux. Frege prend la voie inverse : c'est en partant de la définition du fait que deux nombres soient égaux que l'on peut obtenir la définition du nombre ou, plus exactement, en partant de la définition du fait que deux concepts sont équinumériques que l'on peut définir le nombre : « ce concept [le concept de nombre cardinal] doit recevoir sa détermination de notre définition de l'identité de deux nombres » (Frege 1969, 189 ; Frege 1884, 74, §63). Cette définition est donnée ainsi : « […] je dirai que le concept *F* est *équinumérique* au concept G si nous sommes en possession d'une telle correspondance [correspondance biunivoque] » (Frege 1969, 194 ; Frege 1884, 79, §68). « Je définis donc : le nombre qui appartient au concept *F* est l'extension du concept : "équinumérique au concept *F*" » (Frege 1969, 194 ; Frege 1884, 79-80, §68).

En suivant cette présentation de Frege, Richir considère comme problématique le fait que Frege présuppose comme si nous connaissions en quoi consiste l'extension du concept.

> La difficulté est qu'il faut présupposer les extensions de concepts, c'est-à-dire ce fait que les classes d'objets subsumés par *F* ou *G* soient toujours déjà constituées comme classes d'individus toujours déjà déterminés – comme si les concepts permettaient d'effectuer des découpages déterminés dans un domaine toujours déjà constitué d'individus […]. (Richir 1983, 82)

Ici, il met le doigt sur le fait que Frege présuppose la possibilité de la détermination complète des éléments qui appartiennent à l'extension du concept. Cette présupposition peut être justifiée si l'extension forme un ensemble fini, mais elle peut ne pas se justifier s'il s'agit d'un ensemble infini. Comme nous avons vu plus haut, la présupposition de la détermination complète implique la présupposition de l'idéal transcendantal qui est une apparence transcendantale. Selon Richir, ceci provoque des paradoxes dont l'un est le paradoxe de Russell.

Reprenons le texte de Frege. Après avoir défini le nombre comme extension du concept, Frege définit le concept de relation. Dans une phrase, nous pouvons dissocier les concepts, d'une part, et les complémentaires des concepts, à savoir les relations qui lient les concepts, d'autre part (Frege 1969, 196 ; Frege 1884, 82, §70). Frege explique que celles-ci appartiennent à « la logique pure », puisqu'elles sont totalement indépendantes de l'intuition (Frege 1969, 196 ; Frege 1884, 83, §70). Il peut ainsi formuler, en utilisant ce « concept de relation », comme suit :

> [L]'expression : « le concept *F* est équinumérique au concept *G* » a [la] même signification que l'expression : « il existe une relation Φ qui associe biunivoquement les objets qui tombent sous le concept *F* et les objets qui tombent sous le concept G ». (Frege 1969, 198 ; Frege 1884, 85, §72)

Après avoir ainsi exposé les conceptions basiques, Frege aborde ensuite la définition du zéro qui est, elle aussi, totalement indépendante de l'intuition : « Puisque rien ne tombe sous le concept : "non identique à soi-même", je pose par définition : 0 est le nombre cardinal qui appartient au concept "non identique à soi-même" » (Frege 1969, 200 ; Frege 1884, 87, §74). Quel que soit l'objet dont il s'agit, il est évident, par le principe de non-contradiction, qu'il est impossible qu'il soit non-identique à soi-même. Ainsi, il n'y a aucun objet qui appartienne à ce concept.

Richir discerne dans cette définition de zéro un certain passage : dès que l'on définit comme zéro le nombre qui revient au concept « non-identique à soi », on pose l'extension de ce concept comme l'ensemble vide. L'ensemble vide est ainsi posé :

> Cette non-identité à soi caractérisant le vide ou le Rien est elle-même identifiée à soi comme le Rien ou le vide d'une extension dont le concept est le nombre zéro en tant que concept découpant, dans le domaine de tous les concepts possibles, les concepts dont l'extension est vide - par où le nombre zéro devient lui-même un individu identique à soi. (Richir 1983, 87)

Nous confirmons ici que Richir saisit l'individuation du nombre zéro comme identification du soi (au niveau de l'ensemble) à travers sa non-identité à soi (au niveau du contenu qui appartient à cet ensemble). Dès que l'on définit comme « zéro » le nombre qui revient au concept « non-identique à soi », ce « zéro » s'individualise comme « zéro » (identique à lui-même).

Après avoir défini le zéro, Frege aborde la définition de la succession immédiate dans la suite naturelle des nombres.

> « [I]l existe un concept *F* et un objet *x* qui tombe sous ce concept tels que le nombre cardinal qui appartient à ce concept est *n* et que le nombre cardinal qui appartient au concept "qui tombe sous *F* mais n'est pas identique à *x* est *m*". » veut dire la même chose que « *n* suit immédiatement *m* dans la suite naturelle des nombres. ». (Frege 1969, 202 ; Frege 1884, 89, §76)

Si nous pensons le cas où le concept *F* est « identique à 0 », le concept « appartenant à *F* mais non identique à *x* » est « identique à 0 mais non identique à 0 ». Comme il n'y a pas d'objet qui tombe sous ce concept, le nombre qui revient à ce concept est zéro. Ainsi, m = 0. Or, il n'y a qu'un seul objet qui tombe sous un concept « identique à 0 » : il s'agit du nombre « zéro ». Donc, en définissant comme 1 ce concept « identique à 0 », nous obtenons la conséquence selon laquelle 1 suit immédiatement à zéro (Frege 1969, 203 ; Frege 1884, 90, §77).

Richir fait la remarque suivante concernant le concept *F* indiqué dans le paragraphe 76 :

> […] la définition proposée par Frege [de la succession immédiate] fait intervenir autre chose que le jugement de recognition par identification, à savoir la non-identité de x à l'égard de tous les autres objets tombant sous F (ou la non-identité à x de tous les autres objets). (Richir 1983, 89)

Poser le concept « appartenant à F mais non identique à x » présuppose déjà la possibilité de distinguer x de tous les autres objets appartenant à F. Cette non-identité en question, c'est-à-dire la non-identité d'un objet à un autre objet, présuppose de surcroît le fait que x est identique à soi-même et que les objets autres que x sont identiques à eux-mêmes en leur part, donc en somme leur individuation. Ainsi, dit Richir, « ce qui sous-entend à son tour que *tous* les objets tombant sous F soient préalablement reconnaissables, distinguables, à savoir individués » (Richir 1983, 88).

En mettant ainsi en relief cette présupposition fregéenne, Richir attire l'attention sur le mouvement analogue que nous avons confirmé au moment de l'identification du nombre zéro : « Il y a donc dans le concept de successivité le jeu d'une différence entre identités ou individus » (Richir 1983, 89). Le mot « jeu » désigne ici le mouvement de l'identification des objets appartenant à F mais non identiques à x comme tels. Ces objets, à leur tour, sont pris comme ceux qui tombent sous le concept F auquel le nombre m revient. « C'est donc bien à partir de F et de x (et donc de n) qu'on peut réfléchir le concept auquel revient le nombre m (précédant immédiatement n), et par là, identifier le nombre m comme précédant immédiatement le nombre n » (Richir 1983, 90).

Ainsi, selon Richir, Frege présuppose le fait que tous les objets tombant sous le concept F sont complètement déterminés et distingués clairement les uns des autres. D'où surgit le problème suggéré : il est légitime de présupposer ceci lorsqu'il s'agit du concept contenant des objets finis, mais illégitime lorsqu'il s'agit du concept contenant des objets infinis.

Revenons sur le texte de Frege. Après avoir défini la succession immédiate, Frege fait encore un pas en avant pour définir ce qu'il avait décrit en empruntant la notion d'« hérédité » dans *Begriffsschrift* (1879) (Frege 1993, 61). Ici, il désigne par « la φ-suite » la suite des termes liés avec le concept de relation φ.

> La proposition : « Si tout objet avec lequel *x* a la relation φ tombe sous le concept *F* et si, quand *d* tombe sous le concept *F*, il suit que, quel que soit *d*, tout objet avec lequel *d* a la relation φ tombe sous le concept *F*, alors *y* tombe sous le concept *F*, quel que soit le concept *F* » veut dire la même chose que « *y* succède à *x* dans la φ-suite » et que « *x* précède *y* dans la φ-suite ». (Frege 1969, 204-205 ; Frege 1884, 92, §79)

En suivant cette définition où la relation φ est définie comme relation héréditaire, Richir remarque ainsi : « définir l'ensemble ou la classe des *d* revient à définir la succession φ héréditaire et réciproquement ; ou mieux encore la définition de l'un ne va pas sans la définition de l'autre [...] » (Richir 1983, 103). Ce qui pose un problème aux yeux de Richir est que l'ensemble des *d* est défini en s'appuyant sur la succession φ, et que la définition de la succession φ s'appuie sur la détermination complète de l'ensemble des *d*. Si l'ensemble des *d* est l'ensemble fini, il n'y a pas de problème. Mais l'attention de Richir porte sur le fait que Frege emprunte les expressions « quel que soit *d* » et « quel que soit le concept *F* ». Ainsi, remarque Richir :

> […] l'on fait *comme si*, par la quantification universelle sur l'objet *d* ainsi que par la quantification universelle sur le concept *F*, on disposait d'avance de la totalité déterminée de la suite φ, alors qu'en réalité, nous le savons, nous n'en disposons jamais, si du moins la suite est infinie, comme c'est le cas pour la suite *N*. (Richir 1983, 104).

Le problème se pose, pour Richir, dans la mesure où cette quantification universelle ne pose aucune limite à la validité de la définition. Comme nous l'avons confirmé, l'idéal transcendantal dépassant notre limite de la connaissance, il est impossible de poser préalablement la possibilité de l'individuation intégrale de l'ensemble des *d*. Ainsi dit-il : « On procède donc comme si, dans le cas de la suite *N*, on connaissait d'avance toutes les propriétés des nombres – ce qui est impossible [...] » (Richir 1983, 104). Pour Richir, Frege présuppose la connaissance de l'idéal transcendantal. En ce sens, la définition fregéenne de la suite de l'hérédité présuppose l'inconnaissable.

ii. Le schématisme transcendantal dans la définition fregéenne de la suite naturelle des nombres

Revenons sur le texte de Frege. Après avoir ainsi défini la suite héréditaire, Frege aborde, dans le paragraphe 82, la définition de la suite naturelle des nombres qui commence par zéro et qui continue sans se terminer. Pour cela, il commence par prouver que la suite naturelle des nombres n'a pas de fin : « Il faut maintenant montrer que – sous une condition qui reste à donner – le nombre qui appartient au concept : "appartenant à la suite naturelle des nombres qui se termine par *n*" suit immédiatement *n* dans la suite naturelle des nombres ». Si l'on peut prouver que le nombre appartenant au concept « appartenant à {0, 1, 2, ..., *n*} » suit immédiatement *n*, l'on peut dire qu'à tout *n* il y a un nombre qui suit immédiatement dans la suite naturelle des nombres, et donc qu'il y n'a pas de fin dans cette suite (dans la discussion qui suit, pour la commodité, nous désignerons le concept « appartenant à la suite naturelle des nombres qui se termine par *n* » comme {0, 1, …, *n*}, ou bien, à l'instar de Richir, tout simplement comme $H_{n.}$).

Comme Frege n'esquisse qu'un plan général de cette preuve, nous l'aborderons en la suivant pas à pas. La preuve se divise en trois étapes :

Ėtape (1) :

Prémisse 1 : *a* suit immédiatement *d*.

Prémisse 2 : le nombre qui revient au concept « appartenant à {0, 1, ..., *d*} » (H_d) suit immédiatement *d*.

Conséquence : le nombre qui revient au concept « appartenant à {0, 1, ..., *a*} » (H_a) suit immédiatement *a*.

Ėtape (2) : (1) est affirmée lorsque $d = 0$, $a = 1$.

Ėtape (3) : La conséquence de (1) est affirmée pour tous les nombres *n*.

D'abord, il y a une correspondance biunivoque entre le concept « appartenant à {0, 1, ..., *d*, *a*} mais non identique à *a* » et le concept « appartenant à {0, 1, ..., *d*} » (H_d), donc ces deux concepts sont équinumériques. Frege précise ici qu'on présuppose préalablement comme suit : « […] aucun objet appartenant à la suite naturelle des nombres commençant par 0 ne peut se succéder à lui-même dans la suite naturelle des nombres » (Frege 1969, 207 ; Frege 1884, 95, §83), à savoir que $d \neq a$. Par la prémisse 2, le nombre qui revient au concept H_d suit immédiatement à *d*. Par la prémisse 1, ce nombre qui revient au concept H_d est identique à *a*. Ainsi, le nombre qui revient au concept « appartenant à {0, 1, ..., *d*, *a*} mais non identique à *a* » (que nous désignons dorénavant à l'instar de Richir comme H_a-*a*) est aussi *a*.

Par la définition de la suite immédiate dans le paragraphe 76, le nombre qui revient au concept *F* « appartenant à H_a » suit immédiatement le concept « appartenant à *F* mais non identique à *a* ». Or, comme le nombre qui revient au concept « appartenant à *F* mais non identique à *a* » est *a,* le nombre qui suit immédiatement *a* est le nombre qui revient au concept *F* « appartenant à H_a ». Ainsi, l'étape (1) est prouvée.

Puis, lorsque $d = 0$, $a = 1$, le nombre qui revient au concept « appartenant à H_0 », à savoir 1, suit immédiatement 0. Ainsi, par (1), le nombre qui revient au concept « appartenant à (H_1) » suit immédiatement 1. Ainsi est prouvée l'étape (2).

Nous définissons ce nombre qui revient au concept H_1 comme 2. Il s'ensuit que le nombre qui revient au concept H_2 suit immédiatement 2. Définissons comme 3 le nombre qui revient au concept H_2, et nous obtenons le fait que le nombre qui revient au concept H_3 suit immédiatement 3, et ainsi de suite. En général, le nombre qui revient au concept H_n suit immédiatement *n*. Ainsi l'étape (3) est prouvée.

En attirant notre attention sur l'esquisse donnée par Frege de cette preuve, Richir remarque que le nombre qui revient au concept H_n est individué à travers la répétition de l'individuation des nombres qui commence par la fixation du zéro :

> [...] c'est seulement la fixation du 0 comme origine ou premier élément de la suite qui permet d'associer univoquement la propriété *F* héréditaire [...] à tout nombre de la suite. [...] un nombre *n* ne peut être individué que par rapport à l'individuation du 0 [...]. (Richir 1983, 117).

C'est donc comme si le nombre qui revenait au concept H_n jouait le rôle de « mémoire » (Richir 1983, 118) de l'accumulation des termes de la suite des nombres commençant par zéro et se terminant avec *n*. Mais cette accumulation de proche en proche des termes individués en succession est possible seulement lorsque l'on présuppose que « la suite soit toujours déjà constituée d'individus distincts » (Richir 1983, 119 et seq.), présupposition que Richir ne partage pas avec Frege.

L'attention de Richir est donc portée sur *a* dans le paragraphe 82 :

> [...] *a* ne s'individue comme nombre qu'à partir du moment où il est envisagé comme lieu de réflexion permettant d'excepter *a* de la section considérée de la suite et de réfléchir H_a-*a* en l'extension ou le cardinal *a*. (Richir 1983, 122 *et seq.*)

Il considère donc qu'un acte de réflexion est en œuvre dans la fixation du nombre cardinal de H_a-*a*, à savoir *a*. Richir y voit de surcroît ce qu'il qualifie de « clignotement » : un passage de la non-identité à soi vers l'identité à soi : « Donc *a* en tant que phénomène individué se phénoménalise comme lieu de clignotement entre son identité à soi […] et sa non-identité à soi […] » (Richir 1983, 124).

Cette affirmation de Richir, difficile à suivre à première vue, devient compréhensible par un changement de point de vue : prendre *a* comme chaque terme qui suit immédiatement à soi-même (*a* précédant qui est pris dans la succession actuelle comme *d*). Nous saisissons ici le clignotement de *a* comme un mouvement. Pourquoi est-ce un « clignotement » ? Puisque ce mouvement constitue l'identification de *a* qui se distingue du *a* précédant (donc *d* actuel), à savoir l'identification du soi qui s'identifie comme ce qui se distingue de soi-même (comme nous l'avons confirmé plus haut, Frege pose comme condition préalable que *a* soit différent de *d*).

Dans la même phrase citée ci-dessus, Richir précise que, dans son identité à soi, *a* « disparaît comme phénomène et apparaît comme imminence du concept », et que, dans son non-identité à soi, *a* « apparaît en tant que phénomène distinct de son concept mais disparaît du même coup comme phénomène individué » (Richir 1983, 124). Le *a* qui « disparaît » est le *a* précédant, à savoir le *d* actuel, il est déterminé et déjà enregistré dans la suite des termes dans le passé, le *a* qui « apparaît » est le *a* qui arrive, il est encore un phénomène qui se distingue du *a* précédant déterminé et qui sera soumis au concept.

C'est dans ce clignotement que Richir voit le « schématisme transcendantal » en œuvre : il s'agit d'une rencontre entre le penser et le phénomène dans le *a* que Richir qualifie de « maillon de la chaîne » :

> [...] en lui [le maillon de la chaîne], [...] pris en tant que maillon relativement indéterminé comme cela se passe dans l'individuation du phénomène *a*, se rencontrent le penser en tant que le réfléchissant ou le mouvement même de réflexion et la pensée en tant que le phénomène *a* individué mais non encore assigné comme nombre cardinal de la section H_a-*a*. (Richir 1983, 129 *et seq.*)

Il s'agit de la rencontre du « penser » et de la « pensée », de l'acte de penser et du phénomène pensé, le lieu du schématisme transcendantal kantien : « […] le nombre devient, comme pour Kant, schème transcendantal de la quantité : […] » (Richir 1983, 132).

Richir qualifie ce schème de « rythme de la répétition » (Richir 1983, 136). En tant que ce rythme même, le nombre est défini, individualisé à partir du zéro et en établissant le rapport de l'hérédité entre zéro et 1, ensuite 1 et 2, et ainsi de suite, comme on noue un maillon de chaîne l'un à l'autre, et cette individuation de *a* n'a pas de fin. Frege construit donc un nombre en concevant par l'acte de penser le phénomène *a*. Le nombre ne peut pas être construit sans cette rencontre du penser et la pensée, sans l'acte de la connaissance.

Richir discerne dans ce mouvement non seulement la temporalisation transcendantale, mais aussi l'espacement transcendantal : « Et c'est par ce pur mouvement qui donne à la répétition répétée son statut de phénoménalité, […] que se constitue par conséquent l'espacement et la temporalisation transcendantale […] » (Richir 1983, 134 sq.). Il annonce donc bien la thèse kantienne selon laquelle l'acte de compter est la production du temps :

> […] [le nombre] est une représentation qui embrasse l'addition successive de l'unité à l'unité (homogène). Donc, le nombre n'est autre que l'unité de la synthèse du divers compris dans une intuition homogène en général, rendu possible par le fait que je produis le temps lui-même dans l'appréhension de l'intuition (Kant 2006, 227 ; Kant 1911, 1904/1911, A142-143/B182).

Ce retour de Richir vers Kant implique pourtant aussi un certain écart par rapport à ce dernier. Pour Richir, l'acte de compter n'est pas seulement la production du temps, car, pour que les nombres soient comptés, il faut que l'on garde en mémoire le nombre compté précédemment et le juxtaposer avec un autre nombre qui va être compté. Il s'agit de mettre ceux qui se succèdent (ceux qui sont temporels) en coexistence (dans leur spatialisation[3]).

Ainsi, nous avons vu que, aux yeux de Richir, ce que Frege décrit n'est autre que le schématisme transcendantal. Il nous reste à voir : comment Richir considère-t-il la critique faite par Husserl envers Kant dans la *Philosophie de l'arithmétique*, en soutenant que la constitution du nombre n'est pas influencée par l'ordre temporel ?

Bien que Richir n'évoque pas ce problème, nous pouvons le résoudre ainsi : Kant distingue la « *série du temps* » comme schéma transcendantal de la quantité (unité, pluralité, totalité), et l'« *ordre du temps* » comme schéma transcendantal de la relation (inhérence et subsistance, causalité et dépendance, communauté) (Kant 2006, 163, 229 ; Kant 1904, A80/B106, A145/B184 *et seq*.). Les catégories de la quantité n'obtiennent une concrétude pour la sensibilité que par la procédure de l'addition sous la forme de la série du temps, à savoir la suite du temps. Les catégories de la relation n'obtiennent une concrétude pour la sensibilité que par l'ordre du temps : persistance temporelle, succession temporelle, simultanéité temporelle (Kant 2006, 228-229 ; Kant 1911, 1904/1911, A144-145/B183-184). Le nombre en tant que suite du temps est donc indépendant de l'ordre du temps.

Conclusion

La mathématique d'aujourd'hui, en acceptant l'impossibilité de la preuve de l'existence de l'infini, pose cette existence comme axiome. Si nous partons de cet axiome, nous pourrons légitimement poser comme acquis la possibilité de la détermination complète des éléments de l'ensemble infini. Ainsi, cet axiome étant accepté, nous sommes libérés de la tâche de fonder l'arithmétique, en même temps que se légitime la définition fregéenne de la suite de l'hérédité.

Néanmoins, la phénoménologie ne peut pas poser sans examen critique ce qui n'est pas fondu dans le vécu de la subjectivité. La problématique husserlienne de la *Fundierung* (fondement), déployée dans la *Crise*, montre bien que l'*Idealisierung* (idéalisation) accompagnée de l'oubli du fondement (*Fundierung*) dans l'intuition sensible, peut provoquer la *Versponnenheit* (superficialité) de la science (Husserl 1954, 347). Tenir pour superflue la tâche du fondement est synonyme d'accepter l'impossibilité de la réactivation de ce qui avait été oublié.

Richir reprend cette tâche non seulement en retournant au schématisme transcendantal kantien, mais aussi en discernant celui-ci dans la définition logiciste fregéenne de la suite naturelle des nombres. Il s'agit du « schématisme de l'idéalité » qui rend possible « la construction du concept dans l'intuition pure » (Richir, Carlson 2015, 90).

La contribution de Frege, de ce point de vue phénoménologique, consiste à avoir rendu clair les deux caractéristiques de la suite des nombres : d'une part, « à tout nombre donné de la suite il y a un successeur », et d'autre part, « la suite est infinie » (Richir, Carlson 2015, 89). À Richir d'ajouter : cette suite reste toujours un infini potentiel.

NOTES

[1] Kant distingue, à l'intérieur du *quantum* au sens large, *quantum* au sens étroit (grandeur extensive) et *quantitas*. Le premier, étant spatialisé comme image à travers son schème, sera appliqué à l'objet de l'intuition dont la forme est l'espace. Le deuxième, étant temporalisé à travers son schème qui est le nombre, sera appliqué à l'objet de l'intuition dont la forme est le temps : « L'image pure qui présente toutes les grandeurs (*quantorum*) au sens externe est l'espace, tandis que celle de tous les objets des sens en généal est le temps. Mais le *schème* pur de la *grandeur* (*quantitatis*), envisagée comme concept de l'entendement, est le *nombre* […] » (Kant 2006, 227 ; Kant 1911, 1904/1911, A142/B182).

[2] Nous pouvons considérer, du point de vue de Frege, que la possibilité de la détermination complète des éléments de l'ensemble infini sera assurée par la pensée, et non par l'intuition. En ce sens, la position de Frege a aussi sa propre cohérence.

[3] C'est Bergson qui nous donne de la matière à penser sur ce point : « [L]orsqu'on ajoute à l'instant actuel ceux qui le précédaient, comme il arrive quand on additionne des unités, ce n'est pas sur ces instants eux-mêmes qu'on opère,

puisqu'ils sont à jamais évanouis, mais bien sur la trace durable qu'ils nous paraissent avoir laissée dans l'espace en le traversant » (Bergson, 2007, 59).

RÉFÉRENCES

Leibniz, Gottfried Wilhelm. 2002. *Monadologie und andere metaphysische Schriften*. Hrsg. von Ulrich Johannes Schneider. Hamburg: Meiner.

Kant, Immanuel. 1900. [*Akademie Ausgabe*]. *Gesammelte Schriften*. Bde. I-XXIII. Hrsg. von Preussischen Akademie der Wissenschaften. Berlin: Reimer. [Abréviation : AA].

________. 1911. [AA, A] „Kritik der reinen Vernunft (1. Aufl. 1781)“. In *Akademie Ausgabe*, Bd. IV, Hrsg. von Benno Erdmann. Berlin: G. Reimer [Abréviation : A].

________. 1911. [AA IV *Prol*.]. „Prolegomena zu einer jeden künftigen Metaphysik, die als Wissenschaft wird auftreten können“ (1783). In *Akademie Ausgabe*, Bd. IV, Hrsg. von Benno Erdmann. Berlin : G. Reimer. [Abréviation : *Prol.*]

________. 1904/1911. [AA, B]. “Kritik der reinen Vernunft (2. Aufl. 1787)“. In *Akademie Ausgabe* Bd. III, Hrsg. von Benno Erdmann. Berlin : G. Reimer. [Abréviation : B].

________. 2006. Kant, Immanuel. *Critique de la raison pure*. Traduit par Alain Renaut, Paris : GF-Flammarion. (Pour les citations, nous utilisons la traduction française).

Lange, Friedrich Albert. 1894. (Erstdruck 1877). *Logische Studien*. Leibzig: Baedeker.

Frege, Gottlob. 1993. *Begriffsschrift und andere Aufsätze, Zweite Auflage, Mit E. Husserls und H. Scholz' Anmerkungen*. Hrsg. von Ignacio Angelelli. Hildesheim/Zürich/New York: Olms.

________. 1884. *Grundlagen der Arithmetik. Eine logisch mathematische Untersuchung über den Begriff der Zahl*. Breslau: Verlag von Wilhelm Koebner.

Frege, Gottlob. Imbert, Claude (tr.). 1969. *Les fondements de l'arithmétique. Recherche logico-mathématique sur le concept de nombre*. Paris : Seuil. (Pour les citations, nous utilisons la traduction française).

Husserl, Edmund. 1954. *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie*. Hrsg. von Walter Biemel. Den Haag: Martinus Nijhoff.

________. 1970. *Philosophie der Arithmetik. Mit ergänzenden Texten (1890-1901). Husserliana Bd. XII.* Hrsg. von Eley, Lothar. Den Haag: Nijhoff.

Bergson, Henri. 2007 [1889]. *Essai sur les données immédiates de la conscience*. Edité par Arnaud Boaniche. Paris : P.U.F. [« Quadrige »].

Derrida, Jacques. 1990. *Le problème de la genèse dans la philosophie de Husserl* [1953-1954]. Paris : P.U.F.

Richir, Marc. 1983. « L'Hérédité et les nombres ». *La liberté de l'Esprit* n° 4 (*Qu'est-ce qu'un père ?*) : 77-137. Paris : Balland.

________. 2004. *Phantasia, imagination, affectivité*. Grenoble : Millon.

Richir, Marc. Sacha, Carlson. 2015. *L'écart et le rien – Conversations avec Sacha Carlson*, Grenoble : Millon.

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Wittgenstein's Affirmation of Mysticism in his "Private Language" Argument[1]

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Abstract

Although the view that mystical experiences are ineffable is present in many mystics in both East and West, mystical *philosophies* are rare in the West, where "scientific," common sense, or ordinary language philosophies dominate. One exception is Wittgenstein's *Tractatus* which holds that there are ineffable mystical things about which one "must be silent". Indeed, Wittgenstein, throughout his career admired India's Tagore, who held mystical views. However, many scholars agree with Nieli, who argues that Wittgenstein, in his "Private Language Argument" replaces his earlier *Tractatus*-mysticism with the view that *all* genuine language and experience is public. The paper argues that Nieli is incorrect. Mysticism is still present in Wittgenstein's *Philosophical Investigations* but in a more subtle form than that of the *Tractatus*. Specifically, the mysticism of *Tractatus* is what has been called an "autobiographical" mysticism while that of Wittgenstein's *Philosophical Investigations* is what has been called a "radiant" mysticism. Given Wittgenstein's longstanding admiration for Tagore, the persistence of mysticism into Wittgenstein's *Philosophical Investigations* should be no surprise. The paper argues that rather than denying the mystical, Wittgenstein's private language argument actually spell out the place for the ineffable more precisely than had been done in the *Tractatus,* thereby helping to explain Wittgenstein's longstanding admiration for Tagore.

Keywords: Wittgenstein, Tagore, mysticism, Private Language, ineffability

> To prepare, in a spirit of reverence and by a life of discipline, for the world-life in which the soul is to attain maturity amidst her daily work of self-dedication and find at the serene end of her physical existence her own perfect revelation in a world of ineffable light and life, – is the only way the soul can attain to existence and meaning (Tagore, "The Fourfold Way of India", 503).

Although there are many marks of a mystical state, James (2012, 267-68) identifies ineffability as the "handiest" mark by which he classifies a mental state as mystical, and many mystics, in both the East and the West and in most major religions, claim that mystical states are ineffable. For example, this claim is found in the ancient *Taittirỹa Upanishad* (Matilal 1975). It is also found in various forms in the modern Indian philosophers and mystics Rabindranath Tagore and Sarvepalli Radhakrishnan (Sinha 1994, 109-119; Radhakrishnan 1991, 26; Brightman 1991, 413-15).[2] It is generally held that mystical states of consciousness are private states that can normally only be achieved by extraordinary individuals (Stace 1960, 33, 52, 55, 59, 76; Underhill 1999, 34-36, 328; King 2002, 17-18). However, though the view that mystical experiences are ineffable is present in many religions and in many mystics in both East and West (Otto, 2016), genuinely mystical *philosophies* are rare in the West, where "scientific", ordinary language or common-sense philosophies tend to dominate. One exception is Wittgenstein, who, in his early *Tractatus-logico-philosophicus* (hereafter *TLP*), first published in German in 1921, defended the mystical view that there are things that are beyond "the limits of language" and, therefore, about which one "must be silent" (*TLP*, Preface, 6.522, 6.54, 7).[3] *TLP's* doctrine of silence is also reminiscent of Tagore's philosophical views. For, according to Sarinindranth Tagore (2014 268), Rabindrananth Tagore's nephew, the very last works in his uncle's life are, "filled with the image of silence further extending the metalinguistic conviction that language his no final vocabulary with which to answer the question of being". However, in Wittgenstein's "Later Philosophy" (hereafter, WLP), beginning with his *Philosophical Investigations* (hereafter, *PI*), published posthumously in 1953, the mystical dimension appears to have disappeared altogether or, at least, to have been radically deemphasized. For *PI's* "Private Language Argument" (hereafter PLA) argues against the possibility of a private language and, therefore, it would seem, against the possibility of private (mystical) experiences. It is natural to assume that *PI* (293) sees mystical experiences as just another private "beetle" each mystic has in their own unopenable mental "box" that, since it cannot be put into words of public language, "cancel's out, whatever it is," so that "the box might even be empty." This anti-mystical reading of WLP is argued in Nieli's *Wittgenstein: From Mysticism to Ordinary Language*. Nieli's thesis is that WLP abandons the mystical philosophy of inwardness from his early *TLP* and replaces it with the view that *all* genuine language and experience is ordinary public language and experience, which is why WLP inspired "Ordinary Language Philosophy" (Biletzki and Matar 2018, §'s 1, 3; Parker-Ryan 2018, [I]). Since, on Nieli's view, everything in WLP becomes outward, public, and expressible in "ordinary language," there is no place left for *TLP's* mystical "inner" self. The present paper argues that mysticism is still present in WLP, but in a more subtle form than that in *TLP*. Since Tagore had his own doctrine of the ineffable (see epigraph above and Kannath, 2004), and given Wittgenstein's longstanding admiration for Tagore, from his early *TLP*-period to his "later period" (Monk 2012, 410), the persistence of mysticism into WLP should be no surprise. The present paper argues that rather than denying the mystical, PLA attempts to spell out the place for the ineffable more precisely than it had been in *TLP*. Thus, WLP's PLA can be seen as clarifying the conceptual place for the ineffable that Wittgenstein saw in Tagore and others.

§ I sketches the orthodox view of Wittgenstein's mysticism. § II explains Nieli's view that PLA banishes private or mystical experiences from WLP. § III argues against Nieli that PLA explicitly endorses the view that there are private ineffable *experiences*. § IV refutes the main textual objection in *PI* itself to the present view and argues that PLA actually attempts to specify the nature of the ineffable more precisely than had been done in *TLP*. § V clarifies WLP's view on the question whether or in what sense mystical experiences can be expressed in language.

1. Wittgenstein's Mysticism

> There are indeed things that cannot be put into words. … They are what is mystical. (Wittgenstein, *Tractatus*, 6.522)

The view that there are things, in fact, the most important things, that are ineffable, is, arguably, the central theme of *TLP*. For in the Preface to *TLP* Wittgenstein states that "The whole sense of the book" is that "what can be said at all can be said clearly, and what we cannot talk about must be passed over in silence" and that, therefore, "the aim of the book is to draw a limit to thought, or rather—not to thought but to the expression of thoughts; for in order to draw a limit to thought ... we should have to be able to think what cannot be thought." That is, the whole aim of the book is to delineate what can be "said," roughly, the propositions of natural science, from the truly important matters that cannot be "said." *TLP* specifies the domain of the mystical to include everything of "authentic value", that is, "ethics, aesthetics, religion – all that is 'transcendental'" (Black 1970, 373-4). Since the "transcendental" includes everything is *a priori* (*TLP*, 6.13, 6.3-6.34), this would include the "metaphysical" propositions (*Sätze*) of *TLP* as well. Indeed, Wittgenstein's remark to Ficker that there are two parts to *TLP*, the part that can be written (the account of the logic of factual propositions), and the part that cannot be written (the "mystical" part), where the latter is the most important part (Monk, 2012, 178), clarifies *TLP's* attempt to set the "limits of language". For, whereas Carnap (1969, 435-6) reads *TLP's* attempt to demarcate the limits of language as an attempt to safeguard the factual (scientific) propositions by segregating them from the mystical "nonsense" that might confuse them, *TLP's* real point was the reverse, namely, to quash the illusion that the genuinely "authentic" issues are solved when one solves scientific problems: "We feel that even when all *possible* scientific questions have been answered, the problems of life remain completely untouched" (*TLP*, 6.52). Thus, Wittgenstein is quite serious when he told Ficker that *TLP* is a book on "ethics," that is, "the mystical", and that this ethical part of the book cannot be written. *TLP's* positive attitude towards the mystical explains why philosophers such as Russell and Carnap, as great as these were in their own specializations, were incapable of understanding Wittgenstein's real intentions in *TLP*.

The place of the mystical in WLP is, admittedly, harder to see, but it is not entirely absent. At *CV* (10), written in 1931, Wittgenstein still invokes the *TLP*-doctrine that there are limits to language. However, this is not particularly significant since in 1931 Wittgenstein had just returned to philosophy and had not yet made the decisive moves towards his WLP. But Wittgenstein also refers to the limits of language doctrine at *PI* (119): "The results of philosophy" are achieved by the "bumps that the understanding has got by running its head up against the limits of language" and it is these bumps that "make use see the value of the [philosophical] discovery". However, Wittgenstein does not, as he had done in *TLP*, go on in *PI* to attempt to describe what is beyond the limits of language. He does not go on to specify that the mystical includes ethics, aesthetics, religion, and metaphysics. That is, whereas *TLP* had been criticized because it states that one cannot "say" mystical things, and then proceeds to attempt to "say" them (Carnap 1969, 435), WLP reaffirms that there are limits to language but, *prima facia*, more consistently than *TLP*, keeps silent about them.

It is for this reason that almost all of the discussions of Wittgenstein's mysticism focus on *TLP*. Very few commentaries on Wittgenstein mention the mystical in connection with WLP at all because the bare acknowledgement that he still retains the idea that there are limits on language is very little on which to ground a mystical doctrine. Indeed, Hallett (1977, 209-210) argues that *PI's* (119) reference to the old *TLP*-doctrine that there are limits to language is a holdover from his earlier *TLP*-period and does "not belong" in in WLP.

2. Nieli's View that Wittgenstein abandons *Tractatus*-Mysticism in his "Later Philosophy"

> In his attack on universal essences, Wittgenstein, it would seem, is consciously trying to dispense with the two great enemies of his post-Tractatus period—destructive positivism on the one hand, mysticism, metaphysics, and the potential torments of the inner life, on the other. (Nieli, *Wittgenstein: From Mysticism to Ordinary Language*, 234)

According to Nieli, WLP does not merely reject *TLP's* mysticism but actually sees it as WLP's "enemy". Nieli gives several kinds of explanations for Wittgenstein's alleged reversal on the mystical. First, Nieli (1987 246-47) holds that Wittgenstein was personally motivated by his own inner torment, and thought that producing a philosophy that denies the "inner" might help those with inner demons to lead a normal life free of torment from psychic conflicts. Second, on a more conceptual basis, Nieli (1987, 220), holds that Wittgenstein's "private language argument" (PLA) rejects "at least for the purposes of a philosophy of language, the notion of a private or personal sphere inaccessible to public view." According to Nieli (1987 197, 222), following Strawson (1954), WLP's message is "Forget about the private object so that you can direct all your attention to publicly observable things and events". For Nieli, the Wittgenstein of PLA is not merely a philosopher interested in the philosophy of language but a psychic healer who holds that an "inner life" is bad for one. Since WLP's reasons for his hostility to the private sphere are set out in its PLA, it is necessary to consider, at least briefly, that argument. The core of PLA is set out at *PI* (258),

I want to keep a diary about the recurrence of a certain sensation. To this end I associate it

> with the sign "S" and write a sign in a calendar for every day [when] I have the sensation. … [A] definition of the sign cannot be formulated. But … I can give myself a kind of ostensive definition. … I speak or write the sign down, and … concentrate my attention on the sensation—and so, as it were, point to it inwardly. But what is this ceremony for? For that is all it seems to be! A definition surely serves to establish the meaning of the sign. Well, this is done by [concentrating] my attention; for in this way I impress upon myself the connection between the sign and the sensation. But "I impress it on myself" can only mean: this process brings it about that I remember the connection right in the future. But in the present case, I have no criterion of correctness. One would like to say: whatever is going to seem right to me is right. And that only means that here we can't talk about "right".

Wittgenstein's objection to the indicated "ceremony" is that concentrating one's mind on the connection between the sensation and its description does not establish *criteria* for the correct use of the sign. If his memory fails about which sensation he called "S", he has no criteria for determining that his memory is incorrect. He could consult other of his memories, but that would be like looking at one copy of the newspaper to verify what's in another copy of the same newspaper (*PI*, 265). This problem does *not* arise for public languages. Suppose Wittgenstein sees a green patch, which he calls "G", and determines to record in his diary every time he sees the same shade of green again. If a member of his linguistic community observes him record an "G" in his diary upon seeing a yellow patch in normal light, they can correct him. It is precisely the impossibility of this kind of independent check that is lacking in his private sensation-language: "The balance on which impressions are to be weighed is not the impression of a balance" (*PI* 259). In brief, a person's private memory is only the "impression" of a criterion. But since genuine languages require actual criteria for the use of words, and since private languages do not have them, but only impressions of them, private languages are not genuine languages. Thus, these alleged private words do not have any meaning at all, *not even for the alleged private language user*. That is, it is not merely that we in the public world cannot understand what the private language user means by their words, but that the private language user cannot understand their own private words because there is no meaning there to understand (Nieli 1987, 222).

This would appear to be bad news for mysticism. For PLA does not merely deny the possibility of private languages in the quite reasonable sense, with which most people could agree, that private experiences are "too intimate or personal to be expressed through public language," but, rather, as part of PLA's "war against the inner, private object" (Nieli, 1987, 222, 224). That is, PLA wants to "eliminate the private object and the activity of mind reflecting upon itself" (Nieli 1987, 223).

In place of a philosophy, like that in his own earlier *TLP*, that allows a significant place for the inner (mystical) life, Wittgenstein's WLP has tried to create "a philosophy of language in which the only reality that is to count is public reality" (Nieli 1987, 225). Private feelings and mental states are "banished from our field of apperception by being denied linguistic expression" to the point that "one does not talk about them even to oneself" (Nieli 1986, 225). That is, PLA not only denies the possibility of a private language but also denies private experience and the private object altogether thereby condemning the human self to absorption in the public (ordinary) world.

3. The *Affirmation* of Private Experience in Wittgenstein's "Private Language Argument"

> The essential thing about private experience [privaten Erlebnis] is really not that each person possesses their own exemplar, but that nobody knows whether other people have this [LW's emphasis] or something else. The assumption would be possible—though unverifiable—that one section of mankind has one sensation of red and another section another. (Wittgenstein, Philosophical Investigations, 272)

Although PLA argues that a private language is impossible, the reader is surprised when at *PI* (270), clearly referring to private sensations, Wittgenstein makes an unexpected admission,

> Let us now imagine that … I discover that whenever I have a particular sensation a manometer shows that my blood pressure rises. So that [thereafter] I shall be able to say that my blood pressure rises without using any apparatus. This is a useful result.

Garver (1994, 214-15) points out, this "perplexing" passage seems to take back everything Wittgenstein has been arguing in PLA. For Wittgenstein here admits that one might establish a useful correlation between private sensations and public manifestations after all. However, Garver goes on in the same passage to point out that that the moral of *PI* (270), taken in its *entirety*, does *not* contradict the central conclusion of PLA. For the later parts of *PI* (270) point out that in this hypothetical case "it seems quite indifferent whether I have remembered the sensation *right* [LW's emphasis] or not. … We, as it were, turned a knob which looked as if it could be used to turn some part of the machine, but it was a mere ornament, not a part of the mechanism at all". The "machine" or "mechanism" here is the correlation between the private sensation and the public reading on the manometer – and *PI* (270's) claim is that talk of this "machine" is a mere "ornament" because the alleged *mechanism* connecting private and public states plays no role whatsoever in our language.[4]

Thus, the alleged "useful" correlation between the private sensation and the manometer reading is not so useful after all because it actually reduces, upon further examination, to a correlation between writing "S" in one's diary and one's blood pressure rising, which are both public events.

It cannot be denied that Garver makes a good point here. *PI* (270), taken in its entirety, does take back the "perplexing" admission that one might have to countenance correlations between private states and public physical manifestations. However, Wittgenstein does *not* take back everything in that passage. At *PI* (272) (see above epigraph), Wittgenstein explicitly admits that there is no problem with the view that human beings have "private experiences [*privaten Erlebnis*]". Far from denying that there can be private experiences, *PI* (272) *explicitly affirms that there can be private experiences*, and though Wittgenstein is here discussing private sensations, there is no reason why the point cannot be generalized to include a wide range of private experiences, including mystical experiences.

PI (272) does, however, place a significant restriction on these private experiences, namely, that "nobody can know whether other people have *this* experience or something else", where the use of the demonstrative "this" in the formulation is crucial. Consider the hypothetical case in which Wittgenstein has a private experience (which might be a private pain or a mystical experience). *PI* (272) does *not* deny that Wittgenstein can tell his peers that he has such an experience. It only denies that he can tell them that he has "*this*" private experience. This restriction makes perfect sense. The word "this" is a demonstrative, and the reference and meaning of demonstratives, like "I", "this," and "that" are dependent on the *context* in which they are used (Georgi, [I], §'s 2. b, 3. b & c, 4. d; 5. a & b). However, there is a significant difference between "I" and "this". Although the word "I" is a demonstrative, the word "this" is not merely a demonstrative but is what is called a "true demonstrative" in the sense that its reference, in any given context, normally requires *more* than the mere uttering of it (Braun, 1996). Consider the following example! When Wittgenstein, sitting in a coffee shop, says "I knew Bertrand Russell", one does not require anything more to determine that Wittgenstein is referring to himself, the person that uttered that statement. When, however, in the same coffee shop, he says "This is my favorite coffee", one normally requires more information to know to what "this" refers. Suppose, for example, Wittgenstein had *mistakenly* ordered the wrong coffee that day but that his friend had ordered the kind of coffee Wittgenstein had meant to order. In this case, Wittgenstein cannot merely say that "this" is my favorite coffee, but must, perhaps by *gesturing with his arm at his friend's coffee*, say "this" is my favorite coffee. It is only by means of that gesture, in context, that he successfully indicates which coffee is his favorite.

The important point, for present purposes, is that since the word "this" which plays such a prominent role in *PI* (272) is a "true demonstrative," it requires *a great deal of context*, even more than is required for ordinary demonstratives, to determine what it means. In the aforementioned case, it requires knowledge of the spatial context (the coffee shop), but it also requires knowledge of Wittgenstein's gestures as he utters the word "this". But these matters of context are precisely what is, *by definition*, lacking in putative private uses of words. When Wittgenstein has his hypothetical private experience of a certain sensation, or a private mystical experience, he cannot convey to his linguistic community which private experience he means because that experience, being private, is, so to speak, in a different "space" from the public "space" he shares with his linguistic community. It is not in the "space" of the coffee shop or the "space" in which clarifying arm gestures are made. Wittgenstein cannot indicate which private experience he means by gesturing at it, as he does in the case of his friend's coffee. If he gestures at his sensation with his public arm, that gesture does not, by definition, penetrate into his private mind and pick out the right private experience. However, if he privately "gestures" at his private experience, perhaps by focusing his mental attention on it, once again, this "private pointing" cannot, by definition, penetrate into the public space of his linguistic community and fix the reference of his private word for that public community.

The fact that *PI* (272) acknowledges that human beings can have "private experiences [*privaten Erlebnis*]," but cannot coherently say that they have "*this*" private experience, has another significance. Although many, but not all different sorts of words are believed, in one way or another, to "'hook onto' bits of reality", it is often held that the demonstratives and indexicals are *paradigm* cases of this sort of "referring" language that "hooks onto" reality (Michaelson 2019, § 1). Kaplan (1989), whose account of demonstratives and indexicals has set the baseline for all subsequent accounts, also holds that some demonstratives and indexicals achieve "direct reference" to the relevant object in the world. If this is the case, then, when *PI* (272) states that one cannot coherently refer to "this" private experience of one's own, he is denying that by talking about one's private experiences one is coherently talking about something in *reality* to which one has *direct reference* denied to anyone else. His reasons are not hard to find. For WLP holds that the word "this" functions in our language a very particular way. *PI* (380), warning against the tendency to posit a "private ostensive definition," states that it must be possible to ask of any putative "this": "This?—What?" That is, a "this" is always a "this-such". It is this human being, this tree, this airplane, etc. Furthermore, the word "this" "hooks onto" reality via the appended "such". If Jones, at a party, points at a potted plant, and says, "This is my wife", he has not succeeded in picking out the appropriate object because the criteria for calling something a wife are massively different from those for calling something a potted plant. Further, the criteria for the application of these various sortal terms, "wife", "tree", or "airplane", to items in the world are, as WLP puts it, embodied in the "grammar" of the public language (*PI*, 353). Since, however, this appeal to these *public* criteria for the application of demonstrative referring expressions are, by definition, lacking in the case of a putative private experience, the idea that one can do an end-run around public language in which such demonstratives operate within specific sorts of restrictions and somehow manage to use them to refer (or directly refer) successfully to a private object is incoherent. The "such" that is required to mediate the connection of the "this" with the object in reality is lacking in private uses of "true demonstratives." Thus, *PI* (272) does not deny that one can have private (mystical) experiences but only that one can use the bare demonstrative "this" (minus the "such") to put such *wordless private or mystical* experiences into words (whether private or public words) – thereby completely distorting *both* the logic of the situation *and* the nature of private experiences.

Rather than denying the possibility of private experiences, mystical or otherwise, what PLA actually does is separate the notion of private experiences from *the sort of* factual language, whether this language be private or public*,* that depends on the sort of contact with reality that is established by means of the "true demonstratives".[5]

But this is the same claim made by many mystics. Significantly, it is also claimed by R. Tagore in *Shesh Lekha 13,*

> Today, my sack is empty
> I have given completely
> whatever I had to give.
> In return if I receive anything
> Some love, some forgiveness—
> Then I will take it with me
> When I step on that boat
> That crosses to the festival of the wordless end
> (quoted in S. Tagore 2014, 268-69).

This is not merely the idea that death is "wordless" (the end of the "words" of this material life), although that that is involved as well, but is that the wordlessness of death itself symbolizes the idea that "the telos of existence is thought to be wordless" (S. Tagore 2014, 268). S. Tagore (2014, 269) goes on to point out correctly that Tagore's language here recalls Wittgenstein's final remark in *TLP* (7) that "Whereof one cannot speak, thereof one must be silent." Nieli is, therefore, incorrect that WLP abandons the mysticism of *TLP* in WLP. That same mysticism is present, precisely where one might think it most unlikely to find it, in WLP's PLA. For in PLA, where Wittgenstein might seem to deny the possibility of private or mystical experiences, he actually *affirms* it, but stresses that it is a "wordless" experience (irrespective of whether these are "pubic" or "private" words). That is, *PI* (272) not only reaffirms *TLP's* (6.522) mystical view that there are things that cannot be put into words", but even attempts to specify the precise nature of this "wordless" experience, and the associated limits on language more precisely than *TLP* had done. Nieli has, in effect, confused WLP with the "ordinary language philosophies" defended by some of Wittgenstein's disciples, like John Wisdom, Norman Malcolm and J.L. Austin, with WLP's much more spiritual view.[6]

4. Reply to an Objection

> The great difficulty here is not to represent the matter as if there were something one couldn't do. As if there really were an object from which I derive its description, but I were unable to shew [zeigen] it to anyone. Wittgenstein, *Philosophical Investigations*, 374)

Although it is argued in the previous section that WLP affirms the possibility of private (mystical) experiences, some might claim that *PI* (374) contradicts this? For if, as *PI* (272) implies, one can have "private experiences", then, since the private is, by definition, not something one can express to anyone else, there *is*, contra *PI* (374), something that one cannot do, namely, put one's private experiences into words. How are *PI* (374) and *PI* (272) consistent with each other?

There is no contradiction. For when *PI* (374) stresses that it is important not to represent the matter as if there is something one cannot do, *PI* (272) actually *explains why there is nothing that one cannot do*. Recall that *PI* (272) allows that there can be private experiences but adds that what is not possible is that "no one can know whether I have *this*" private experience (where the demonstrative is crucial to the formulation). Thus, *PI's* (374) point is that when asked if there is something one cannot do (i.e., express private experiences into words), WLP replies that one cannot coherently specify for any "this" that *this* is what one cannot do!

The real common thread between *PI* (272) and *PI* (374) is that both are, as *PI* (374) indicates, opposed to the idea that *the proper model of private or mystical experiences* is that "there [is] an object from which I derive its description but I were unable to show anyone". That is, WLP is opposed to the use of the name and object-named model in these kinds of cases. Thus, it is not private or mystical experience *per se* that WLP finds objectionable, *but only that specific distorting model of private or mystical experience*. Specifically, WLP objects to the view that a private or mystical experience is like a beetle in a the unopenable box of the mind that can, somehow, be called "this" beetle, but cannot be shown to anyone. Similarly, the language that expresses private or mystical experiences is not like the language that describes objects in containers (experiences in the mind or beetles in boxes). If one uses this name-object model, which *is* appropriate in other contexts, like most scientific contexts, in order to conceptualize mystical experience, one will only produce a caricature of private or mystical experiences that can then easily be "refuted" by "scientific" minded philosophers: "So you mystics are claiming that you each have a private mystical beetle in your various mental boxes that you each cannot show to anyone else – even to other mystics? Then how do you know that what you mean by your 'mystical experience' is the same as what other mystics mean by their 'mystical experiences'?" What Wittgenstein objects to in PLA is this name-object model of private experiences, not "*privaten Erlebnis*" as such.

Garver's correct point that in *PI* (270) Wittgenstein takes back his earlier suggestion in *PI* (270) that there could be a "useful" correlation between private experiences and readings on a manometer should not be surprise. Garver is right that Wittgenstein, further down in the passage, suggests that the private object drops out as irrelevant because it does not matter if he remembers it incorrectly. But still further down, in the very last paragraph of *PI* (270), referring to our language-game of sensations, Wittgenstein makes clear that this does not mean that he completely rejects a sensation-language,

> And what is our reason for calling "S" the name of a sensation here? Perhaps the kind of way this sign is employed in this language game.—And why a particular sensation, that is, the same one every time? Well, aren't we supposing that we write "S" every time?

Wittgenstein here reaffirms that we do possess a "sensation-language" in which we talk about our own sensations but that it is a very different kind of language-game from the language-game of physical objects. We talk about our sensations as we talk, as he will say two paragraphs later in *PI* (272), about "our own [*privaten*] exemplar", *not* as we talk about the "this" beetle. Whereas the name-object model is appropriate in the natural sciences, the language of mental states is, with some important qualifications, properly modelled on *expression*-language

(McDonough [I], § 7), not on the sort of "referential' language that involves "true demonstratives". Roughly, when one speaks about one's private or mystical states, one is not *naming* objects hidden in the unopenable box of the mind, but rather giving *expression* to one's private states.

Finally, the last paragraph in *PI* (270) explains a crucial feature of WLP's notion of concept formation. Specifically, WLP holds that we do *not* form our concepts by deriving them from objects, but, rather, much the reverse, our concepts (for example of a physical object or a sensation or a mystical state) *are based on the way we use words*. Thus, our concepts of mental states are based on our the very unique ways we talk about them as opposed to the way we talk about physical objects. Thus, our concepts physical states are formed on the way we use that kind of physical language that is essentially parasitic on the use of the "true demonstratives", but our concepts of mental states are dependent on our natural ways of *expressing* those mental states. Thus, in order to understand the concepts of mental states, one must examine the ways the actual "language-games" of mental-state ascription. Similarly, in order to understand our concepts of mystical states, one must look at the actual "language-games" in mystical communities. There will, of course, be a concept of those mystical states because there are *de facto* communities of people express such states in virtually all times and cultures and there are notable similarities about the way these very different communities talk about these states (Stace 1960, 31-40, 134-152). There is, therefore, no more need for Stace (1960, 13-18) to argue that the enquiry into the nature of mystical states or language about those states is worthwhile than there is to argue that the enquiry into the nature of physical states or scientific language about those states is worthwhile. For the justification for the enquiry into the nature of mystical states is supplied by the existence of these widespread "forms of life" and the associated "language games" (*PI*, 83, p. 226). There is no need to "justify" an interest in mystical language any more than there is a need to justify an interest in physical language. Both kinds of language are *equally* grounded the "natural history" of human beings (*PI*, 25, 415, p. 230). Neither has any more basic priority over the other: "What has to be accepted, the given, is—so one could say—*forms of life* [LW's emphasis]" (*PI*, p. 226). That includes, of course, mystical "*forms of life*."

5. Ineffable in What Sense?

> Although declaring that the heart of their experience is ineffable, [some mystics] have much to say about it, pouring out their story in journals, letters, poems, essays, sermons, and confessions—telling all. Other mystics find the inner light within the silence of their own souls, and let their light shine without the accompanying clamor of words. (Brightman, "Radhakrishnan and Mysticism", 393)

It is often said that many mystics, like the author of *TLP*, claim that their mystical experiences are ineffable, and then spend a great deal of time talking about them, e.g., Carnap (1969, 435) criticizes *TLP* on the grounds that Wittgenstein "seems to me to be inconsistent in what he does [in *TLP*]. He tells us that one cannot make philosophical statements and that whereof one cannot speak thereof one must be silent; and then, instead of keeping silent, he writes a whole philosophical book". However, one can escape Carnap's objection by making certain distinctions about what it means to "say" something or "put something into words". One "puts something into words" in one sense when one makes a factual statement of the sort that is parasitic on the "true demonstratives" and one "puts something into words" in a very different sense when one writes a poem or allegory. The present section attempts to clarify WLP's (*PI*, 270-272) account of these two different senses.

Brightman (1991, 402) states that "the mystic experience is ineffable only in the sense in which the taste of good food is ineffable". However, this gets one no further absent an explanation of the sense in which the taste of good food is ineffable. Phillips (1991, 153) attempts an explanation: "Givenness is always ineffable. It can be pointed at by words, but words cannot convey it directly. All discourse ultimately points back to an ostensive step [at which point] we must simply behold." That is, since "givenness" is always ineffable, it can only be "pointed at" in an original "ostensive step" in which one "simply behold[s]" the given. This would be news to Sellars

(1997, 13-14) who, referring to that "great foe of immediacy", Hegel, holds, that it is a myth that anything is simply "given" to consciousness. Sellars sees it as one of the most pervasive and destructive myths in philosophy that there is some "this" that is "purely given" in ostension independently of *mediation* by sortal terms like "human" or "tree" – and it is the "myth of the given" that underlies Phillips' seductive but ultimately *impotent* notion of the mystical as a pure "this" that one can only "simply behold".

Although one might not think so at first glance, Sellars' critique of the "myth of the given" is good news for the mystic. For if Phillips were correct that one can only "simply behold" the "given," it would be hard to understand how *any* words that purport to convey mystical experiences, even poems, have any meaning whatsoever. Further, WLP *(PI*, 272) agrees with Sellars both that there are no pure "thisses" and that the word "this" succeeds in referring to items in reality only via the appended sortal terms. Thus, Phillips' view is precisely the opposite of the account of the mystical implicit in WLP (*PI*, 270). Since, for WLP, the grammar of these sortals is enshrined in the grammar of *public* languages, there are can be no private uses of "this" that succeed in referring to *objects* in reality. Since, however, WLP (*PI*, 272) admits that we can talk about private experiences, but not "this" private experience, it follows that our talk about private experiences is not factual language. The language in which WLP (*PI*, 272) allows that we can describe our mystical experiences is not like the factual language in which one describes "this" tree. That is, when one describes one's mystical experience, one is not referring to an *object* (like a beetle in some unopenable box) and subsuming it under some public sortal term like "tree". However, WLP (*PI*, 272) allows that one *can* describe one's mystical experience in the sort of non-factual language that is not parasitic on the "true demonstratives", which explains why mystics often employ poetry or allegory to convey their mystical experiences.

One can, therefore, specify the sense in which, for *PI* (272), one *can*, and the sense in which one *cannot*, put mystical experiences into words. One *cannot* put mystical experiences into words in the sense in which one can put ordinary or scientific "factual" discourse that is parasitic on the "true demonstratives" into words. That is, in mystical discourse one cannot say of some possible "this" that it falls under some sortal expression whose meaning is described in the grammar of public language. One *can*, however, put a mystical experience into words in the sort of non-factual language, like poetry or allegory, that does *not* "hook onto" reality by means of "true demonstratives" like "this". There is, therefore, no contradiction between the fact that mystics often claim that mystical experiences cannot be "said" in words and then go on to "say" a lot about them. For there are two different notions of "saying" involved. The basic point is that there are two very different sorts of language and of their relation to "reality"[7] involved here. There need not, therefore, be any "inconsistency" in mystical discourse of the sort that Carnap finds in Wittgenstein's *TLP,* and the same conclusion can be extended to mystical discourse in other philosophical, religious, and literary figures. One can, however, also infer from this that the Wittgenstein of *TLP* is the first "autobiographical" sort of mystic mentioned by Brightman (1991, 393-94) in the epigraph above, that is, the kind that says that the mystical is ineffable, but then proceeds to say a lot about it, while the Wittgenstein of WLP is the second sort of mystic mentioned by Brightman, namely, the "radiant" kind that "finds the inner light within in the silence of their own souls" and "let[s] their light shine without the accompanying clamor of words." Thus, the reason Nieli cannot find any mysticism in WLP is that he is looking for the "autobiographical" mysticism found in *TLP*, but the mysticism of WLP is of the latter "radiant" sort exhibited by so many of the great mystics of the East who let "their light shine without the accompanying clamor of words".[8]

NOTES

[1] By Wittgenstein's early philosophy is here meant his Tractatus. By his "later philosophy" is here meant his Philosophical Investigations (PI). The sole reference in the present paper to his Culture and Value (CV) is a 1931 remark during the transition from his "earlier" to his "later philosophy". References to TLP are either to the Preface or to proposition number. References to CV are to page number. References to PI are either to paragraph number, i.e., (PI, 435), or to page number, i.e., (PI, p. 230) as required.

[2] For a discussion of Tagore's and Radhakrishnan's views that mystical insight concerns a higher "reality" see Tagore (1924) and Conger (1991).

[3] It is surprising that though Stace (1960, 46, 291) mentions Wittgenstein several times concerning broadly logical or linguistic points, he never once mentions *TLP's* mysticism.

[4] For an account of Wittgenstein's general opposition to mechanistic views of language in WLP see McDonough (1989).

[5] It does not follow from this that mystical language is not in some sense about a "higher reality" but only that it is not about a "reality" that can only be spoken about by means of *"true demonstratives*".

[6] For a discussion of ordinary language philosophy see Parker-Ryan 2018, [I]! However, one should not assume from the fact that that the ordinary language philosophers do not acknowledge the mystical in their philosophical views that they are *necessarily* antithetical to it. The present author was once visiting Norman Malcolm in his university offices when he opened a large package in his mail containing an illustrated edition of the *Tao Te-Ching* that he had previously ordered. Malcolm exclaimed "Magnificent!" and stated how impatient he had been for its arrival. It is noteworthy that Malcolm was, in his day, one of the rare genuinely religious philosophers, an Anglican (McDonough [I]. §1), and the Anglicans do have a significant mystical tradition (Ball 2007, Chap. 5).

[7] See notes 2 and 5 above!

[8] It is noteworthy that when Harry Frankfurt (2013, 112) met Wittgenstein while visiting Malcolm at Cornell in 1949 (McDonough, [I], § 1), he stated that Wittgenstein "shown … with a very remarkable, nearly incandescent inner light - a light of single minded and uncannily concentrated pure dedication to a search for clarity and truth" and seemed "almost supernaturally dedicated to these ideals". Wittgenstein's person, so to speak, "radiated" his spiritual earnestness.

REFERENCES

The abbreviations in the text refer to: Wittgenstein 1958 (PI); Wittgenstein 1961 (TLP). The author also uses some other common abbreviations (such as WLP = "Wittgenstein's Later Philosophy", PLA = "Private Language Argument).

Ball, Peter. 2007. *Anglican Spiritual Direction*. Harrisburg-New York: Church Publishing.

Biletzki, Anat, and Matar, Anat. 2018. "Ludwig Wittgenstein". *Stanford Encyclopedia of Philosophy* URL: https://plato.stanford.edu/entries/wittgenstein/

Brightman, Edgar Sheffeld. 1991. "Radhakrishnan and Mysticism." In *The Philosophy of Sarvepalli Radhakrishnan,* edited by Paul Arthur Schlipp, 393-415. LaSalle: Open Court.

Black, Max. 1970. *A Companion to Wittgenstein's Tractatus*. Ithaca: Cornell University Press.

Carnap, Rudolf. (1969). "Philosophy and Logical Syntax." In *Readings in Twentieth Century Philosophy*, edited by William Alston and George Nakhnikian, 424-460. New York: The Free Press.

Conger, G.P. 1991. "Radhakrishnan's World". In *The Philosophy of Sarvepalli Radhakrishnan,* edited by Paul Arthur Schlipp, pp. 82-112. La Salle: Open Court.

Frankfurt, Harry. 2013. "Reflections of my Career in Philosophy". In *Portraits of American Philosophy*, edited by Steven Cahn, 105-128. Lanham, MD: Rowman and Littlefield.

Garver, Newton. 1994. *This Complicated Form of Life: Essays on Wittgenstein.* Chicago: Open Court.

Georgi, Geoff. [I]. "Demonstratives and Indexicals". *Internet Encyclopedia of Philosophy*. https://www.iep.utm.edu/dem-indx/

Hallett, Garth. 1977. *A Companion to Wittgenstein's Philosophical Investigations*. Ithaca: Cornell University Press.

James, William. 2012. *The Varieties of Religious Experience*. London: Routledge.

Kannath, Sheila. 2004. "Mystical Consciousness of Bhakti in Tagore". *Journal of Dharma: Dharmaram Journal of Religions and Philosophies* 29 (3): 371-386.

Kaplan, David. 1989. "Demonstratives: An Essay on the Semantics, Logic, Metaphysics, and Epistemology of Demonstratives and Other Indexicals'". In *Themes from Kaplan*, edited by J. Almog, J. Perry, and H. Wettstein, 481-563. Oxford: Oxford University Press.

King, Richard (2002), *Orientalism and Religion: Post-Colonial Theory, India and "The Mystic East".* London: Routledge.

Malcolm, Norman. 1986. *Nothing is Hidden*. Oxford: Blackwell.

Matilal, B. 1975. "Mysticism and Reality: Ineffability". *Journal of Indian Philosophy* 3 (3-4): 217-252.

McDonough, Richard. 1989. "Towards a Non-Mechanistic Theory of Meaning". *MIND* 98 (389): 1-21.

________. 2015. "Monk on Russell's Heart of Darkness". *Russell: The Journal of Bertrand Russell Studies* 35 (2): 29-42.

________. "Norman Malcolm (1911-1990)." *Internet Encyclopedia of Philosophy*
URL: https://www.iep.utm.edu/malcolm/

Michaelson, Eliot. 2019. "Reference". In *Stanford Encyclopedia of Philosophy*. URL: https://plato.stanford.edu/entries/reference/

Monk, Ray. 1990. *The Duty of Genius*. New York: Penguin.

Nieli, Russell. 1987. *Wittgenstein: From Mysticism to Ordinary Language*. Albany: State University of New York Press.

Parker-Ryan, Sally. 2018. "Ordinary Language Philosophy". *Internet Encyclopedia of Philosophy*.
URL: https://www.iep.utm.edu/ord-lang/

Phillips, Bernard. 1991. "Radhakrishnan's Critique of Naturalism". In *The Philosophy of Sarvepalli Radhakrishnan*, edited by Paul Arthur Schlipp, 115-171. La Salle: Open Court.

Radhakrishnan, Sarvepalli. 1991. "Autobiography of Sarvepalli Radhakrishnan". In *The Philosophy of Sarvepalli Radhakrishnan*, edited by Paul Arthur Schlipp, 3-82. La Salle: Open Court.

Sellars, Wilfrid. 1997. *Empiricism and the Philosophy of Mind*. Cambridge: Mass: Harvard University Press.

Sinha, Harendra Prasad. 1994. *Religious Philosophy of Tagore & Radhakrishnan*. Delhi: Motilal Banarsidass.

Stace, W.T. 1960. *Mysticism and Philosophy*. Los Angeles: Jeremy Tarcher, Inc.

Strawson, Peter. 1954. "Review of Ludwig Wittgenstein's *Philosophical Investigations*." *MIND* 63: 70-99.

Tagore, Rabindrananth. 1935. *Forward to Sarvepalli Radhakrishnan's* The Philosophy of the Upanishads. London: George Allen and Unwin.

________. 1994. "The Fourfold Way of India." In *The English Writings of Rabindranath Tagore: A Miscellany*, edited by Sisir Kumar Das, 495-503). Delhi: Sahitya Akademi.

________. 2014. "The Unanswered Questions: Some Remark's on Tagore's Late Style". In *Rabindranath Tagore in the 21st Century*, edited by Debashish Banerji, 265-276. Berlin: Springer.

Underhill, Evelyn. 1999. *Mysticism: The Nature and Development of Spiritual Consciousness*. Oxford: Oneworld.

Wittgenstein, Ludwig. 1958. *Philosophical Investigations*. Translated by G.E.M. Anscombe. Oxford: Blackwell.

________. 1961. *Tractatus-Logico-Philosophicus*. Translated by David Pears & B.F. McGuiness. London: Routledge Kegan Paul.

________. 1980. *Culture and Value*. Translated by Peter Winch. Chicago: University of Chicago Press.

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Cognitivism and Motivation Internalism

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Abstract

As a central thesis in metaethics, motivation internalism promises to explain the connection between moral judgement and motivation. Generally, it is defended both by non-cognitivists and cognitivists. While not accepting non-cognitivism, I ultimately reject any form of cognitivist position, which claims that cognitivism is not compatible with Hume's psychology of motivation. Precisely, I argue that the connection between moral judgement and motivation is neither necessary nor internal. Based on the possibility of moral indifference, I counter the claim, namely that moral judgement is essentially motivational. Although, my goal is not to offer any positive explanation of the connection in question, but the result of my argument has implications on why we are better off accepting Hume's psychology as cognitivists.

Keywords: motivation, internalism, metaethics, moral judgement, cognitivism

[O]ne can be indifferent to morality without error.
Philippa Foot, 1978

I. Introduction

The internalist debate about actions has taken various turns in recent years. However, as a result of different characterizations of internalism, philosophers seem to be talking past one another (e.g. Brink 1989; Darwall 1992; Tresan 2009). Against this background, I will proceed by clarifying its major manifestations as it allows us to situate this paper in a context. First, internalism refers either to *justifying reasons* or *motivating reasons*. In the former, the argument is that what counts for or against an action is internally embedded on the agent's subjective psychological profile. For example, Bernard Williams (1981) argues that an agent' reason for action is always *internal* insofar as such reason is rooted in the subjective motivational set or desires. On this view, an agent necessarily acts accordingly just in case she has a reason (Rosati 2014). This position is also referred to as *reason internalism* or *internalism about practical reason*.[1] In the latter, the main concern is about motivating reasons for action. Unlike reason internalism, this form of internalism focuses on the connection between moral considerations and motivation, hence *motivation internalism* (hereafter, MI). While it is generally agreed that moral considerations make practical claims on us (e.g. Brink 1989; Smith 1994; Railton 2006), the motivation internalists argue that what moves us is internal to moral considerations. That is, it is the case that moral considerations motivate agents necessarily (e.g. Nagel 1970; McNaughton 1988; Bromwich 2013). Although reason internalism and motivation internalism are *prima facie* distinct positions, it would be incorrect to claim that they do not overlap in some cases (Pettit and Smith 2006). Zangwil (2008) argues that one set of issues is sometimes appealed to in arguments concerning the other. Regardless of this overlapping tendency, I will focus mainly on motivation internalism (hereafter MI).

That said, MI is defended on the bases of the psychological states underlying the moral considerations in question. Normally, it is explained either in terms of conative states or cognitive states. In the former, it is the thesis that non-cognitive states such as pro-attitudes motivate agents. Precisely, non-cognitivism strongly claims that conative states are motivational states. On this construal, to judge is to express these mental states; and they in turn motivate necessarily. This position is considered compatible with Hume's understanding of psychology of motivation (Coleman 1992; Stroud 1977; Mackie 1980; Darwall 1983). Hume claims that moral motivation is guaranteed by desires. The defenders of the latter position appeal to cognitive states to explain the necessary connection between moral considerations and motivation. On this view, cognitive-based moral considerations motivate not only necessarily, but also sufficiently (Korsgaard 1996; Nagel 1970; Scanlon 1998; Platts 1980; Bromwich 2010, 2013). I refer to this view as cognitivist motivation internalism.

In this paper, I reject any form of cognitivist position, which claims that (1) cognitivism is not compatible with Hume's psychology of motivation. Precisely, I argue that the connection between moral judgement and motivation is neither necessary. While my goal is not to offer any positive explanation of the connection in question, but the result of my argument has implications on why we are better off accepting Hume's psychology as cognitivists. In the remainder of the paper, I provide some background before specifying the form of motivation internalism relevant to our discussion. Next, I present Olivia's case as an argument against the claim that moral judgement is motivationally efficacious. Finally, I conclude with some remarks.

II. Features of Motivation Internalism

One of the features driving MI is the idea of necessity.[2] The necessity claim is the view that the connection between moral considerations and motivation is unconditional and not contingent. As we shall see later, the necessity claim of MI comes in metaphysical and conceptual forms. The cognitivists construe the content of moral considerations differently, for example it can refer to moral properties, moral facts, moral judgements, moral beliefs, etc. Suppose we take moral property, say moral goodness, as an instance of moral consideration. The cognitivist of this tradition would argue that moral property necessarily guarantees motivation. For example, Plato once held that 'knowing' the good is necessarily 'doing' the good. We can formulate this claim as follows:

> Plato's MI: Necessarily, if an agent *knows* that φ is morally good, then she is moved to act accordingly.

According to John L. Mackie,

> Plato's Forms give a dramatic picture of what objective values *would have to be*. The Form of the Good is such that knowledge of it provides the knower with both a direction and an overriding motive;

> something's being good both tells the person who knows this to pursue it and makes him pursue it. (Mackie 1977, 37)

Plato's understanding of MI is based on *moral ontological* and *epistemological grounds*. On Mackie's reading, Plato's moral goodness is not only action-guiding, but also it provides moral agents with the motivating reasons for action. Notice that motivation is overridingly tied to moral goodness, such that if an agent knows that something is morally good, he is necessarily motivated to do it. The knowledge of moral goodness rules out the possibility of moral motivation being toppled by other competing desires, motives etc. Bromwich describes this form of MI as *decisive internalism* (Bromwich 2008).

However, Plato's MI is disturbing at least on three fronts. First, ontologically, it assigns a rather strange character to moral values. Indeed, Mackie was right when he said, "If there were objective values then they would be entities or qualities or relations of a very strange sort, utterly different from anything else in the universe" (Mackie 1977, 38). Second, epistemologically, it reduces the accessibility of moral properties to special intuitive faculty. Again, Mackie says, "Correspondingly, if we were aware of them, it would have to be by some special faculty of moral perception or intuition, utterly different from our ordinary ways of knowing everything else" (Mackie 1977, 38). In other words, this will require moral agents to possess special intellectual faculty for perceiving moral properties. Needless to saying that such a demand would render agents lacking such faculties motivationally unfit. Third, although Plato's claim seems to secure the connection between moral property and motivation, it fails to account for motivational failures, which are as well part of our moral experience. In other words, it is unable to justify some cases (e.g. weakness of will, overridingness of stronger emotions) which are significant parts of our moral experience.

Above all, Plato's MI is non-constitutive, that is, it does not involve an agent's formation of first-person moral beliefs. This applies to other forms of MI, which hold that perception of right and wrong necessarily motivates (e.g. Price 1965; McDowell 1979, 1981). For the purpose of this paper, I focus on the claim that moral beliefs necessarily motivate. This form of cognitivist motivation internalism involves agent's (first-person) expression of moral judgement, hence constitutive. In other words, I will not pursue the claim, namely that knowledge of, actual consciousness of or cognitive contact with moral property" motivates necessarily beyond this point (Darwall 1992, 157).

III. Moral Judgement and Motivation Internalism

While the content of moral judgement is understood differently in metaethics, I will restrict myself to the cognitivist understanding. On this construal, moral judgement is on par with the ordinary act of judging. Hence, by judging something to be morally wrong, an agent is affirming a state of affair in the world. For example, by judging that torture is morally wrong, Peter is both asserting and affirming something about the wrongness of torture. Normally, such affirmations are said to entail belief. That is, Peter *affirms* that torture is morally wrong, because he *believes* that the act is morally significant. Since to judge is to express one's belief about something, which can be true or false. I will use 'moral judgement' and 'moral belief' interchangeably.[3] To this end, the cognitivist MI claims that such moral beliefs motivate necessarily. Precisely, they motivate agents independent of any antecedent or mediating desires. Generally, we can formulate this view of MI as follows:

> Cognitivist Motivation Internalism (CMI): Necessarily, if agents *judge* or *believe* that they are morally required (or morally ought) to φ, they are motivated to φ.

Unlike Plato's view, this form of MI provides space for agents' engagement in "deliberative process of practical reasoning and judgement" (Darwall 1992, 158). Brink refers to it as *appraiser internalism,* the claim that

> It is in virtue of the concept of morality that moral belief or moral judgement provides the appraiser with motivation or reasons for action. Thus, it is a conceptual truth about morality, according to appraiser internalism, that someone who holds a moral belief or

> makes a moral judgement is motivated to, or has reason to, perform the action judged favourably (Brink 1989, 40).

The content of moral beliefs makes practical claims on agents holding them. Hence, CMI is to be understood as the thesis that moral belief guarantees motivation insofar as agents hold the content of such belief as true. The guaranteed motivation "rests upon the nature of belief itself and upon the content of the belief that one is (oneself) morally required to…." (Mele 1996, 729). It is by believing that something is morally bad (or morally good) that agents are said to be *judges* or *appraisers*; and the cognitivist motivation internalist (hereafter, internalist) claims such moral judgement necessarily motivates moral agents to act accordingly.

Furthermore, the necessity claim of MI carries some sort of metaphysical commitment. Roughly, it is supposed to apply to *all persons* and *possible worlds* sharing the concept of morality. Tresan (2009, 54) argues that "to get internalism we must posit accompaniment, not just actually, but throughout possible worlds. That is, 'Entail' indicates that the accompaniment is necessary". When applied to CMI, it implies that *all* agents judging or believing that φ is morally required are motivated to φ (at least if φ is understood as *normatively unqualified).* This claim amounts to the following:

> It is necessary that any agent in any possible world who judges or believes that φ is morally required is motivated to φ.

This claim holds provided the agents' content of belief is the same (or at least similar) across possible worlds. However, notice that it does not say anything about the agents' psychological profiles. Suppose we characterise their psychological profile as 'normal'[4], the content of such moral beliefs is said to motivate globally. Assuming the necessity claim sticks, the internalist is claiming that if any agent believes φ to be morally required, she is necessarily motivated to φ, regardless of the world in question. I refer to this claim as a core feature of CMI. Moreover, if the internalists claim is correct, they would be "advancing a reformative conception of cognitivist belief and alternative to a Humean theory of motivation" (Mele 1996, 736). However, we shall see shortly that this view is false.

In the *Treatise on Human Nature*, Hume argues that "morals excite passions and produce or prevent actions. Reason of itself is utterly impotent in this particular. The rules of morality, therefore, are not the conclusions of our reason" (THN 457/294). Since reason is considered as a faculty for forming beliefs, we formulate Hume's claim as follows:

(1) (Moral) beliefs do not motivate because they are inert in this regard.

For the internalist to prove that Hume's constraint on belief is false, he has to justify that moral beliefs motivate in the first place. However, such justification has to be at least on the same strength of attack levelled against beliefs. Notice that Hume's attack is not just that moral beliefs do not motivate, but that genuine beliefs do not motivate at all. Therefore, the internalist rebuttal must not be that moral beliefs or some of them *can* motivate, but that they *must* motivate (Shafer-Landau 2000, 279). Bromwich, for one, argues that internalists have reasons to charge against Hume's constraint without diluting their position. According to her, all moral beliefs motivate *simpliciter*.[5] The success of anti-Humeanism depends on refuting the claim on (1) without admitting defeasibility (Bromwich 2009, 2013).

Assuming Bromwich's claim is correct, then moral belief will not just motivate simply because it is moral, but because it is essentially belief. Based on this, we can impute the following claims to CMI.

(2) Since (moral) beliefs motivate *simplicite*r,
(3) It is necessary that, for any agent A, and for any action φ, if A judges that she is morally required (or that it is right) to φ, then she is motivated to φ.

In other words, the internalist can only show that (1) is false by proving that (2) is true, however not on per *ceteris paribus* basis. I take this view as the standard construal of MI, namely the claim that the connection between moral judgement and motivation is internal and not defeasible (Brink 1989, 8; Tresan 2009, 53-54). However, I will shortly show why CMI is false, but before then it is important to consider two more core features of MI.

IV. Internality and Conceptuality

Apart from the necessary connection between moral belief and motivation, MI claims that motivation is essential to moral judgement. Mele writes that "…what is guaranteed, more precisely, is that motivation […] is built into any belief that one is (oneself) morally required to [...] and is internal to the belief of that kind in this sense" (Mele 1996, 730). It is this in-built force that explains why agents are necessarily motivated upon believing that they are morally required to do something. In *Obligation and Motivation in Recent Moral Philosophy*, W. K. Frankena points to the essentiality claim as follows: It is so rooted in moral considerations that it is logically impossible for agents not to be motivated even if they lack actual or dispositional motives for doing what is morally required (Frankena 1958, 40-41). Notice that in order to justify (2) the internalist has to root such intrinsic or built-in motivation force in moral beliefs alone. Zangwill was right when he argued that "the internalist needs to claim not just that moral beliefs are necessarily motivating, but that motivation is essential to moral beliefs" (Zangwill 2008, 94). Another way of making the essentiality claim of MI is to ask whether motivation is embedded on the content of moral belief or not. In response to this, Roskies writes that motivation "must hold in virtue of the content of the moral belief itself, not in virtue of some contingent or auxiliary non-moral fact or reason" (Roskies 2003, 52). The necessity claim would make sense just in case motivation is internal, that is, essential to moral judgements. According to Fine (1994), the necessity claim does not entail the essentiality claim, because it is possible for the former to hold without the latter. For example, an internalist can believe that there is necessary connection between moral judgement and motivation, while denying that such motivation springs from moral beliefs. Following Zangwill, I argue that MI should be construed not only in terms of the necessity claim, but also as an essentiality claim because "if motivation is essential to moral beliefs, that would explain why moral beliefs are necessarily motivating" (Zangwill 2008, 95).

The third feature of MI is the conceptuality claim. MI has been largely understood as a conceptual claim. That is, the necessary connection is understood as an essential part of our ordinary language and meaning of moral terms (Strandberg and Björklund 2013). If only roughly, just as it is part of the ordinary meaning of terms, for example, to understand a bachelor as an unmarried man, the internalist sees the concept of moral judgement as motivationally efficacious. For example, Nagel argues that "motivation must be tied to the truth, or meaning, of ethical statements that when in a particular case someone is (or perhaps merely believes that he is) morally required to do something, it follows that he has a motivation for doing it". (Nagel 1970, 7) The conceptual claim seems to reflect the folk intuition about moral motivation. However, it is debated whether such intuition is conclusively on the side of internalism (for more discussion see Strandberg and Björklund 2014; Roskies 2003).

V. Cognitivist Motivational Internalism

The version of internalism relevant to this paper does not understand the motivation force of moral belief as overriding. Rather, it holds that the *necessary* connection between moral belief and motivation is *not defeasible*; and that moral judgement motivates essentially. To illustrate,

> It is *necessary* that, for any agent A, and for any action φ, if A judges that she is morally required (or that it is right) to φ, then she is *efficaciously* motivated to φ by her moral judgement *alone* and not by external desires, feelings or emotions.

While motivation might fail in the face of other competing factors or states, the necessary connection is not defeasible; and motivation is internal to moral judgement. What such a robust MI tends to block is the problem of creeping (actual or conceptual) external factors – for example, desires, non-moral motives, etc. (Bromwich 2010, 19). The internalist concern is to avoid accepting Hume's psychology of motivation. Hence, to secure the necessary connection, his task to show that motivation directly stems from moral beliefs. Assuming this strategy works, then it would be correct to say that CMI "[is] a sort of Holy Grail of meta-ethics. It offers us all we ever wanted from morality. The internalist claim gives morality the psychological "oomph" it needs to motivate action by itself, rather than having to hitch [a] motivational ride on pre-or non-moral motives. The realist thesis makes morality what it seems to be: a discourse about facts—moral facts—which we can discover, about which we can disagree, and of which we can often convince each other" (Noggle 1997, 88).

The argument of the internalist must be effective in explaining that moral motivation stems from the *content* of the agent's moral beliefs alone. In other words, he has to justify how motivation is internal to moral beliefs without relying on any actual or hypothetical psychology that is external to the content of the moral judgements. On the contrary, in what follows, I will argue that the CMI is false. Precisely, I argue that moral motivation is neither internal nor essential to moral judgement. My argumentative strategy is to show that an agent can make genuine first-person moral judgements and yet fail to be motivated.

VI. Olivia's Case: An Argument against Cognitivist Motivation Internalism

Consider Olivia is a professor of moral psychology. Recently, she had a long conversation with Emma, a doctoral student at the department she was visiting. They discussed the dangerous impacts of climate change, especially on women and children from poor countries. Olivia argues convincingly that we are obliged to protect our environment. She strongly believes that any action with harmful impacts on the environment is morally impermissible. At the end of the conversation, Emma came to share her moral conviction: *Environmental harm is morally unjust*. Later, Emma and some of her friends filed a petition against the university authority on the grounds of some of its environmental unethical practices. Their target was to collect 1000 signatures. Within the space of three weeks, the petition gained an overwhelming support from both the professors and students, thanks to Olivia's moral conviction. However, when Emma approached Olivia to get her signature on the petition, she *declined* to sign it. She never doubted whether the university's policy was an instance of environmental injustice. She continues to believe that environmental harm is morally unjust and the policy in question is morally impermissible. Even at that Olivia does not seem to care about the issue at stake. In other words, she is indifferent about signing the petition – she is indifferent about the moral issue in question.

The phenomenon of indifference is part of human experience. We witness cases where people remain indifferent to various issues, ranging from simple to complex everyday issues. It is not rare to encounter people who do not *care* about what they believe. Indifference is, as well, an essential part of our moral experience. It is not queer to claim that people exhibit indifference in the face of moral demands or issues. Even though it is a contestable position, moral indifference is defended in philosophy (Foot 1972; Stocker 1979; Milo 1981; Brink 1989; Mele 1996; Svavarsdȯttir 1999; Zangwill 2008).[6] Zangwill writes: "it certainly seems that moral indifference is no mere abstract philosopher's possibility, but a common actual phenomenon" (Zangwill 2008, 102).

The idea of indifference, if only roughly, is about the degree of people's interest or care about what they believe. "Intuitively, we want things more than others, and we believe some things to a greater degree than others. (We are more confident of some claims than others.) Our mental world is not black and white." (Zangwill 2008, 95) This experience replicates in the domain of morality, we care about things more than others as well as believe things in different degrees. And the degree with which agents care about moral demands or issues determines to a large extent their motivation. Hence, our strategy is to show that CMI fails to capture this phenomenon in its psychology of moral motivation. Alternatively, if it is the case that the argument from indifference succeeds, then the claim that motivation is internal to moral belief is false given the possibility of holding a genuine moral belief and yet not caring about morality.

Moral indifference is the belief that it is, in fact, possible for someone to *know* or even *believe* that he or she is morally required to do something and yet not *care* about it. The phenomenon of indifference differs in its various construal of caring about the requirements of morality. For example, an agent might be presented as either 'not caring at all' or 'not caring very much' or 'caring less' about moral requirements (Zangwill 2008, 101). Zangwill rightly pointed out that we must not present indifferent agents as people who reject morality. The temptation of painting moral indifference as rejection of morality is seen in the case of *amoralism*. On the one hand, some externalist might think that it is only such a strong position of amoralism that guarantees a decisive counterexample to internalism. On the other hand, internalism seems to attack externalist cases of indifference from the perspective of rejection of morality, thinking that such a position flies in the face of categoricity of moral requirements. However, we are not claiming that moral demands do not apply to indifferent agents. Rather it is argued here that agents are not motivated by them because they do not care enough about moral issues (we shall return to this issue in the next chapter). That said, given that we do not need to construe indifference in such a strong term – complete indifference, we shall take it as "*the phenomenon of not caring very much about the demands of morality*" (Zangwill 2008, 101). Following Zangwill's framework of indifference, our goal is to argue that people's interest, care or desires come in various strengths; and that indifference is actually possible because people care varyingly about moral issues. Assuming this argument works, then it serves as a counterexample to CMI's efficacy claim.

Zangwill's indifference argument is premised on the idea of degrees of beliefs as well as strengths of desires. He rightly pointed out one of the often-overlooked elements of moral motivational debate is the "Proportional Determination Thesis", the view that the "strength of moral desire is proportionately determined by degree of moral belief" (Zangwill 2008, 95). The internalist claim implies, among other things, that motivation is essential to moral beliefs, that is, they motivate efficaciously. Given this, it is argued that if two persons are alike in their moral beliefs, it is necessary that they will be motivated alike given the claim that moral beliefs are motivationally efficacious independent of any additional desires. In other words, it is not a matter of contingency that motivation follows directly given that their moral beliefs are alike in every respect. It would only amount to inconsistency should the internalist claim that the content of belief of one of the persons is motivational efficacious, whereas the other not. If motivation is essential to moral beliefs as internalist claims, then the content of moral beliefs of agents with equal cognitive dispositions *must* motivate them alike. On the contrary, it is actually possible for agents to share similar cognitive states, dispositions, beliefs and yet motivationally respond differently. Consider the Augustine's example in *De Civitate Dei*:

> Suppose that two men, of precisely similar disposition in mind and body, see the beauty of the same woman's body, and the sight stirs one of them to enjoy her unlawfully, while the other continues unmoved in his decision of chastity. What do we supposed to be the cause of an evil choice in the one and not in the other? What produced that evil will? …The mind? Why not the mind of both? For we assumed them to be alike in both mind and body […] What other reason could there be than his will, given that their dispositions were precisely the same, in body and mind?

An agent might hold a genuine moral belief, but if he does not care about the desirability of the belief that he ought to do the action, he will not be motivated by his moral belief. In other words, given the different intensities of individual's care about moral issues, it is possible that the phenomenon of indifference might occur between two persons sharing similar moral beliefs. More so, Zangwill argues that indifference can as well "be a matter of a person ceasing to care as much as he used to while his moral beliefs remain unchanged. Or it might be the possibility that a person at a time cares less than he actually do at that time while moral beliefs remain constant" (Zangwill 2008, 101).

VII. Explaining Olivia's Behaviour

Olivia exhibits features of indifference: She does not seem to care, at least, about the moral issue at stake. Although, she is capable of forming and holding genuine moral judgements, she remains unmotivated or unmoved by them. We can attempt explaining her behaviour based on the two main categories outlined by Zangwill, namely the trans-personal and trans-temporal cases of indifference. In the former case, recall the incident between Olivia and Emma. Both share the moral belief that environmental harm is morally unjust. However, while Emma was motivated, Olivia remained unmoved in the face of the same moral belief. The internalist thinks that her behaviour is odd given that motivation is essential to moral beliefs as well as the fact that their moral beliefs are alike. However, Olivia's behaviour is not odd. It is actually possible that the strengths of her interests or care about moral issues vary. To illustrate this, imagine that Olivia was once highly active and took part in various environmental actions. However, recently she experienced that all their efforts made no (substantial) difference at all. Increasingly, her motivation to engage in such actions starts to dwindle, although she still strongly believes that the cause is morally right and even warrants actions. Now, she is completely worn out to act accordingly.

Furthermore, assuming we rule out the cases of errors related to cognition and applications of moral concepts; and that they share precisely similar dispositions in mind and body. It is possible that she was not moved because not of her moral belief was less genuine than that of Emma, but because she does not care very much about the moral issue in question or moral demands in general. As we illustrated above, it is possible that she once cared about such actions, but now such a motivation is longer there. Given this, it might be claimed, contrary to the internalist claim, that:

> If agents A and B judge that φ is morally required, it is possible for A and B to be motivated differently (hence, not necessarily to φ) given their respective degrees of care about φ, while their moral belief φ remains unchanged.

In the latter case, namely, the trans-temporal case of indifference. Suppose Olivia used to care about morality, but of lately she started caring less about moral issues. It might as well be that she cares about moral issues, but of lately she started caring not very much about environmental matters related to morality. On this level, her care about moral demands has become less than usual. As in the first case, she not only grasps the content of moral belief, but also, she genuinely believes that environmental harm is morally unjust and yet she has no motivation to sign the petition. Given this, it might be claimed, contrary to the internalist claim, that:

> If an agent A judges that φ is morally required, it is possible for A not to be motivated given a change in her care about φ, while his moral belief φ remains unchanged.

Notice that in both cases that Olivia did not completely reject moral demands, at least, she continues to hold her moral beliefs. Notice also that other concerns did not matter more to her than morality. In other words, she is indifferent to her moral belief, because her care about the moral issue in question is not proportionately determined by the degree of her moral belief. Alternatively, it is possible that a change in Olivia's moral belief will not necessarily provide a change in her care about a new belief as the internalist claims. This is because we seem to care more or less about morality regardless of the genuine contents of moral beliefs we hold. In sum, Olivia might share moral beliefs with the rest of us, but if she cares less, she will be indifferent to morality. So also, she might have cared about morality (like the rest of us), but if she cares less now than usual, she will be indifferent to the moral demands that she used to care about, while her moral beliefs remain unchanged.

Nevertheless, it might be argued that moral beliefs and caring to act accordingly do not come apart. That is, to believe that X is morally required is inevitably to be motivated to X. Given that moral beliefs are taken to be best practical judgements of reason; it is argued that agents cannot fail to be motivated by what they judged as morally required. However, do we necessarily adhere to (even the best of) our moral judgements? The mere fact that we want certain things more than others or believe certain thing to have greater degrees than others, if only roughly, seems to show, at least, the possibility of caring less about what we judge as good. In other words, denying this possibility seems to amount to the following claim, namely, 'to believe something is necessarily to care about it'.[7] Such a denial is problematic, for it might place the idea of moral agency under a grave risk. If agents lack the possibility of choosing freely, morality would become a suspicious enterprise. In addition, such a move might lead to determinism, the sort that eliminates the possibility of freedom to choose. It is against this background that Henry of Ghent in his *Quodlibet* argues thus: "We must assume that [there is] over and above the freedom in reason to judge [*libertas arbitrandi*] and there is in the will a freedom to choose what is judged [*libertas eligendi arbitratum*], so that the will does not choose with any necessity even what reason judge after deliberation"[8] (Henry, *Quodl.* 1. 16, 5:102; Hoffmann, 2008).

Olivia has the possibility of (freely) choosing to care more or less about moral issues. She can as well choose not to care as she used to in the past. The case of indifference, hence Olivia's case, is actually possible given that people have the possibility of choosing to care or not to care at all; and there are cases where people freely decide to be indifferent to moral issues. Consider one of the three examples presented by Zangwill:

> [A] mercenary I once met on vacation exuded moral indifference. He was in control, reflective and articulate. Everything he said convinced me that he was perfectly aware that his vocation was genuinely morally wrong, not merely what people conventionally call 'wrong'. He fully understood the wrongness of his vocation. Nevertheless, he was not very concerned about that. He was more concerned with his immediate interests and concerns, that is, colloquially, looking after number one. There was no moral cognitive lack. He made that quite clear. Indeed, he insisted on it. The mercenary was unusually indifferent to the demands of morality; but he shared moral beliefs with the rest of us, and with his former self. He insisted on that (Zangwill 2008, 102).

Like Olivia, the mercenary in Zangwill's example is not suffering from psychological impairments. He knows fully well that his 'vocation' is morally wrong, hence knows what morality demands, but he freely chooses to be indifferent to those demands. In fact, he freely chose this vocation. Cases of moral indifference are part of our ordinary moral experience; and Foot elegantly captures the possibility of this phenomenon in the following words: . . . one [*can*] be indifferent to morality…. (Foot 1978, xiv). The emphasis is on *can* – normal people *can* freely choose to reject morality or care more or less about moral issues. We can be indifferent.

VIII. Conclusion

The fact that we can decide against our best practical judgement of reason explains why we can actually desire to be morally indifferent or even bad. In his work '*Desiring the Bad: An Essay in Moral Psychology'* Michael Stocker argues that these phenomena are actually part of our moral experience. Stocker argues that there are cases where people fail to be motivated or act according to their best decision or intention (Stocker 1979). In essence, it is not case that moral judgement of what one believes to be good or morally required to do necessarily motivates accordingly. Stocker writes that: "motivation and evaluation do not stand in a simple and direct relation to each other, as so often supposed. Rather, they are interrelated in various and complex ways, and their interrelations are mediated by large arrays of complex psychic structures, such as mood, energy, and interest" (Stocker 1979, 738-9). In other words, cases such as Olivia's seem to show that it is possible to hold moral beliefs and yet not be motivated accordingly. In other words, the necessity and essentiality claims are false. Although, it is not my goal to defend any positive explanation of the connection between moral judgement and motivation, I think that we are better off accepting the Hume's psychology of motivation as cognitivists. Such a combination allows us to integrate the important roles of desires, emotions, self-identity etc.; and thereby, better explaining the (moral) motivational profiles of agents by (de Sousa 1987; Colby and Damon 1992; Frankfurt 1998).

NOTES

[1] For more discussions on argument for internal reason see Goldman 2005; Manne 2014.
[2] These features are not peculiar to cognitivist motivation internalism. They also apply to non-cognitivist version of MI just in case moral judgements are understood as expressions of conative states.
[3] However, it is understood as a cognitive state.
[4] Such agents are said to be normal in the absence of psychological conditions such as depression, weakness of the will, spiritual exhaustions, etc.
[5] She puts this thus: "In defence of this thesis (that is, cognitivist internalism) it is tempting to either argue that the Humean constraint only applies to non-normative beliefs or that moral beliefs only motivate *ceteris paribus*. But succumbing to the first temptation places one under a burden to justify what is motivationally exceptional about moral beliefs and succumbing to the second temptation saddles one with a thesis that fails to do justice to the practicality intuition that cognitivist motivational internalism is supposed to capture" (Bromwich 2009, 2).
[6] Although Frankena defends an internalist position, he believes that moral indifference is possible. He writes thus: "It has not seemed to me inconceivable that one should have an obligation and recognize that one has it and yet have no motivation to perform the required action" (Frankena 1958, 42-43).
[7] On the contrary, belief is different from caring, the former is cognitively-laden, whereas the latter is an emotional capacity.
[8] Super libertatem ergo arbitrandi in ratione oportet ponere libertatem eligendi arbitratum in voluntate, ut voluntas nulla necessitate eligat etiam

quod ratio sententiat....” (*Quodl.* 1. 16, 5:102; Hoffmann, Tobias (2008). Henry of Ghent's Voluntarist Account of Weakness of Will. In *Weakness of Will from Plato to the Present*. Catholic University of America Press).

REFERENCES

Brink, David Owen. 1989. *Moral Realism and the Foundations of Ethics*. Cambridge: Cambridge University Press.

Bromwich, Danielle Nicole. 2008. *Belief Internalism*. Retrieved from TSpace Repository/School of Graduate Studies – Theses. 2009-01-20T19:35:44Z.

Bromwich, Danielle. 2010. “Clearing conceptual space for cognitivist motivational internalism.” *Philosophical Studies* 148 (3): 343-367.

Bromwich, Danielle. 2013. “Motivational Internalism and the Challenge of Amoralism.” *European Journal of Philosophy* 24 (1): n/a-n/a.

Colby, A. & Damon, W. 1992. *Some do care: Contemporary lives of moral commitment.* New York: Free Press.

Coleman, Dorothy. 1992. “Hume's Internalism.” *Hume Studies* 18 (2): 331-347.

Darwall, Stephen L. 1983. *Impartial Reason*. Cornell University Press.

Darwall, Stephen L. 1992. “Internalism and agency.” *Philosophical Perspectives* 6: 155-174.

De Sousa, Ronald B. 1987. *The Rationality of Emotion*. MIT Press.

Fine, Kit. 1994. “Essence and modality.” *Philosophical Perspectives* 8: 1-16.

Foot, Philippa. 1972. “Morality as a system of hypothetical imperatives.” *Philosophical Review* 81 (3): 305-316.

Foot, Philippa. 1978. *Virtues and Vices and Other Essays in Moral Philosophy*. Oxford: Oxford University Press.

Frankena, W. K. 1958. "Obligation and motivation in recent moral philosophy." In *Essays in Moral Philosophy*, edited by A. I. Melden, 40-81. Washington: University of Washington Press.

Frankfurt, Harry G. 2006. *Taking Ourselves Seriously & Getting It Right*. Stanford University Press.

Frankfurt, Harry G. 1998. *The Importance of What We Care About.* Cambridge: Cambridge University Press.

Hoffmann, Tobias. 2008. "Henry of Ghent's Voluntarist Account of Weakness of Will." In *Weakness of Will from Plato to the Present*, 115-137. Washington DC: Catholic University of America Press.

Hume, David. 1739/2000. *A Treatise of Human Nature*. Oxford: Oxford University Press.

Jaworska, Agnieszka. 2007. "Caring and full moral standing." *Ethics* 117 (3): 460-497.

Korsgaard, Christine M. 1996. *The Sources of Normativity*. Cambridge University Press.

Mackie, J.L. 1977. *Ethics: Inventing Right and Wrong*. New York: Penguin.

Mackie, J.L. 1980. *Hume's Moral Theory*. London: Routledge & K. Paul.

McDowell, John. 1979. "Virtue and Reason." *The Monist* 62 (3): 331-350.

McDowell, John. 1981. "Following a Rule and Ethics." In *Wittgenstein: To Follow a Rule*, edited by S. Holtzman & Christopher M. Leich, 141-162. London: Routledge.

McNaughton, David. 1988. *Moral Vision: An Introduction to Ethics*. London: B. Blackwell.

Mele, Alfred R. 1996. "Internalist moral cognitivism and listlessness." *Ethics* 106 (4): 727-753.

Milo, Ronald D. 1981. "Moral Indifference." *The Monist* 64 (3): 373-393.

Nagel, Thomas. 1970. *The Possibility of Altruism*. Oxford: Oxford Clarendon Press.

Noggle, Robert. 1997. “The nature of motivation (and why it matters less to ethics than one might think).” *Philosophical Studies* 87 (1): 87-111.

Pettit, P., & Smith, M. 2006. “External reasons.” In *McDowell and his critics*, edited by C. Macdonald & G. Macdonald, 142-169. Oxford: Blackwell Publisher.

Platts, Mark de Bretton. 1980. Moral Reality and the End of Desire. In Mark Platts (ed.), *Reference, Truth, and Reality*. Routledge and Kegan Paul.

Railton, Peter. 2006. “Humean theory of practical rationality.” In *The Oxford Handbook of Ethical Theory*, edited by David Copp, 265-81. Oxford: Oxford University Press.

Rosati, Connie S. 2006. “Moral motivation.” *Stanford Encyclopedia of Philosophy*.

Roskies, Adina L. 2003. “Are ethical judgments intrinsically motivational? Lessons from acquired sociopathy”. *Philosophical Psychology* 16 (1): 51-66.

Scanlon, Thomas. 1998. *What We Owe to Each Other*. Cambridge (MA): Belknap Press of Harvard University Press.

Shafer-Landau, Russ. 2000. “A defense of motivational externalism.” *Philosophical Studies* 97 (3):267-291.

Smith, Michael. 1994. *The Moral Problem*. London: Blackwell.

Stocker, Michael. 1979. “Desiring the bad: An essay in moral psychology.” *Journal of Philosophy* 76 (12):738-753.

Strandberg, Caj & Björklund, Fredrik. 2013. “Is Moral Internalism Supported by Folk Intuitions?” *Philosophical Psychology* 26 (3): 319-335.

Stroud, Barry. 1977. *Hume*. London: Routledge.

Svavarsdȯttir, Sigrun. 1999. “Moral cognitivism and motivation.” *Philosophical Review* 108: 161-219.

Sytsma, Sharon E. 1995. “Ethical internalism and moral indifference.” *Journal of Value Inquiry* 29 (2): 193-201.

Tresan, Jon. 2009. “Metaethical internalism: Another neglected distinction.” *Journal of Ethics* 13 (1): 51-72.

Williams, Bernard. 1981. "Persons, Character, and Morality." In *Moral Luck*, edited by James Rachels. Cambridge: Cambridge University Press.

Zangwill, Nick. 2008. "The indifference argument." *Philosophical Studies* 138 (1): 91-124.

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Interpreting Pain: Gadamer on Rilke

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Abstract

The paper discusses Gadamer's interpretation of Rilke, distinguished by its respectful depth and hermeneutical mastery. Through the principle of mythopoietic reversal, his philosophical approach does not force the poetical utterances, but highlights a meditative project, whose centre is an epic poem on human limits, as admirably represented in the world of the Duino Elegies. In this context, the hermeneutical issue focuses on a few key human experiences, which identify and define man's being in the world. One of the most relevant experiences is pain, conceived no longer as simply one feeling among others, but as an essential horizon of comprehension. The value of this interpretation is demonstrated by the interpretation of the iconic figure of the angel: in opposition to both traditional angelology and even the Heideggerian reading, this figure is interpreted innovatively as a sign not of transcendence but of immanence.

Keywords: Gadamer, Rilke, Thinking and Poetizing, Interpretation, Hermeneutics, Pain

1. Introduction. The Task of Understanding

At the beginning of the 1940s, at a time when one feared looking one's enemy in the face, for fear of glimpsing oneself, and language was characterized by a logic of intolerance, Hans-Georg Gadamer presented a seminar at the University of Leipzig on one of the poets of twentieth century who conceived that same German language in a relevant and innovative fashion: Rainer Maria Rilke. This choice was neither fortuitous nor oriented by didactic convenience: in the cold, dark rooms of the university building, lacking heating and electricity, Rilke's work was a necessity. In him, the philosopher saw more than a mere representative of a literary epoch, and in his poetry, he saw much more than the proof of an elegant capacity of expression: in Rilke's poetry Gadamer found a language of resistance. The interpretation of the *Duino Elegies* (hereafter *DE*) was a way to resist history and Nazism, and to at least conceive a different world (see Gadamer 1985). This biographic episode shows the relevance of Rilke's work not only to the academic profession, but above all to Gadamer's formation. Rilke was neither an icon nor an object of study, but a master pointing out a road in the darkness; that is, he showed the way to interior freedom in the darkness of the absolute lack of freedom. His poetry, which more than any other opposed "conformity" (Gadamer 1994, 155), i.e. the acceptance of a unique and certain truth, explored the main questions of human existence by celebrating its greatness and misery, without embracing eternal or deterministic principles. In the Rilkian poetic "I", which undertakes the insane challenge of questioning the angel, man is represented in the extreme solitude of one who cannot belong to any order – order both in terms of command and of *Ordnung*, i.e. hierarchy, spheres of belonging (*DE*, I, 1-2). For this, too, constitutes a difference between man and angel: while angels belong to the community of their heavenly sphere, man is alone in his life, just as the poet is alone before his inspiration and his blank page. We will see that all certainties have been broken before the wall that is the angel, before the question of the meaning of what remains and of what flees runs away, - meanings that, for man, dramatically coincide. Thus, in an age where all truth was subjected to the political authority, Rilke expressed all the laborious freedom of one who accepts his own duty and does not shrink before the unsettling presence of the "strong night" (Rilke 1976, 293).

Gadamer considers Rilke a deeply philosophical poet, though not for the same reason his master Heidegger did, for whom Rilke's poetry was philosophical only because it was metaphysical, by remaining "in the shadow of a tempered Nietzschean metaphysics" (Heidegger 1971, 166). For Gadamer, Rilke's poetry is *essential* because it reveals the horizon of comprehension of human limits, which not even philosophy can express in so compelling and impressive a manner. Beginning in the mid-1950s, Gadamer penned several important essays[1] on the German poet, within the context of a broader philosophical project that has always found in poetry the privileged space for realizing *im Vollzug*[2] the central perspectives of hermeneutics. However, these essays would emerge only sporadically, because Gadamer never wrote the commentary to the *Elegies* that he had wanted to write since the 1930s, when he was first exposed to Rilke. Thanks to this exposure, he learned that in poetry the hermeneutical praxis always realises itself as an ever-dynamic and open interpretive investigation. One of the most relevant achievements of hermeneutics is the affirmation that knowledge is never immediate, direct, and simply intuitive, but always a mediated, profound, and clouded, an activity which needs to be passed through, not just perceived. Interpretation allows one to achieve this type of "knowledge practice" that is hermeneutics, and since poetry can only be understood when it is interpreted, hermeneutics finds in the poetic saying an essential experience.

These essays show Gadamer's intimacy with Rilke's poetry in interpreting it, and his sober respect for the poet's words, which are not inserted into a predetermined speculative framework, but placed within involved in an open context of sense. The philosopher is not afraid to discuss the heart, feelings, love, etc.: philosophy does not lose its prerogatives before these existential themes; on the contrary, it is enriched by the vividness of the human tension and by practical involvement. For this reason, Gadamer's interest is purely hermeneutic: the challenge of interpretation has no peculiar truths to prove; it must question a truth already present in the words themselves, without seeking external foundations. The original hermeneutic task, in fact, involves explaining what is incomprehensible is: this may appear an easy task, but is actually the hardest. It is difficult to avoid reducing our attitudes and thoughts to assumptions used to demand the legitimation of what we want to understand. However, understanding is not only an intellectual operation, an act of mental speculation, but is a primary relationship to the world, and philosophy finds in poetry a privileged access to reach it. Among contemporary poets, Rilke seems to Gadamer one of the best to understand this relationship.

2. The Task of the Human

To understand Rilke's poetry and its penetrating hermeneutic significance, especial attention is required to comprehend the *Elegies*, a supreme masterpiece by means of which the poet intended to furnish an epochal fresco of the human being. In this regard, Gadamer considers Rilke's replacement of the poem *Anti-strophes* (Rilke 1989, 116-117), meant to be the fifth *Elegy*, with the actual "Elegy of the saltimbanques"[3] an important hermeneutic issue. In this work, written in February 1922, when the poet was "prisoner of himself" in the "propitious" and "beneficial solitude in Muzot"[4], chronologically following the tenth *Elegy* – which was conceived thought as the conclusion of the "most important work" (Rilke 1937, 246) since 1913 – there is an essential reference, totally absent in the *Anti-strophes*, to death (*DE*, V, 101)[5]. We can go beyond Gadamer's observation and underline that not only does the fifth *Elegy* refer clearly to death, but also, the theme of the unavoidable uncertainty of human existence is quite far removed from the world of the *Anti-strophes*, as the latter is a tribute wholly dedicated to loving women who, surpassing the miserable possibilities of males, can show the power of true, absolute feeling (*Fühlen*). In this poem, a balance has been achieved thanks to this unconditional reverence for the greatness of those women whose heart is so immense it overshadows the span of "distances out to the outermost star".

The main difference between the *Anti-strophes* and the fifth and other *Elegies* is precisely this character of equilibrium, of fulfilment, of stasis from conflict, in which the poet does not have to fight for meaning, as his task is only to recognize and praise one that is already present[6]. The world of the *Elegies* does not lie in the shadow of this grace but is a world full of enduring tension. Whereas in the loving women of the *Anti-strophes* all appears quieted as in the "bread on the altar", in the final version of the fifth *Elegy* this tension emerges in all its corrosive power, erupting into human destiny for which "love and separation" (*DE*, III, 67) coincide, excluding them from their truth and showing the impossibility of reaching any perfection, because perfection is only "god's affair" (*DE*, III, 73).

For this reason, according to Gadamer, the main theme of the *Elegies* is "something universal", namely "the weakness of the human heart, its failure to surrender completely to its feeling" (Gadamer 1994, 156). All the figures populating the extraordinary world of the Duino manifest the impossibility of any coincidence between men and their own task: in contrast to the angel, who is always "identical with its mission" (Gadamer 1994, 158) and always capable of surrendering to its own feeling, man cannot recognize that dedication (*Hingabe*) is, as we will see, his task, due to the intrinsic inability of the human heart to be fully itself, i.e. to accept (and not refuse) the extreme and problematic absoluteness of feeling. This defeat becomes evident in love and before death, two experiences with which the human heart is rarely in contact, as they can destroy the one who does not understand the strength of his limits, and the fact that precisely these limits constitute his identity. Rilke's purpose is to ensure that, in poetry as in life, "nothing is ever lost!..." (Rilke 1969, 273): the human heart must be able not to dissipate or lose anything, not to shrink before that which alone can enable us to understand authentic feeling; however, this very capacity not to shrink before such a reality is unachievable for man. This need for authenticity is incessant in the *Elegies*, as it represents the desire for unity and totality of feeling, which had always moved the soul of Rilke's poetry since the works of his youth[7]. This type of poetry must recuperate through words what the heart cannot achieve. However, no unity exists for man and the poetic "I" of the *Elegies* laments the abyss that exists between its ambitions and their impracticability: "Our nature's not the same", "Wir sind nicht einig" (*DE*, IV, 2).

Since the unity of man and world is impossible, the *Elegies* seek to ascertain the possibility of the unity of feeling, outside of any transcendence, which must occur only in a "purely earthly, deeply earthly, blissfully earthly consciousness" (Rilke 1969, 309); this consciousness is possible - even if never entirely - only very rarely, in the profound wisdom of young dead people, in the brave integrity of children, in the unconditional abandon of loving women, because since due to their intrinsic "disunity and violence of all human behaviour" (Gadamer 1994, 161), men are destined intended to be torn.

3. The Task of the Angel

This ordinary incapability of the human heart to live the risk of a full feeling is expressed profoundly in the extreme confrontation, tragically impractical and at the same time ineluctable, with the angel. The Gadamerian interpretation manifests about this main topic of the *Elegies* all its originality and significance. In both past and present interpretations of the Duino cycle, a common point of view identifies the angel with a being representing heavenly transcendence, "made" of a different nature than that of man. Gadamer intends to break the consolidated traditional connection that places the figure of the angel within an iconographic and cultural angelology that views it as a superior, unhuman, ontologically different being. Traditional angelology is insufficient to explain the Duino angel, actually a philosophical concept: and as such, something revolutionary. Gadamer writes that the angel is the "most extreme conception of our own being" (Gadamer 1994, 158), definitively changing the manner of interpreting the Duino epic. The philosopher is not interested in outlining the unhuman essence of the angel, or even its superiority, but what it might mean for man. The angel is "a supreme possibility of the human heart itself, a possibility never fully realized, one that the heart cannot achieve because the human being is conditioned in so many ways, rendering him incapable of a clear and total surrender to his feeling" (Gadamer 1994, 157).

The angel as a possibility of the human heart does not mean it is something at its disposal; conversely, man is unable to use that absoluteness that could call him into his own. To grasp the significance of this interpretation of the Duino angel, we might recall the first lines of the *Second Elegy* (1-2): "Every angel's terrifying. Almost deadly birds / of my soul, I know what you are, but, oh, / I still sing to you!". Here Rilke is very clear: angel is an essence that knows neither half-sentiments, nor division of feeling, because it is a soul creature. To be sure, there are biblical and iconographic traditional influences of common memory, but these are incapable of revealing the intrinsic identity of the Duino angel. They cannot highlight the essence of what the poet wishes to show. When by "angel" the poet means the supreme possibility of the human heart, its capability not to be conditioned or limited in its own feeling, one could assume that man could have a balanced relationship with this angel, because he could somehow attain it. However, this cannot happen; indeed, man feels crushed under the weight of this higher capability, as he is unable to achieve it, because he is accustomed to look for his limits outside himself, to blame something that transcends him, that remains external to his weakness.

Within the Gadamerian interpretation, it is clear that man's incapacity to be an angel is due to nothing but himself alone, because the bounds hindering real feeling lie within him. Rilke does not blame the angel for man's inferiority, since this inferiority is rooted in man's inescapable essence. Man's gravest defect, instead, is his claim to possession. Man relates to his feelings as if they were at his disposal, things he can possess, but the angel, by manifesting a perfect correspondence with feelings, signals the defeat of every claim, the failure of every "endless desire for possession" (Rilke 2016, 146). This desire is visible above all in love, when man wants to possess his lover like an object. In this regard, Gadamer harshly criticises Romano Guardini and his explanation of Rilke's "doctrine of love" as the latter is not a theory, but a praxis of learning to love (Gadamer 1994,142). This praxis overcomes the subject-object relationship, because loving someone does not mean reducing him to our identity but respecting the necessary distance.

Thanks to the tension towards the angelic nature, Rilke's poetic "I" shows how human feeling is anonymous, evasive and elusive due to its intrinsic limits, and can find no equilibrium before that which is capable of plenitude and authenticity. For Rilke, poetry is a way to redeem all that fades away in man's life because of his limits; the major purpose of the *Elegies* is always to achieve a recovery or, as Peter Szondi would say, a salvation (see Szondi 1975) through poetry of what is temporary, and thus provisional. It is important to point out that this salvation has no religious connotation, it is a secular and courageous salvation, which accepts its universal loneliness.

Yet how could this recovery take place in reality, beyond the words? For Gadamer, this constitutes at least one inescapable question, as Rilke does not stop before the failure of the human heart, contenting himself with a word celebrating its ruin. To read Rilke's work as simply the umpteenth lamentation on the limitations of human existence is not to do him justice. The poet points to a recovery that does not end with the rhythm of a line, but becomes possible for life itself, when the human heart does not dissipate the extreme and challenging experiences of life but converts them into something of its own.

4. The Task of Mythopoetic Reversal

In this regard, Gadamer proposes a concept with which it is possible to interpret the poetic project of the *Elegies* in depth: the hermeneutical principle of "mythopoetic reversal". He is not afraid to apply hermeneutical proceedings to understand poetry; this philosophical concept does not suffocate the poetic word but respects it and helps achieve the interpretive aim. This conception contains an implicit critique against his mentor Heidegger, who strongly disagreed with the use of all philosophical paradigms to read poetry, though his own interpretations of poets' works were always "violent". Gadamer wishes to read poetry in a manner that respects it, and the principle of mythopoetic reversal helps him understand Rilke's world. He clarifies that this principle is in no way a rhetorical proceeding, a closed, predetermined and useful concept which, starting from an impeccable assumption, continues until reaching a rigid conclusion: the mythopoetic reversal must be followed because the open risk of a meditative correspondence is better than the closed rigour of speculative certainty. This does not mean that philosophy must abandon its rigour when used to interpret compared to poetry, but that philosophy must change if it wants to understand poetry. This becomes clear when we consider that Gadamer does not take up this challenge by constructing a poetics of the Rilkian *oeuvre* but chooses an intervention directly "on the ground". He interprets the lines of the *Elegies* by discussing their refined and even deliberate complexity. The true hermeneutic answer is not speculation about the interpretation, but the interpretation itself; its only purpose is to show how important it is that "the interpreter, who gives his reasons, disappears and the text speaks" (Gadamer 1989a, 51).

The first moment instance of the mythopoetic reversal relates to the Rilkian poetic word, which "demands a clarification of the horizon surrounding it" (Gadamer 1994, 155). Its purpose is to gather the powerful message of the *Elegies* into a unity - one which, however, does not reduce it to a hierarchical structure, but to allow all its vivid projectuality to emerge. The hermeneutical task is not a simple matter of images and metaphors; the reversal consists of the interpreter retranslating into concepts of his own understanding what has been elevated in poetic reflection. All the famous figures of the Duino world – dead young people, sad lovers, artists, mourners' lamentations – act and suffer, and their action and suffering are not extraneous to the reader but are the reader's own pathos. In the reversal, a twofold movement occurs: that which allows the reader to interact with the poetic world in question, after this world has reawakened the power of the poetic saying itself.

When the actions and passions of the figures of the *Elegies* do not indicate nothing other than our own actions and passions, they push us to a peculiar level (*Niveau*), in which it is possible to root a reflection (*Reflexion*). When we look at a man who, by entering a church in Naples or Rome, is terrified before the absurdity of the death of young people (*DE*, I, 62-63) – who should be a promise of the future, not the tragic evidence of the lack of any future – before the destiny of one who knows that death overcomes all injustices because it is the only justice which exceeds man by imposing upon him an incapacity to choose, in that moment the poetic word carries us to a *Reflexionsniveau* that amplifies our interpretative possibilities, by going beyond the misery of our fears.

This reversal is described as "mythopoetic"; it is important to underline that this mythopoiesis of Gadamer is not a theoretical construction that can combine the logical connection between myth and poetry into a single, unique concept; it indicates a hermeneutical approach able to open up a horizon of visibility onto the unity of the Rilkian *oeuvre*. In this work, the term "myth" does not mean that poetry recounts the actions of heroes or the sagas of gods, but that it is able to reawaken the consciousness that finds its truth in nothing but its own being-said. For Gadamer, a myth is that which can be considered valid and persuasive, even if needs no rational demonstrations to confirm it, or scientific proceedings to support it[8]. Poetry is a type of myth because it is based on its own word, it must not be taken within a context of cultural genres or aesthetic frameworks, because this is the task of the literary science (*Literaturwissenschaft*) and not of real thought. In this critique, Gadamer follows his mentor Heidegger (Heidegger 1991, 77), who sees in aesthetics as philosophy of art an articulation of the metaphysical program of the interpretation of the essent and of reality as such, but in contrast to Heidegger, Gadamer's interpretation is free from predetermined philosophical assumptions, because his interest is only in what Rilke says, not in what Rilke's saying represents for philosophy.

What unites myth and poetry? For Gadamer, the answer is simple: the word, the "authentic word", which he defines admirably as the "universal human task" (Gadamer 2007, 88). This definition, too, manifests the difference from the Heideggerian "poetic word"; the latter is the centre from which the truth of being irradiates itself, authenticity as the warehouse of the *Event* as *Ereignis* (i.e. the only being outside an ontic theory) (see Heidegger 1972); the former, instead, is a task, the hermeneutical search for authenticity. Myths establish the primacy of the saying over the demonstration, of the word over the fixed concept, and poetry indicates that this primacy of the word has been directed toward the truth – not understood as a normative prescription, but as an essential openness.

The relevance of this hermeneutical principle consists in its indicating a point of view that cannot be something objective, such as a principle of determination, but only something subjective, taken up by the observer, transforming him and involving him. It all takes place thanks to this principle according to which in approaching poetry, there is no claim to bring the poetic sense closer to the reader, and that on the contrary, it is possible to bring the reader closer to the poetic sense. This principle does not insist on transforming the poetic saying in something easier to comprehend but aims at transforming how one relates to it. The poetic saying remains at its distance, in the richness of its continuous reference to something not at one's disposal. Such a proceeding does not upset the essence of the world of meanings we must reverse in our own heart to understand it. To better understand this mechanism, consider the reversal that, in the process of sight, the brain effects on the image, turning it upside-down on the retina. The brain must overturn the image before it, not to possess it, but simply to process it and convert it into something in which it can take part, that is, simply to see it: "the world of our own heart becomes, for poetic saying, objectified for us as a mythical world, that is, a world of acting beings. Whatever surpasses the range of human feeling appears as the Angel; the terrible shock over the death of young people appears as one recently diseased; the lament that fills our heart and pursues the deceased appears as a creature pursuing the one just deceased. In short, the full range of experience in the human heart is poetically liberated as the activity of one's own personal existence" (Gadamer 1994, 159).

The reversal as an approach to Rilkian poetry is actually already employed in Heidegger's interpretation[9], and we find some references even in Romano Guardini's[10], but in neither of these it is thematised or chosen as an interpretive principle, primarily because it already entails the substitution of one point of view with another; in Gadamerian reversal, our heart is led before a mythological matter, which in the poetry involves our limits, making us walk the street opened by the words.

It should be pointed out that Heidegger does not address the *Elegies*, save for some mention of the eighth; he is convinced this should not be attempted because contemporary thought is not yet capable of do it[11]. It may seem strange for an author who believes philosophy can find its future in the dialogue with poetry to ignore one of the most relevant poetic experiences of the twentieth century. He perhaps is hindered by his own prejudice that Rilke's work is influenced by the "derailed Christianity" (Heidegger 1992, 158)[12] – in terms of his concepts of animal and man – entailed by Nietzschean metaphysics. This prejudice prevents him from seeing the peculiar character of this poetry, which, though possessing accents typical of the religious and mystical tradition, is, as Gadamer clearly shows, not subject to any religious authority.

5. The Task of Pain

From this point of view, too, it is clear that with the principle of the reversal, Gadamer reaches the centre of Rilkian poetry without violating it, attaining a high level of discourse in addressing the question of pain, one of the most important questions not only of the *Elegies*, but of the entire Rilkian *oeuvre* from the time of his young poems and the novel about Malte. Gadamer recalls that in the tenth *Elegy* Rilke proposes an impressive definition of men as *Vergeuder der Schmerzen*, squanderers of pain (*DE,* X, 10), because they habitually see pain as an enemy, a threat, without recognizing its real essence. In a letter to Ilse Blumenthal-Weiß dated back to 29 December 29, 1921 Rilke talks about "his" Marianna Alcoforado (the Portuguese nun whose letters to her unfaithful lover he had translated) and writes that women have the art of reaching a full activity of the heart; men, instead, are always distracted and amateurish, or worse, they are *usuriers* of feeling. Before the shocking intensity of feeling, Rilke can only define men as usurers of pain (Rilke 1937, 77). However, pain is actually a horizon of comprehension (*Verständnishorizont*) of human life; without pain, the truth of existence would elude us. Pain is thought as an essential experience that enables us to discern and ponder. "Pain is inevitable. Suffering is optional" (Murakami 2008, 4): we have to learn how to earn our pain.

However, this horizon seems unreachable: when, along his path, man encounters the death of a child or of a young person, he feels devastated before his incapacity to discover the meaning of a destiny that so terribly cuts short the promise and horizon of a life, and so he cannot stop opposing this pain. In the face of the irreversibility of another's death, man feels powerless against what ineluctably transcends him, also because in his ordinary life he is used to the reversibility of his daily affairs. Nevertheless, in this paralysis in which he can nothing, at least he can endure precisely that which he does not bear, *he can pain*. Yet this must not be considered a debilitating condition or an expression of deficiency but reveals itself as a human capability to understand the essential nature of the experience of another's death. What allows us to think of death meditatively, without being tied to a rigid rationality or an elusive emotionality, is precisely pain, because it makes it possible for the human being to do something before the impossibility of everything. Thus, pain is man's *Reflexionsniveau* on death; while feeling it, he can reach a level of reflection that allows him to think of death not as a definitive loss, nor as a hostile forcefulness that cannot be accepted, but as the other, necessary, side of life[13]. Therefore Rilke, in his most famous work about the mutual belonging of life and death, *Sonnets to Orpheus*, explains cogently the essentiality of pain. In sonnet II, 29, the poet asks us to identify the most painful affair, the most terrible experience – "Was ist deine leidendste Erfahrung?": "What experience has been painful to you?" (Rilke 1977, 195). The answer is unusual: he does not refer to a peculiar event, but instead indicates the very centre of this experience. The answer to this fundamental question is: "Ist dir Trinken bitter, werde Wein", "if the drinking's bitter, turn to wine" (Rilke 1977, 195). This line shows us that the reversal is not only a way to read Rilke's work, but even a possible goal and law of his poetry; in these words, indeed, an overturning occurs, the same overturning that the poet must effect if he wishes to open himself to a more authentic relationship with the essentiality of pain. He does not avoid the suffering by trying to escaping its source; rather, he merges himself totally with it, with its truth. From this merger a new comprehension derives, in the search for that depth that can be reached only through pain. This pain is not only a means of knowledge and identification; it is *metamorphosis*, in which man does not reverse the pain into joy (this cannot be an authentic reversal, as it is a mere substitution), but reverses the pain into the authenticity of his own existence. Pain as lamentation shows us that a mythical figure of the poetic imagination becomes the essence of the reader, who can reverse in his own existence the truth of this fundamental experience. It is essential for human life because it is the unique guarantor of the authentic *Hingabe*, that "dedication" that is fundamental[14] to understanding Gadamer's interpretation of Rilke. The *Hingabe* confronts us with all our limits, with all that we cannot reach; it is, indeed, the capacity to abandon ourselves to what it is different and far from us, the unlimited abnegation of people and things, contact with the open that knows no influence, no conditioning. When pain possesses man, he has no space or time for anything else; real pain is so absolute that, despite its unbearable devastating power, it teaches man the authentic *Hingabe*, the authentic absoluteness of feeling. Therefore, when man feels pain, he arrives at a truth of existence that would otherwise have been denied him, a truth in which nothing essential has been lost. Only if man does not avoid pain, and become capable of that abandon that pain requires, will he cease to be a squanderer of own feeling.

6. Conclusion. The Task of Poetry

This question reminds us of the point from which Gadamer started, i.e. the consideration of Rilkian poetry as a deep meditation, inspiring a consciousness that does not bend to the sense of emptiness of totalitarian temptations. Before such a message, it is the task not only of hermeneutics, but, for Gadamer, of all humankind, to explore and defend, against all the ideological damages of homologation, the language of resistance, which can familiarize us with a thought that does not submit to authorities, spreading universal and objective certainties, but which indicates the authentic word. A word such as the one present in the ninth *Elegy*, in which man finally realizes he cannot compare himself to the angel, competing with him, placing himself on the same level. It would be ridiculous for him to want to sing the greatness of the universe, the miracles of the celestial spheres, or even the illusion of his knowledge. The poet reminds us that one cannot try to express the inexpressible to the angel: such an attempt would be poor and useless before the magnificent completeness of the angel's "overwhelming presence" (*DE*, I, 4). One must only speak of simple things to the angel, express the immense simplicity of a little thing:

> You can't impress him with your grand emotions. In the cosmos
> where he so intensely feels, you're just a novice. So show

> him some simple thing shaped for generation after
> generation
> until it lives in our hands and in our eyes, and it's ours.
> Tell him about things. He'll stand amazed, just as you did
> beside the ropemaker in Rome or the potter on the Nile.
> Show him how happy a thing can be, how innocent and
> ours;
> how even grief's lament purely determines its own shape,
> serves as a thing, or dies in a thing – and escapes
> in ecstasy beyond the violin. And these things, whose lives
> are lived in leaving – they understand when you praise
> them.

If we follow and develop the Gadamerian viewpoint, the angel should not be understood as the symbol of a religious tradition, nor taken to represent transcendence in an age when spirituality appears always more distant and inaccessible, because "in no way does it appear as a messenger or representative from God. And it certainly does not testify to any kind of transcendence in the religious sense" (Gadamer 1994, 157). Gadamer convincingly proposes an interpretation of the angel not by utilizing an ontological or theological foundation, but by presenting it as a sign of the human limits that hinder the absoluteness of feeling. The angel of the *Elegies* does not represent any cultural or traditional paradigm, but is a limit signalling the incapability of men to give themselves to the fullness of their own feelings. Men are accustomed to hiding themselves before the extremity of their experiences, "always insufficient in giving love, uncertain in making decisions, and powerless regarding death" (Rilke 2012, 13). The angel is the extreme limit of human feeling, the sign not of transcendence but of immanence.

In this way Gadamer is able to overcome the prejudicial reading offered by Heidegger, who includes Rilke's poetics in the field of modern western metaphysics also because of the angel: "Rilke's Angel, despite all difference in content, is *metaphysically the same* as the figure of Nietzsche's Zarathustra" (Heidegger 1971, 131). Heidegger, of course, does not propose a mere superimposing of the figures, but sustains they belong to the same main conception and tradition, which avoids thinking outside of metaphysics. This claim notwithstanding, Heidegger's interpretation is wrong, because

Zarathustra is indeed one who, while maintaining the integrity of his solitude, walks among people, teaching them; he looks on their imperfections and points out their weaknesses. The angel is just the opposite. He lives high above men, and does not look upon their deficiencies, nor point out anything to them, but simply *is*. Zarathustra mourns human incompleteness, and sometimes ridicules it; the angel, in contrast, states this incompleteness, grounds it as real and ineluctable, without feeling the need to dispute or exalt it. Zarathustra is a bridge (Nietzsche 2006, 7), the angel is a wall. The former is someone who still must realize himself, the latter, one who is already realized and who does not involve men in this untouchable realization. Zarathustra tries to overcome man, while the angel attests that all such overcoming is impossible for men. The angel is the sign of extreme human finitude, which, as nothing, cannot fight against it to will itself, as stated by Nietzsche. Both are points of arrival, but while man could one day *become an* overman (*Übermensch*), he can never *be the* angel.

In its unicity as an insurmountable wall and multiplicity as an order of angels among whom one cannot tread, the figure of the Rilkian angel is free from a characteristic element of traditional angelology, i.e. he is free of the element of mediation between men and gods (mediation always present, even in the tremendous angels of the biblical *Book of Revelation*). The Duino angel loses entirely this function of approaching and interaction, condemning man to the abyss of a fatal loneliness. He is very different than the angels of Hölderlin's poetry, which maintains the traditional angelology intact, as in the poem *Homecoming* (*Heimkunft*) (Hölderlin 2004, 31-32). Hölderlin's angels are "preserver", mediators between sky and earth, making men feel safe. For this reason, they are "Angels of the House", and even "Angels of the Year", because they can fill the void of human precariousness that arises from man's dislocation in space and time. The eternity of these angels does not terrify, even if it states the impossibility of human eternity. In Hölderlin's angels man can find a meaning for his existence; in Rilke's angels, only the absence of a univocal meaning. Rilke's angels are less comforting and more frightening, they cause a deep and indelible anguish while indicating not a chance, but the extreme limit of being excluded from any reassuring contact with the divine. Hölderlin's angels can relieve human weaknesses; Rilke's angel is the insurmountable and rigid guarantee of the impossibility of any transcendent reassurance. Instead of brining man closer to the gods, the Rilkian angel repels every demand to possess the divine. Thus, in losing the main function of mediation typical to angels, the Rilkian angel has also lost the trait of being a source of consolation.

Rilke's angel indicates an absoluteness impossible for men, influenced as they are by mediocrity and fear; it indicates a task to be performed. *Task* (*Aufgabe*) is one of the most important concepts of Gadamerian philosophy (see Gadamer 1989b), because this philosophy is not meant to be an intellectualistic contemplation of ideas but practiced as an activity of interpretation immersed in the world it means to understand. This task is, for hermeneutics, essentially the task of the word, especially the poetic word. This is the great and unique chance of humankind, the treasure before which even the angel's haughty perfection might concede an expression of wonder, before that treasure which cannot be a property but is an interrupted and tenacious task: the word we call poetry.

NOTES

[1] Gadamer's essays focused on Rilke are the following: *Rainer Maria Rilke's Interpretation of Existence: On the Book by Romano Guardini*, dated back to 1955 (Gadamer 1994, 139-151); *Poetry and Punctuation*, dated back to 1961 (Gadamer 1994, 131-137); *Mythopoietic Reversal in Rilke's* Duino Elegies, dated back to 1967 (Gadamer 1994, 153-171), and *Rainer Maria Rilke nach funfzig Jahren*, dated back to 1976 (Gadamer 1993, 306-319).

[2] It is no coincidence that the subtitle of the volume *Ästhetik und Poetik II* is *Hermeneutik im Vollzug*. In Gadamer's works we find a continuous confrontation with classical poets such as Hölderlin, Goethe and Kleist, a massive dialogue with Paul Celan, and numerous essays dedicated to Stefan George, Eduard Mörike, Hilde Domin, Karl Immermann and Ernst Meister.

[3] Rilke recalls this important substitution, for example, in a letter to Lou Salomé of February 20, 1922; see Rilke 1969, 242-243.

[4] The quotations are from Rilke 1937, 193, 109, 122.

[5] Gadamer 1994, 156. Gadamer argues that the *Antistrophes* are too "direct" and "immediate", as opposed to the artistically mediate world of the *Elegies* (Gadamer 1994, 155). In this regard, we may recall how Romano Guardini

emphasizes that the name of *Lamort*, the main character of the sixth strophe, must be read as "La mort", i.e. the person of death; see Guardini 1961, 154.
[6] According to Rilke and his *Sonnets to Orpheus*, the poet's essential task is to praise: *To praise, that's it!* (I, 7); see Rilke 1977, 98 (all the translations of the *Elegies* and the *Sonnets* come from this edition).
[7] See, for example, the lines of the first strophe of the *Spanish Trilogy*, in which the poet asks God to make him and all the things "one thing", i.e. to bring all the fragments populating the earth to the unity of the earth itself; see Rilke 1989, 83.
[8] "All mythical consciousness is still knowledge"; see Gadamer 2004, 286.
[9] Heidegger 1992, 151-161. In his interpretation of some lines from the eighth *Elegy*, Heidegger discusses "reversal" in reference to the visual representation of the animal, as a being that is able to see the open, in contrast to man, who has too many obstacles to see the horizon of the open without prejudices. Heidegger mentions the eighth *Elegy* also in *Heraklit*; see Heidegger 1994, 210-211, 220.
[10] See Guardini 1961. F.J. Brecht, too, referring to lines 81-86 of the fifth *Elegy*, indicates the *Umschlag* as basilar characteristics of the reversal of the human capability (*Können*) of the heart; see Brecht 1949, 159.
[11] "We are unprepared for the interpretation of the elegies and the sonnets, since the realm from which they speak, in its metaphysical constitution and unity, has not yet been sufficiently thought out in terms of the nature of metaphysics. (…) We are not only unprepared for an interpretation of the elegies and the sonnets, but also we have no right to it, because the realm in which the dialogue between poetry and thinking goes on can be discovered, reached, and explored in thought only slowly"; see Heidegger 1971, 87-140.
[12] In his maturity, Rilke defines himself a fervent anti-Christian, however, according to Heideggerian interpretation, an anti-Christian still moves within a Christian horizon precisely when he insists on abandoning it.
[13] "Affirmation of life-AND-death appears as one in the 'Elegies'. To grant one without the other is, so it is here learned and celebrated, a limitation which in the end shuts out all that is infinite. Death is the side of life averted from us, unshone upon by us"; see Rilke 1969, 309.
[14] Even Hans Urs von Balthasar recognizes the *Hingabe* as the highest point of the Rilkian meditation; see von Balthasar 1939, 316.

REFERENCES

Balthasar, Hans Urs von. 1939. *Apokalypse der deutschen Seele*. Salzburg: Pustet Verlag.

Brecht, Franz Josef. 1949. *Schicksal und Auftrag des Menschen*. Basel: Reinhardt.

Gadamer, Hans-Georg. 1985. *Philosophical Apprenticeships*. Translated by Robert R. Sullivan. Cambridge, MA: MIT Press.

________. 1989a. "Text and Interpretation". In *Dialogue and Deconstruction. The Gadamer Derrida Encounter*, edited by Diane P. Michelfelder & Richard E. Palmer, 21-51. Albany: State University of New York Press.

________. 1989b. *Das Erbe Europas. Beiträge*. Frankfurt a.M.: Suhrkamp.

________. 1993. *Ästhetik und Poetik II*. Tübingen: Mohr.

________. 1994. *Literature and Philosophy in Dialogue: Essays in German Literary Theory*. Translated by Robert H. Paslick. Albany: SUNY PRESS.

________. 2004. *Truth and Method*. Translated by Joel Weinsheimer & Donald G. Marshall. London: Bloomsbury.

________. 2007. "The Universality of the Hermeneutical Problem." In *The Gadamer Reader. A Bouquet of the Later Writings*, edited by Richard E. Palmer, 72-88. Evanston, Illinois: Northwestern University Press.

Guardini, Romano. 1961. *Rilke's Duino Elegies: An Interpretation*. Translated by Kenneth Graham Knight. Chicago: Regnery.

Heidegger, Martin. 1971. "What Poets are for?" In *Poetry, Language, Thought*, translated by Albert Hofstadter, 89-139. New York: Harper & Row.

________. 1972. *On Time and Being*. Translated by Joan Stambaugh. New York: Harper & Row.

________. 1991. *Nietzsche*. Volumes I and II. Translated by David F. Krell. New York: HarperCollins.

________. 1992. *Parmenides*. Translated by André Schwer & Richard Rojcewicz. Bloomington & Indianapolis: Indiana University Press.

________. 1994. *Heraklit. Gesamtausgabe Band* 55. Edited by Manfred S. Frings. Frankfurt a.M.: Klostermann.

Hölderlin, Friedrich. 2004. *Poems of Friedrich Hölderlin*. Translated by James Mitchell. San Francisco: Ithuriel's Spear.

Murakami, Haruki. 2008. *What I talk about when I talk about running. A Memoir*. Translated by Philip Gabriel. New York: Knopf.

Nietzsche, Friedrich 2006. *Thus spoke Zarathustra. A Book for All and None*. Translated by Adrian Del Caro. Cambridge: Cambridge University Press.

Rilke, Rainer Maria. 1937. *Briefe aus Muzot 1921 bis 1926*. Leipzig: Insel Verlag.

________. 1969. *Letters of Rainer Maria Rilke. 1910-1926*. Translated by Jane Bannard Greene & Mary D. Herter. New York: The Norton Library.

________. 1976. *Selected Works. 2. Poetry*. Translated by James B. Leishman. London: Hogarth.

________. 1977. *Duino Elegies and The Sonnets to Orpheus*. Translated by Alfred Poulin Jr. Boston: Houghton Mifflin Company.

________. 1989. *The Selected Poetry of Rainer Maria Rilke*. Translated by Stephen Mitchell. New York: Vintage International.

________. 2012. *Letter on God and Letters to Young Woman*. Translated by Annemarie S. Kidder. Evanston: Northwestern World Classics.

________. 2016. *The Notebooks of Malte Laurids Brigge*, Translated by Robert Vilain. Oxford: OUP.

Szondi, Peter. 1975. "Rilkes Duineser Elegien". In *Das lyrische Drama des Fin de Siècle*, edited by Henriette Beese, 379-510. Frankfurt a.M.: Suhrkamp.

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Alexandru Dragomir's Quest for Identity

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Abstract

This article is a hermeneutic attempt to think through some of Alexandru Dragomir's philosophical fragments that focus on the problem of identity. To this purpose, the first part examines Dragomir's existential strategy in asserting the pre-eminence of the *need for identity* in his analysis of the mirror, as well as in his reinterpretation of the myth of Narcissus. The second part tackles the inner configuration of Dragomir's *ownness-strangeness dialectic* along with the function it holds in his understanding of philosophy as a perpetual self-questioning. The final section addresses Dragomir's confrontation with the metaphysical tradition regarding the nature of individual *uniqueness*.

Keywords: Alexandru Dragomir, Romainan philosopher, Martin Heidegger, ontology of the self, autology, existential identity

Alexandru Dragomir's[1] interest in the problem of identity can be traced in his writings as early as February 1945, in an article called *On Mirror*[2]. As far as we know, this article and a Romanian co-translation with Walter Biemel of Heidegger's 1929 conference *What is Metaphysics?* are the only texts Dragomir ever wrote for publishing. In this brief text Dragomir argues that the mirror functions in most cases as *a place of meeting* between ourselves and our image, thus providing us with the occasion to realise how the others see us. Glancing in the mirror, we try to see ourselves through the eyes of strangers, we look for our "alienness" in order to critically evaluate our guise and eventually to correct any irregularities.

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The mirror is typically placed near the exit, hence, we can check our appearance before entering the public space. As scrutiny occurs at a fast pace, the mirror satisfies a purely functional role and answers to our need to integrate in society. Besides this *external attitude*, Dragomir coins a different type of relation to the mirror: *an inner attitude*, a frantic search for our "ownness". In this case, the mirror transcends its purpose as a place of meeting and becomes *a place of conversation*. The gaze evolves into existential scrutiny as the orientation is no longer objective, but dependent on everything that we are.

The drive to grasp our image, to be in a state of *eye to eye with ourselves*, indicates the profound urge to find an expression to our ego, persona and identity. For Dragomir, the myth of Narcissus illustrates the essence of the mirror and manifests a *limit situation* involving the inner attitude. The mystery of the mirror and its power of attraction rests on the assumption that it can reveal something of the self's inner side, that it can *objectify the subject*[3]. Narcissus considers the contemplation of himself in the mirror as an act of most intimate knowledge, and is charmed by the possibility of an immediate connection with his own persona. After several extended attempts, Narcissus understands the mirror's illusion, he realises the impossibility of a total assimilation between the individual and his expression. In the end, yearning for a finality of his efforts, Narcissus chooses an absurd unification, gently closing the distance that separates him from his reflection in the water, to the point of vanishing.

Contrary to the main versions of the myth of Narcissus, which tend to focus either on the aesthetic problem regarding the limits of beauty, either on the ethical dilemma of a misplaced affect, Dragomir gives it an ontological interpretation, thus saving the dignity of its character. The demise of Narcissus does not originate in his sublime beauty nor in his misplaced love for himself but in the dramatic endeavour to satisfy the basic human need of finding oneself, in an improper and misleading milieu of the mirror. Reflecting our external image, the mirror can function as a conversation starter as well as it initiates a return to the self. However, it also plays with our sense of curiosity and wonder (lat. *miror*) in creating the illusion that our spirit can take a tangible form, yet, ultimately, the mirror cannot fulfil our *existential thirst* for identity. Narcissus's mistake consists in the aspiration to find an aesthetic solution to an existential need for self-consciousness and in the obstinacy to follow this path to the end. By exemplifying the extreme conditions in which the search for identity can lead to self-negation, Dragomir's elucidation of the myth brings forth the fundamental urgency to find our selfhood. The orientation towards the self implies all together a double intentionality: a recognition of our *ownness*, the part of ourselves that establishes our most basic familiarity, and a grasping of our *strangeness*, the previously hidden part that still belongs to us. The cardinal necessity for identity seems to be, in Dragomir's view, the inherent consequence of the permanent tension generated by the dialectic movement between the recognition of our *ownness* and the assimilation of our *strangeness*.

Alexandru Dragomir offers a more detailed reflection on the problem of identity in relation to the ownness-strangeness dialectic in a fragment named *The Banal Strangenesses of Mankind*[4]. The fact that "life goes on by itself" represents a striking triviality, certainly not worthy of any attention. However, if one pauses to analyse it, its uncanny meaning begins to reveal itself. "That for which I try to give evidence is the intimate strangeness that resides in me, which is *my life*. My life cannot be hurried, stopped, or delayed. My life, the basic fact of my ownness, is also something strange to me, which means I am fundamentally split, always having to follow the stranger that lies in myself (*daß ich immer mitgehen muß*)." (Dragomir 2005, 107) We are not the origin of our life, we cannot change its essence, thus life contains, in a sense, an independent passage or motion that exceeds our power. Of course, it stands in our power to put an end to it, as Narcissus does, but the mere negation cannot change the essence.

At this point, one could wonder about the meaning of the "I" implied by "my life". Dragomir states that no matter how we understand the self (soul, persona, ego, subject or conscience), even if we reduce the metaphysical entanglement surrounding it to a simple *point*, everything that happens in life is *a priori* related to *the fact of the point of reference*. In other words, the relational dimension of the self remains given in spite of abstracting the content meaning. *The point of reference* signifies the *non-spatial centre* toward which every single thing is orientated. "Everything comes to me and leaves from me, and I cannot make it otherwise, not even in a dream." (Dragomir 2005, 205) In addition, any intention to comprehend the primal self-relation as a sentiment, sensation or consciousness fails on the count of it being their condition of possibility.

According to Dragomir there is a third layer, that of *banal strangenesses* which involve our presence in the world. The orientation towards the self implies identity, while the orientation towards the world implies alterity. Besides the *self-relation,* we are similarly situated in a *world-relation* that is also given and independent of our will or power. The world follows its own course, obeys its own laws and remains indifferent to our presence. Utterly overwhelmed by the world, we take part in it and are at the mercy of its power. Nevertheless, the self stands fundamentally open to the world, it is free to perceive and understand it. The difference between the two lies in the fact that the understanding is not given, but involves effort and choice. The *aporetic strangeness* of the fact that we simultaneously are an insignificant part of the world and something that encircles the entire world through understanding constitutes our "*ex-centricity* in the world" (Dragomir 2005, 117).

The merge between *the point or reference* and the *strangenesses of life* always takes place in *the fact of living itself* and therefore the content meaning of the self takes the expression of the way we live, while our identity depends of the expressions we give to this relation: self-knowledge (*γνῶθι σεαυτόν*), self-contemplation, self-preocupation (*ἐπιμέλεια ἑαυτοῦ*), self-interogation (Augustine: *Mihi quaestio factus sum*, Heidegger: *Selbstfrage*), self-sufficiency (*αὐτάρκεια*), self-deceiving, self-forgetting, self-love (*φιλαυτία*), etc. The quest for the self depends on the answer given to the fundamental question: *how should one live the life that has been given?* Dragomir believes that by asking this question we stand in the openness of our original freedom, a space where the concern for the meaning of life appears.

> "Firstly, the original freedom is not an act of the will, but one of meaning. Of course, we have no clue what "meaning" is, even less the meaning of life. It is enough to know that "meaning" signifies anything whatsoever, like the act of distinguishing my thoughts. For example, in our case, meaning signifies advancing the banality that *life is lived* to the status of a problem. The fact that I live my life is primary. Thus, the relation between the permanent going that constitutes the life independent of «me» and the orientation to myself, in the sense of the life given to me, in and through which I am what I am, is also primary. But then again, if I strive to advance my life to the status of problem, instead of living by chance and bringing into play my minimal freedoms, then the horizon of my original freedom and the prospect of the meaning of my life, appear." (Dragomir 2005, 110-111)

This fragment illustrates how Dragomir adapts the Socratic imperative of self-questioning to his Hegelian inspired *ownness-strangeness* dialectic and makes use of it in a post-Heideggerian existential scenario. It is in this context that Dragomir shows that the limits of our freedom can be found exclusively in confrontation with our given strangenesses which in a sense constitute "the trap we live in[5]". It is the constant activity of understanding *the trap we live in* that bears the name of philosophy.

In a text named *On Uniqueness*[6], Alexandru Dragomir takes another path to examining the problem of identity, one that gets him closer to the metaphysical tradition but not further from his belief that "to philosophize means thinking about the facts known to everyone." (Dragomir 2010, 134) Dragomir observes that there are many types of *individual uniqueness*: some of them are innate (our prints) some change with age (our way of walking) or with disposition (our voice) etc. Each individual is unique in a multitude of ways. But a question concerning *the grounding uniqueness* arises against this unsettling plurality. At this point, the metaphysical terminology involving *uniqueness* requires clarification. The *individual* has only an indicating sense; the *specific* designates the species, not the individual; the *proper* (lat. *proprium*) refers to the *common* elements belonging to the genus and the species, not to the individual. Thus, the usual language of *uniqueness* emphases the common elements of a class, a direction assignable to the Platonic and Aristotelian paradigm that established *uniqueness as essence.* For Aristotle, besides the essential *uniqueness* of the *genus* or the *species*, there is only a numerical unicity (κατ᾽ ἀριθμόν ἐστιν ἕν, τὰ δὲ κατ᾽ εἶδος, τὰ δὲ κατὰ γένος. *Met.* 5.1016b), that also retains only an indicating sense.

> "According to Plato, following the line of thought of the *essence* leads us to the last species, and not to the individual, which is *alogos* (*Philebos* 16 b-d). But then, my *uniqueness* (*meine Einmaligkeit*), my proper self is not essential but accidental in nature. It can only be determined from outside, by applying of the space-time «*forms*». The existence in itself is overlooked and the only issue that remains is the essence (*ousia*), the pure forms etc. However, the *uniqueness* needs to be fundamentally tied to the existence. Heidegger solved the matter by distinguishing between *beings* (*Seiendes*) and *being* (*Sein*), but he took these things too easily (*er hat es sich leicht gemacht*)." (Dragomir 2005, 244)

In search for a *grounding uniqueness,* Dragomir agrees with Heidegger's solution to take the existence as a starting point. The banality that each person has its own singular manifestation and reality could be translated as the *immediateness of each individual with itself*. Dragomir believes that our immediateness, the fundamental *datum* of our existence, was not properly unfolded due to its aporetic nature, although it has been frequently called into question in the history of metaphysics, for example in Augustine's famous question: "What is closer to me than myself?" ("Quid antem propinquius meipso mihi?" *Confessions* X, 16) or in Heidegger's understanding of *mineness* (*Jemeinigkeit*). According to Dragomir, at the same time and respect, the *uniqueness of oneself for oneself* is *given*, *evident*, *immediate* and *continuous* for oneself, but also *inexpressible*, *irreducible to a concept, logically inaccessible* and *inscrutable* in itself. As long as we accept the perspective that each human being is unique in itself and for itself, we recognise a common trait of the uniqueness, but this commonality refers strictly to the class, not the content of the uniqueness itself. The *uniqueness of oneself for oneself* can only be indicated as a matter of existence, as we cannot actually exceed the *existential structure* and the others cannot pervade.

The fundamental *uniqueness* makes the basic distinction between the self and everything else that "stands outside" possible (*the uniqueness of oneself for the others*, our ownness, our strangeness, the alterity, the world) and is thus the core structure of our identity. Furthermore, Dragomir points out a side of our existential uniqueness that can be recognized and identified only from an external point of view, but remains inaccessible to us: a *uniqueness of oneself for the others*.

"This particular uniqueness ex-poses me in such a way that the other can see me, can hear me, and so on, but I cannot. The others know my individual specificity, but I do not. I could hardly recognize my voice if I heard it in a recording; my accidental reflection in a mirror surprises me; I am the only one who does not know the way I walk, the way I talk, even though all of these are accessible to the others." (Dragomir 2005, 192)

The reason behind our impossibility to obtain an understanding of our *uniqueness for others* lies in the fact that there is no "outside" or a *meta* point from where we can access and get to know ourselves. Dragomir notices our enigmatic condition of owning two different *uniquenesses* and stops to ask about their possible relation, but provides no answer. In this ontological analysis Dragomir questions the underlying relationship between *uniqueness* and *commonness*, a relationship that in a sense grounds each area of the self and of the entire mankind. In his assessment, our identity configures itself in the permanent confrontation with the dialectic structures that make up our life: *uniqueness* and *commonness*, existence and essence, ownness and strangeness.

From an ontological standpoint, Alexandru Dragomir could be classified as an *autologist*, a thinker, belonging to the metaphysical tradition[7], preoccupied with the fundamental structures we find ourselves. As a proper autologist, Dragomir writes only for himself and thinks only for himself, being totally disengaged from any cultural life[8] that could limit his freedom. Having spent most of his adult life in a totalitarian society, Dragomir practices philosophy as a secret individual activity and aims to uncover his existential situation through a way of life based on self-questioning. As a first consequence, his philosophy always starts from that which is closest to us but usually overlooked: our *banalities*. Secondly, Dragomir cultivates unrestricted kinship with the philosophical tradition, his fragments being the result of "a dialogue with the great dead" (Pleșu 2004, 68): Plato, Aristotle, Augustine, Descartes, Kant, Hegel, Schelling and Heidegger. Thirdly, an exclusively private philosophy disregards the category of originality that belongs to the cultural and academic. Hence, there is more to be gained from thinking through the content of Dragomir's fragments than from a genealogical tracing of their origins. Alexandru Dragomir epitomizes an authentic Narcissus of the philosophy, an autologist detached from culture, driven by an existential need for identity, who takes great liberties with the means of the metaphysical tradition in order to reflect on the self. The difference consists in the fact that Dragomir's traces remain to be thought and understood, while Narcissus's remain to be seen and smelled.

NOTES

[1] The main philosophical biography of Alexandru Dragomir was written by Gabriel Liiceanu (2004, 17-65). See also: Pleșu (2004, 65-73), Patapievici (2004, 73-79), Ciomoș (2004, 79-91), Bondor (2006, 116-129), Ciocan (2007, 63-79), Partenie (2004, 91-102), Partenie (2012-2013, 455-463), Ferencz-Flatz (2017, 45-55).

[2] *On Mirror* was published in *Five departures from present. Phenomenological* exercises (Dragomir 2005a, 13-20). See also the French translation (Dragomir 2005b).

[3] An observation made by Mircea Vulcănescu (1904-1952) (Dragomir 2005a, 18).

[4] *The Banal Strangenesses of Mankind* was published in Dragomir (2005a. 106-120). See also Dragomir (2005c).

[5] "It is for this reason that you cannot live without doing philosophy. In a way, we can live without thinking about the infinite, but we cannot live without thinking about our trap. For the simple reason that we live in it. Philosophy is thinking about the trap in which we live. I agree, of course, that there are many ways out of this trap; the principal escape routes are religion, philosophy, science and art. In the case of philosophy, I escape from the trap exactly to the extent that I want to *understand* it. You can, of course, live in this trap content that «they» give you warmth and food, I mean without feeling any need for philosophy. But for me that is not a life that I can choose. No! I want to understand my world. And this is called doing philosophy." (Dragomir 2004b, 181).

[6] *On Uniqueness* was published in Dragomir (2005a, 190-209). See also the French translation (Dragomir 2004, 121-135).
[7] "Philosophy (Metaphysics) has been my home for the past 70 years." (Dragomir 2008b, 182).
[8] "The toil of self-understanding does not belong to the cultural dimension." (Dragomir 2008a, 37).

REFERENCES

Bondor, George. 2006. "'The good life' and the Problem of Philosophy. The Phenomenology of Time at Alexandru Dragomir". *Hermeneia* 2006 (special issue): 116-129.

Ciocan, Cristian. 2007. "Philosophy without Freedom: Constantin Noica and Alexandru Dragomir." In *Phenomenology 2005*, Vol. III, Part 1, *Selected Essays from Euro-Mediterranean Area*, edited by Ion Copoeru & Hans Rainer Sepp¸63-79. Bucharest: Zeta Books.

Ciomoș, Virgil. 2004. "Théorie et pratique de la phénoménologie. Une rencontre manquée". *Studia Phaenomenologica* IV/3-4: 79-91.

Dragomir, Alexandru. 2004.a "De l'unicité". Traduit par Michelle Dobré. *Studia Phaenomenologica* IV (3-4): 121-135.

________. 2004b. "Utter Metaphysical Banalities." Translated by James Christian Brown. *Studia Phaenomenologica* IV (3-4): 171-183.

________. 2005a. *Cinci plecări din prezent*. Bucharest: Humanitas.

________. 2005b. "Du mirroir." Traduit par Michelle Dobré. *Alter* 13.

________. 2005c. "Banales étrangetés de l'homme." Traduit par Michelle Dobré. *Alter* 13.

________. 2006. *Caietele timpului*. Bucharest: Humanitas.

________. 2008a. *Crase banalități metafizice*. Bucharest: Humanitas.

________. 2008b. *Semințe*. Bucharest: Humanitas.

________. 2010. *Meditații despre epoca modernă*. Bucharest: Humanitas.

________. 2016. *The World We Live In*. Translated by Brown. J.C. Springer (Phaenomenologica 220).

Ferencz-Flatz, Cristian. 2017. "An involuntary phenomenologist. The case of Alexandru Dragomir". *Studies in East European Thought* 69(1): 45–55.

Liiceanu, Gabriel. 2004. "The Notebooks from Underground." Translated by J.C. Brown. *Studia Phaenomenologica* IV (3-4): 17-65.

Partenie, Cătălin. 2004. "Archive relief. Dragomir's perspective". *Studia Phaenomenologica* IV (3-4): 91-102.

Partenie, Cătălin. 2012-2013. "Portrait du philosophe en solitaire". *Revue de métaphysique et de morale* 75 (3): 455-463.

Patapievici, Horia Roman. 2004. "The lesson of Alexandru Dragomir". *Studia Phaenomenologica*" IV (3-4): 73-79.

Pleșu, Andrei. 2004. "Fragments of a portrait". *Studia Phaenomenologica* IV (3-4): 65-73.

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Moral Vision, Outrage and the Contextual Understanding of Values in the World of Tennis

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Abstract

This article elaborates on Jean Baudrillard's ideas about the moral effects of the rise of the consumerist society, and also on Patrick Stokes' conceptual distinctions between different reactions individuals can display when faced with moral decisions. I start from Baudrillard's viewpoint that in the consumerist society, characteristic for the occidental post-modern world, the need (*necessity*) itself has been replaced by the desire to consume per se. The Western individual perceives abundance as a natural right, and this is transforming both the meaning of work and the value of its products. In essence, Baudrillard describes a form of alienation, with effects that transcend the commercial realm of commodity consumption, and which is better understood within the moral domain. Patrick Stokes exploits the Kierkagaardian concept of *interesse* while expressing his view of moral vision. He is designing a thought experiment that reveals a fundamental distinction between radically different moral reactions of hypothetical individuals, even when they are sharing the same cultural, educational, political or religious background. Starting from these two positions, I analyze a few situations and events from the world of contemporary tennis, revealing how universal values get to be ignored, or contextualized under the influence of social prejudice and schemas. My conclusion is that, nowadays, we are witnessing a reshaping of the way people regard and act on their values, especially in the realm of social media. Thus, situations that should be approached by the appeal to values such as truth, justice, and humanity, in fact get to be interpreted in a biased way, due to the existence of some pre-existing patterns of understanding.

Keywords: alienation, consumerist society, moral vision, moral outrage, responsibility, social media, values

1. The Consumer Society, Morality and Values

In his 1970 volume, *The Consumer Society Myths and Structures*, Jean Baudrillard describes what he envisions as a fundamental transformation of the occidental world, and with it, of the (post)modern individual, who seems, in his opinion, to be more preoccupied with objects and the necessity attached to their functions, than with fellow human beings. At the beginning of the chapter entitled *Profusion*, he writes the following:

> There is all around us today a kind of fantastic conspicuousness of consumption and abundance, constituted by the multiplication of objects, services and material goods, and this represents something of a fundamental mutation in the ecology of the human species. Strictly speaking, the humans of the age of affluence are surrounded not so much by other human beings, as they were in all previous ages, but by objects. (Baudrillard 1998, 25)

The individual person itself becomes defined by functionality, similarly to the objects surrounding him. This is, in short, the societal transformation that Baudrillard is alluding to. The individual starts to experience a different time, the time of objects, the time of his owned goods, at a pace that is imposed by those commodities. The contemporary phenomenon that best illustrates this vision is the 'omnipresence' of the mobile devices in our lives. Who could any longer picture their own existence, without a smartphone and all its embedded functions? Smartphones are organizing and guiding our existence, in many instances noticeably replacing the authentic, face-to-face human relating.

A phenomenon that is specific to the consumer society, which puts emphasis not so much on the need itself, but, instead, on the idea of consumption alone, is represented by the ways in which commercialized products and objects are being presented, under the auspices of abundance. The act of consumption, by and of itself, replaced the simple satisfaction of necessities. Respectively, consumption is perpetuated by the abundance of products. The abundance of products, in the form of a multitude of objects found everywhere in the commercial spaces, is witnessed by people as a natural right, to things that everyone is entitled to. Here is where the reader notices a determining attribute of the consumerist society, namely its miraculous status. In essence, this aspect is affecting the way in which the individual understands the meaning and value of work and the casual connection between work and consumption.

> In everyday practice, the blessings of consumption are not experienced as resulting from work or from a production process; they are experienced as a *miracle*. […] Consumer goods thus present themselves as *a harnessing of power*, not as products embodying work. And, more generally, once severed from its objective determinations, the profusion of goods is felt as a blessing of nature, as a manna, a gift from heaven. (Baudrillard 1998, 31, 32)

To illustrate this perspective with an evocative comparison, Baudrillard mentions the experience of a Melanesian tribe when it first came into contact with members of Western cultures. The people of the Melanesian tribe developed a millenarian cult, known as the "cargo cult." According to this system of beliefs, the affluence characterizing white people is due to their capacity, exposed since the times of their people's ancestors, to capture and hijack the goods that were actually destined for them. Only when this "unexplainable" maneuver of the white people would disappear, the forefathers of the tribe would eventually be able to reach them with the miraculous valuable cargo, in this way bringing their people's state of poverty to an end. The contemporary person, member of a consumer society, is taking the right to possess for granted, regardless if this right is deserved or not, resembling the Melanesian tribe's conviction that the goods of their non-natives were actually intended for them.

We believe that the effects of the consumer society surpass the strictly delimited area of commerce, of the patterns of consumption that refers to appropriation of goods and the way in which the individuals interact with these goods. We cannot ignore the fact that the contemporary individual is also an avid consumer of information. The shopping centers, the big malls, and the multitude of commercial websites omnipresent in the virtual world, are granting everyone access to objects and goods, all in one place, one click away. In a similar manner, the information regarding the realities of the world around us are packed in various forms.

In Baudrillard's view, the mall is perhaps the most prominent symbol of the consumer society, embodying the phenomenon of indiscriminate spending, a commercial mixer that homogenizes diverse consumption activities. One can find everything inside a mall, all in one place, for the purpose of shopping, a subtle enough activity that is defined not just by the acquisition of commodities. Shopping is seen by the French philosopher as a form of 'flirting with objects' (Baudrillard 1998, 27), a continuous entertainment that tends to take over our daily existence. Whether you are buying or not, you will ultimately consume. The mall is offering the subtle ambiance of the possibility of consumption through its diverse offer, which brings together all sorts of entertainment goods, including artistic, cultural and sports productions.

Similarly to the mall, mass communication offers the same type of 'supply' when it comes to consumption of information. Sitting comfortably in front of a TV, computer or of a smartphone screen, the modern individual receives all types of information indiscriminately, with an uncritical mindset and in an unfluctuating psychological and moral disposition. In the same way the mall homogenizes work, leisure, nature, and culture, mass communication transforms information in random facts.

> What characterizes consumer society is *the universality of the news item [le fait divers]* in mass communication. All political, historical and cultural information is received in the same – at once anodyne and miraculous – form of the news item. […] What mass communications give us is not reality, but *the dizzying whirl of reality [le vertige de la realité].* (Baudrillard 1998, 33, 34)

The contemporary man, inhabitant of the consumer society, is living behind the camouflage of this whirl of reality, a world of overflowing signs that hide the truth, thus keeping the individual removed from reality, at a protective distance. Genuine interest, responsibility, conscious involvement and moral reactions to events are spared and become dormant.

> The consumer's relation to the real world, to politics, to history, to culture is not a relation of interest, investment or committed responsibility – nor is it one of total indifference: it is a relation of curiosity. On the same pattern, we can say that the dimension of

> consumption as we have defined it here is not one of knowledge of the world, nor is it one of total ignorance: it is the dimension of misrecognition. (Baudrillard 1998, 34)

In this context, curiosity and the false recognition convey the withdrawal from the objective reality, a sort of detachment, or a self-preservation mechanism that is far from serving as a direct and responsible contact with the real world. The realities of the world quickly become anodyne facts. Essentially, this constitutes an alienation process with effects in the moral realm, because this non-involved, passive, and distant way to face the realities of the world have one effect, the mitigation of individual responsibility. The contemporary individual, essentially a consumer, takes contact with the surrounding world through several safety filters: the unidirectional televised image, and the comfortable and anonymous nature of Internet communication. Nothing can affect the individual personally as long as there is a wide gap between 'here' and 'there', between 'me' and 'them'. Baudrillard concludes here:

> At this 'lived' level, consumption makes maximum exclusion from the (real, social, historical) world the maximum index of security. It seeks the resolution of tensions – that happiness by default. But it runs up against a contradiction: the contradiction between the passivity implied by this new value system and the norms of a social morality which, in essentials, remains one of voluntarism, action, efficacy and sacrifice. (Baudrillard 1998, 35)

Lead by Baudrillard's ideas to the moral domain, in the following section I will take a look over the psychological evidence on the phenomena intuited by the French philosopher. Recent research in moral psychology keeps gathering clues converging to the idea that people are habitual seekers and consumers of morally infused information, especially the types of content that points to perceived transgressions, either in the form of news, personal testimonies, or expressed opinions. Consuming information about moral violations is inciting anger, indignation, contempt, or outrage. Moral outrage is among the main reactions to perceived acts of injustice, an emotion driving the impulse to punish the presumed offenders and restate the limits of acceptable conduct. Some researchers in psychology invest efforts in defending the position that this heightened anger and outrage reactivity (and the attached behavioral orientation towards finding and punishing wrongdoers) are actually socially beneficial, leading to the protection of norms – by deterring future threats to these; uncovering and publicly shaming transgressors, is a symbolic act of punishment that is ultimately facilitating collective action against these wrongdoers (Spring, Cameron, Cikara 2018; Kleef, Fischer 2015). However, other voices in this area rush to call this point of view into question, advocating that stirring of intense moral emotions, especially in online media is actually detrimental to social cohesion. In fact, as these authors (e.g. Crockett 2017) argue, the side effects of this phenomenon are: deepening the social divides by dehumanization of ideological opponents, polarization of conflicting parties or escalation of animosities, by making characterial or dispositional attributions when evaluating others, or oversimplifying nuanced, complex issues (more effects of anger are summarized in a review by Lerner and Tiedens 2006). Crockett (2017) points out that, in fact, the most prominent benefits of moral outrage expression are of a personal nature, mostly serving self-presentation purposes. The mentioned author states that beside the already enumerated costs, an unanticipated one is the dulling of the outrage response, which may lead in time to an incapacity to distinguish between abominable and less condemnable acts, something that could make people indiscriminately punitive. If Spring and collaborators (2018) affirm that outrage in online media gives voice to marginalized groups, Brady and Crockett (2019) think that this purported benefit, of motivating social action, is actually limited, hypothesizing it might lead to even more exclusion of marginalized groups. I agree with the latter perspective, especially on the idea that outrage by itself cannot be used as a moral compass.

As Baudrillard considered that indiscriminate consumption leads to an artificial growth, like a cancer that feeds on itself, the habit of expressing moral emotions only generates an increased need to perpetuate them. Expressing these emotions replenishes the need to seek them again, by entering a vigilante state aimed at coining and punishing norm-violators. To bring the analogy with Baudrillard's understanding of consumerism in the realm of moral evaluation, the sought 'status objects' are the morally-charged pieces of information that the individual is pursuing to display, alongside with his disapproval. Thus, the moralizing behavior in online media takes the form of a compulsive or impulsive behavior that seems to lose its social function or other survival benefits. These 'moral objects' are not desired for their utility, but for their symbolic ability to say something about their owner. Mass media and social media perpetuate click-bait news to be consumed. The more morally charged the news, the more spread or shared they become (Brady et al. 2017). If for Baudrillard consumer goods are signs or emblems of distinctiveness, taste and status, in a similar way, people share moral outrage as a proxy for their positive character qualities and display their disapproval as a form of exercising civil duty.

The idea of considering the display of moral outrage as a consumption object is derived from the psychological literature suggesting that often, people post, share and react to morally charged information in a way which is presumably leading to a desire to express even more indignation and anger. It is not norm protection that is mainly sought, but rather more self-serving goals such as presenting oneself as moral, and even improving personal reputation. Displays of indignation and anger when confronted with norm violations are a proxy for or reflection of one's own moral character traits (the implicit statement is 'I am constantly on watch, thus I am an aware, virtuous, trustworthy person.'). In other words, impression management trumps goals like social cohesion. I now dedicate my attention to a philosopher who is attempting to revive a kierkegaardian concept in the service of refining moral psychology's understanding of moral judgment.

2. Moral Vision and Moral Reaction

I now turn to a book written by an Australian philosopher, Patrick Stokes, *Kierkegaard's mirrors. Interest,* *Mirrors and Moral Vision*. At the beginning of this book, the author is provoking his readers to a mental exercise, a thought experiment designed to bring to their attention the inter-individual differences in *moral vision*, and as a consequence of these differences, the variability of reactions people can display when confronted with a decisional situation that has individual consequences. The main idea of this experiment can be summarized as follows: two individuals share the same cultural, educational, moral, political, or religious background. They have similar life expectancies and moral engagements and also similar characters. Based on all these mentioned similarities, one could say that their motivational structures, and generally, their way of understanding and looking at things should be similar. One can describe these two persons as having corresponding personalities, if we were to quantify them. Nevertheless, when confronted with the same decision, these two individuals will act completely different, as we will see in the following hypothetical scenario. Let's suppose, as Stokes guides us, that two such individuals are watching (separately) a TV channel that is broadcasting a material on the effects of the 2004 tsunami that took place in the Indian Ocean, focusing on the humanitarian crisis following the disaster. Up to a certain point, both had the same reactions: they experienced compassion towards the victims, sadness, and sympathy for the suffering. In addition, both individuals are thinking the same thing, namely that something needs to be done to come to these people's aid. However, after this point, the thinking paths of the two individuals start to separate. More specifically, while one of the individuals only expresses an abstract intention of helping, (adhering to an impersonal 'something must be done!'), the other is assuming individual responsibility for this something that must be done and seeks solutions to actively offer his help. In Stokes' own words, the two reactions are described as follows:

> I sit in my chair and ruminate on the horror of what I've seen and the urgency of addressing the problem. You leap from your chair and look up the phone number for the Red Cross, so you can call and find out what you can do to help – make a cash donation? Organize a food drive? Get on a plane and join the relief effort? In effect, you have

> acted, while I have continued to contemplate ineffectually without acting. Crucially, you didn't stop to think whether you are obliged to act, or whether you *should*. You didn't, in fact, stop to think *at all*. (Stokes 2010, 2)

Hence, while the first person in the example remains relatively inert, although he is morally affected by hearing about the people in suffering, the other person straightforwardly decides to act. The first individual is contemplating an abstract moral pattern (in approaching world events), while the other reacts only in conjunction to his own moral imperative, his own concerns, even if, emotionally and cognitively, they are both confronted with the same situations.

Both our characters understand that they are witnessing a tragedy, being equally empathic and experiencing feelings such as pity and compassion etc. One of them perceives the situation as being (morally) compelling, while the other feels himself to be morally compelled to act. What's missing from the first individual's reaction that is present in the other one's? For the latter, the reaction/decision is immediate and direct, while for the former, the reaction is facilitated by a conscious cognitive process, namely by internal deliberation. The moment we are starting to develop a deliberate decisional process over a situation like the one described above, a chasm starts growing between our moral emotions and the possible response in the form of an action. In this context, the author proposed the following position: 'I think we can start to formulate answers to these questions if we articulate a new understanding of moral cognition in terms of normative moral *vision* rather than normative deliberation, good will, and so forth.' (Stokes 2010, 6)

Stokes develops this problem by reinterpreting and re-exploiting the Kierkegaardian concept of *interesse*, through which he develops a distinct model of moral cognition. He formulates the following application of Kierkegaard's concept:

> Under such a model, 'vision' rather than 'deliberation' or 'reflection' stands for what is central to successful moral cognition; the normative locus of moral psychology shifts from practical reason and deliberative intention to distinctive modes of apprehension. Our reading of interesse has begun to scope out a Kierkegaardian model

> of moral cognition which has as its telos the immediate coextensiveness of vision, volition, and action. The perfected moral agent – such as never is and possibly never can be found – sees, judges and acts in one unitary moment. (Stokes 2010, 180)

For the perfect moral agent, the emphasis is put on the immediate transfer between intention and action. There's no separate deliberation phase taking place. Rather, there is an immediate transition from the perception of a situation to the substantive act of helping. The person who is taking action in the previous example is doing it with the same 'naturalness' with which, in Stokes' opinion, the other one is driving his car, based on some automaticity that does not require a constantly involved, active deliberative process. (Stokes 2010, 2)

What is truly relevant for the present situation is one of moral psychology's vulnerabilities, more specifically, when trying to understanding moral cognition as normative deliberation. Along these lines, Stokes emphasized, our internal deliberation process is bringing together not only moral considerations and facts, but also facts of a non-moral nature. The latter have the power to decisively influence our decisions and actions. To support his idea, Stokes is offering a mundane example: 'I should stop to give that hitchhiker a lift, but I'm worried about my safety and I am also running late.' (Stokes 2010, 181) As we can see, the last two aspects of deliberation possess a non-moral nature, but they have influenced a moral decision. These elements of the deliberation are not directly related to the objective situation, namely that someone is waving from the side of the road asking for a ride, thus obviously calling for help.

Of course, several observations can be made in relation to Stokes' proposed example. One can speculate on several situational factors that can interfere with the moral decisional process, for instance maybe the area in the described situation is not very safe and there were a lot of incidents involving hitchhikers reported, or perhaps the driver was hurrying to save another person's life etc. However, for the discussion of the cases that are presented in the following sections of the present paper, I believe that this fusion of moral and non-moral elements, from an ideal decision making perspective, has rather negative effects and contributes to confusions and inadequate moral reactions. What interests me here is this idea of differentiating between moral and non-moral considerations when one is confronted with the decision to act. I agree with Stokes on the premise that this kind of cognitive deliberations that people make when facing moral dilemmas are overshadowing some universal values.

Essentially, this is a matter of responsibility, meaning that the decisions that are taken in the manner described by Stokes are making us feel less accountable as individuals. Thus, when we interpret and judge a phenomenon or situation, we display the tendency to add non-moral elements in the architecture of that decision, elements that might affect the complete or correct understanding of the facts and ultimately, influencing the way in which we decide to act.

A recent series of experiments in moral psychology (Jordan and Rand 2019) suggests that people tend to report more moral outrage and punishment when they lack the opportunity to signal their trustworthiness via prosociality, than when they can express it by sharing. Interestingly, in conditions of anonymity (so when their reputation was not at stake, because when no one was watching, their actions would not signal trustworthiness), more deliberative individuals choose not to punish, if this would be costly to them, even if they report high levels of outrage, thus a desire to punish. The authors suggest that an explanation for this result could be the fact that deliberative individuals would suppress their drive for altruistic punishment in conditions of anonymity because, in the particular situation, the costly behavior would not bring much material benefit. Another explanation would be that, compared to their less reflective counterparts, deliberative individuals may also be better at elaborating moral justifications for their decision to act in a different way than they would if their reputation was at stake.

In the following, in the light of these ideas, I will analyze a couple of situations from the world of contemporary tennis, situations that illustrate a type of reaction that becomes habitual in a post-modern consumer society, with direct implications on the way in which people execute their moral deliberations every day.

Why did I choose tennis? Tennis has always been regarded as a noble, aristocratic sport, a leisure of the elites. This means, from my personal standpoint, that tennis and its audience is creating a space for the promotion and assimilation of certain values and virtues in the first place. Among these, I mention fair-play, respect for the adversary, integrity, developing a certain kind of conduct and, lastly, building the moral character, to mention only some of the defining elements of tennis, in its ideal manifestation.

Nevertheless, the traditional representation of tennis as a sport for the elites, also known as 'the white sport,' lead to the perpetuation of some negative phenomena too. Thus, throughout its history, tennis was promoted as an exclusivist sport, a pastime of the wealthy, a sport that leads to social divides and that, in the common opinion, allowed the emergence of racial, gender, classist or ethnic prejudice. Illustrative for this sinuous history is the fact that, until the feminist activist work of Billie Jean King, a reputed tennis champion in the 60's and 70's, the idea that women could share the right to win the same amount as men in tennis was just an utopia. The idea was rejected on several reasons, including the so called 'biological' reasons, or other rationalizations related to presumed limits of their mental and emotional capacities. In fact, even today, there is a continuous debate regarding the basis for awarding comparable prize amounts for men and women.

Also, it was not only until 1950 that African American athletes had the right to participate in international tennis tournaments, starting with Althea Gibson, who later became a tennis champion. Up to that point, the competitions were closed to any athlete that was not white. Therefore, one can think of plenty of precedents that can be invoked to explain the emergence and development of prejudice and biases in the world of tennis, whether sexist or racists.

In its defining nature, tennis is a sport and a space for character development, by facilitating values awareness, promoting a certain code of conduct and is also supposed to shape moral traits. On the other hand, this space is in the same time, through its exclusivist politics, contributing to the maintenance of the prejudices and biases we mentioned before. This is the context that lead to the appearance of a contradictory phenomenon: the development of a heightened reactivity towards acts that resemble the initial acts of discrimination and prejudice, a sort of 'counter-prejudice,' manifested by immediate emotional, not entirely deliberate, reactions. The contextual inflamed sense of injustice and moral sense give rise to these manifestations. Such reactions reveal a certain type of ignorance, combining a lack of understanding of the situations and personal irresponsibility in acknowledging the limits of one's knowledge, which are visible especially on online media. These types of reactions will be illustrated through a few examples in the following sections.

3. Presumed Racism on Tennis Courts

On February 13th 2018, Ryan Harrison and Donald Young, two American professional players (the former Caucasian, the latter African-American), who have been rivals since their junior years, met in the first round of the New York Open. The match was ultimately won by Ryan Harrison in two sets, 6-3, 7-6. During the first set, the two players had a heated verbal argument. The video cameras did not capture the audio of the incident, thus there was no clear-cut evidence about what really happened on the court during the verbal exchange.

Immediately after the match, Donald Young is posting the following message on twitter: 'I'm shocked and disappointed, Ryan Harrison, to hear you tell me how you really feel about me as a black tennis player in the middle of our NY match. I thought this was supposed to be an inclusive gentleman's sport.' (as quoted in Lutz 2018)

To this tweet, Harrison replies: 'The accusations made by Donald Young tonight following our match are absolutely untrue. I'm extremely disappointed that someone would say this in reaction to a lost tennis match. Any video/audio will

100% clear me and I encourage anyone with the available resources to find it.' (as quoted in Bonesteel and Song 2018)

Unfortunately, the damage had already been done. In response to Young's message, part of the media and especially social media users showed harsh reactions towards Ryan Harrison, accusing him (obviously!) of racism. Over the next few days, Ryan Harrison and his family are being 'brutalized' on social media networks. Even there was no clear evidence supporting the accusations of racism, all the 'elements' of the case have been presented so that they would instantly inflame the public opinion. Which were these key elements? A tennis match between a white and an African-American player, the moments of tension during the game, the white player addressing a few bellicose lines to the black player, the black tennis player losing the game. Following this 'logical' path, something racist must have happened. Michael Bruno, a ball person assisting the match, who stood close to the players during the incident, thus a direct witness to what happened that day, admitted that he did not hear any racial comment whatsoever.[1]

ATP further investigated the case, concluding that there were no racist exchanges or attitudes exhibited during the match.[2] The specter of racial prejudice and the habit of reading such a situation through those lenses, following a 'logical path' that is confirming pre-existing schemas and beliefs, have created more tension around the event than reality itself did. Not to mention the subjective and probably deliberate intervention Donald Young had after the match, that wasn't exactly inspired, but was a perfect fit for the expectations of an over-reactive audience.

In this situation, if we apply Stokes' framework, the moral and non-moral aspects had a compound influence on the way the event was initially received. Thus, Donald Young's decision to post his tweet online was 'intoxicated' by non-moral, subjective reasons that had nothing to do with what happened in reality. Was this caused by the frustration he felt after losing the match or due to the mounting tensions that characterized the historic rivalry with Harrison? One could find plausible the supposition that Donald Young's reaction was a result of a constant frustration he had been experiencing as an African-American tennis player in a white sport; a victimization that could have looked justified' by the exclusivist history of tennis. In any case, neither of these reasons aren't directly connected to the reality of the incident, more specifically, the fact that the exchanges, even if bellicose, did not include any element of racism from Harrison's part. Thus, the way in which Donald Young's tweets have been shared, without a minimal check, pinpoint to a similar pattern. The public's expectations, fueled by an inflamed sense of injustice have been satisfied by the racism allegations, who confirmed their beliefs. Intellectual habits, like reality checking and looking for evidence supporting the claims and weighting them, were eclipsed by the rapid acceptance of the rumors that involved racism. There are also some elements that remind us of the perils of the consumer society: the indiscriminate information consumption and the lack of fact checking, a disregard of individual accountability in using mass communication channels, the effects of the accusations coming from all directions, pointing to Ryan Harrison.

Let's remember another tense moment in the history of tennis that occurred not long ago, which involved a Romanian tennis player, Irina Begu, after a match against Caroline Garcia, representing France. The match took place during the Charleston Open, USA, on April 4th 2016 and was finished with Begu's victory, 6-4, 2-6, 7-6. After the game ended, several individuals started spreading an information of an ambiguous origin on social media, claiming that the French player, in a crucial moment of the match, addressed offensive words to her Romanian competitor, allegedly calling her a 'gypsy s**t'. Although unverified, this information was treated by the international press, and especially the Romanian media as accurate, featuring the news under bombastic titles such as: *Racism at the Charleston Open!, A new scandal in tennis!* etc. Obviously, social media exploded, the tipping point being reached by the verbal assault of Romanian 'supporters' on Caroline Garcia, her Facebook page being flooded by injurious and obscene comments and posts. In reality, Garcia hasn't uttered any of those words, a fact that was confirmed by the subsequent analysis of the video footage of the match, part of an investigation that was performed by the match referees and WTA officials. As of March, 3 2016, WTA through CEO Steve Simon, WTA CEO posted on its website: 'The highest level of professional conduct on court is paramount to the WTA and anything less is unacceptable. After thorough investigation, we have found no evidence to support these allegations. This matter is closed.'

In this second case, only the press and social media users inflamed the spirits. Neither of the two players got involved directly in what followed the match. Some racial prejudice and social perceptions, combined with a generalized inferiority complex of Romanians, overcame other values like truth and more specifically, truth seeking.

It is possible that the emergence and quick diffusion of these false allegations, among the Romanian player's supporters were fueled by a broader phenomenon describing the perceptions of Romanians in Europe, and particularly in France. For decades, the negative stereotypes about Roma migrants in France led to an anti-Romanian sentiment that was, in the perception of many co-nationals, based on their erroneous assimilation with the ethnic minorities, invading the streets of Western European countries like France. Thus, the virulent reaction of the Romanian 'supporters' appeared on a background of identity frustration. None of these situational aspects is directly connected to the reality of the events, which is the fact that there was no racist statement uttered by Caroline Garcia during the match. In addition, not even Irina Begu, the purported victim, did mention anything like this happening on the tennis court. This kind of situations enjoy a lot of publicity not because of their problematic nature per se, but because they readily answer the public's expectations, expectations that have been shaped by prejudice, by habits of interpreting and understanding reality, especially morally charged issues.

As I was mentioning before, tennis has a history of being an exclusivist, elitist sport. This background lead, in many instances, to the signaling of prejudice, of a racial or sexist nature, among other types. Without any doubts, authentic cases of racism, sexism or classism have been reported in tennis. Among the most notorious incidents is the moment experienced by the Williams sisters in 2001, during the Indian Wells Open. From the midst of a predominantly white audience attending in the bleachers, someone addressed Serena with the following words: 'I wish it was '75; we'd skin you alive.'(quoted in Doug 2001) Consequently, the sisters boycotted the Indian Wells Open for 14 years, by refusing to participate in the tournament. Real incidents could nevertheless have generated a heightened level of sensitivity to various forms of injustice, like the ones displayed by Donald Young or Serena Williams during the 2018 US Open final match. Among other things Serena said to the umpire:

> 'How dare you insinuate that I was cheating? You stole a point from me. You're a thief too. This is not right. To lose a game for saying that, it's not fair. How many other men do things? There's a lot of men out here who have said a lot of things. It's because I am a woman, and that's not right.' (as quoted in Eccleshare 2018)

The incident has been widely publicized, reactivating the discussion of racism and sexism in the world of tennis. However, looking at this case from an objective perspective, the referee's decisions were entirely covered by the regulations. In many respects, Serena Williams' impulsive reactions were considered inappropriate for the tennis court and were driven by a personal history and career events that had been scarred by moments of (accurately or not) perceived expressions of racism or sexism. Whichever the case, this type of situations and especially the way they are managed in the public space and on online communication platforms contributes to the propagation of prejudice and they work counter to the spirit and mission of tennis, namely stabilizing and promoting a space characterized by authentic values.

Turning back to understanding today's 'moralization' habits through Baudrillard's account of consumerism, we can understand, through the examples we presented, what are the risks of this indiscriminate consumption of information. As the consumer society ethos can heighten the feeling of deprivation and poverty, especially where the pressure is put on achieving the ideal of the 'good life' through consumption, so can the moralizing instinct be exacerbated by the pervasive circulation of information on social media. This is a reaction to the pressure to conform to a 'righteous life,' that comes in tandem with perceptions of anomy, social cynicism and a lack of distrust in the other members of the group, authorities and society overall. This prosecutorial mindset leads to false positives such as the racism cases presented to perpetuate with such velocity.

Just as compulsive hoarders (a pathology often observed in consumerist societies) purchase and stockpile objects without even getting to use them in many cases, the social media moralist shares information online even before checking or digesting it completely. As consumption is not determined by need, in Baudrillard's view, expressing outrage is determined by a reinforcement pattern similar to the one that leads to the emergence of other habits:

> Just as a habitual snacker eats without feeling hungry, a habitual online shamer might express outrage without actually feeling outraged. Thus, when outrage expression moves online it becomes more readily available, requires less effort, and is reinforced on a schedule that maximizes the likelihood of future outrage expression in ways that might divorce the feeling of outrage from its behavioural expression. (Crockett 2017, 770)

The acquisition of objects (and news to share) is focused on the symbolic rather the utilitarian side of the behavior. If one of the challenges of the consumer society is to sway citizens away from irresponsible choices towards more ethical consumption patterns, the moral philosophers' and psychologists' missions in the information society is developing mindful consumption skills, or trying to persuade social media users to act more responsibly in their interaction with the information they see and share.

4. Truth vs. Celebrity. Sharapova's Case

The case that I am presenting in the following section is not necessarily related to the 'historical' background of prejudice and discrimination in tennis. It isn't about racism or sexism either, but about the inherent conflict between tennis, as a sport that promotes authentic values and the mercantile demands and criteria of the consumer society. It is a struggle between the value of sport and popularity, between merit and the need for consumption, and ultimately, the tension between truth and notoriety.

In February 2016, the famous tennis player Maria Sharapova is suspended for two years, after she tested positive for meldonium during the 2016 Australian Open. The Russian athlete was using the substance for two years based on medical prescription (to keep some respiratory problems under control), but meldonium has been blacklisted starting January 2016. Later, the suspension was reduced to 15 months, allowing the athlete to return in the WTA circuit by March 2017, during the Stuttgart tournament, where she received a controversial Wild Card. This moment marked the emergence of a series of publicly expressed negative reactions and dilemmas. A lot of active or former players have reacted promptly and critically in relation to the organizers' decision, considering that it was unfair to award a Wild Card to a player who had been recently suspended for doping.[3] The tournament organizers usually grant this type of prizes to young players or athletes that have been removed from competitions on fundamentally different grounds than Sharapova's absence, such as health issues or they grant them to local players who are poorly situated in the tennis rankings.

To the already existing controversy surrounding Sharapova's return, mass media coverage has contributed as well, especially through the way reporters conducted interviews with the Russian player. The journalists have repeatedly addressed the suspension period using euphemisms or imprecise descriptions, such as time out, pause or period of inactivity, and avoided referring it by what is was in truth, a suspension over a positive doping test.[4] Additionally, all the other reporters who insisted (some still keep insisting) with the questions on doping were rather ignored by the Russian sportswoman. The tennis public's reaction was divided. A part of the public condemned the return of the player to the WTA circuit, in addition to the preferential treatment she received. However, a larger part of the public received her comeback enthusiastically, considering the event (paradoxically!) an auspicious time for tennis, a sport in need for strong personalities and notorious figures to promote and revitalize its image.

This type of ambivalent attitude within the public, media and even among the tournament organizers, is bringing us back to Patrick Stokes' framework. In this particular case, we can see how non-moral considerations, like Sharapova's notoriety, the previous (and still undisputable) success in tennis of the Russian player, and also the constant promotion of her image in media, have been weighting enough to obfuscate a truth, the fact that Sharapova has been charged with doping, proved guilty and subsequently, temporarily removed from athletic activity.

Also, Sharapova's image, a colossal brand in the feminine tennis, fits in perfectly with the standards of a consumer society like the one described by Baudrillard. The commercial image built around the player, the products that she is advertising along with her image, have always invited to consumption, to shopping. Sharapova's image constantly generated money, and her matches instantly filled tennis courts. Her image is always present in ads and in tournament promotional materials. Surely, the premises of success have always been present for Maria Sharapova, even from the outset of her career. A Russian athlete with classic aesthetic qualities (blonde, tall, with green eyes), who manages to win over Serena Williams in 2004, in the final match of the Wimbledon tournament, the most prestigious Tennis Open in the world. And all that at only 17 years. Thus, the interest for intensely promoting and using Sharapova's image has always existed, especially on the part of tournament organizers and sponsors, who are willing to invest massive resources when they spot a 'winner'. Nevertheless, the way in which the details of her controversial situation have been delivered created confusion and made it difficult to the public to accurately discern the parameters of the problem.

The cosmetic presentation of the player's doping problem is throwing the image of this sport in the realm of uncertainty and compromise, an effect that is substantially more detrimental on the long term than the washing of her public image could have been. This type of phenomenon is eroding the value hierarchies and the moral stance of this sport. In this situation, we can talk about a willful obscuring of a supreme value, the truth. We are noticing a contextualization of truth that is driven by certain interests and non-value criteria. Many of these reactions, conducing to the acceptance of poor standards of conduct are financially motivated, marketing-oriented, as we have already shown. Maria Sharapova's image, her public success, her popularity and notoriety exceeded the importance of protecting values such as fair-play, truth, fairness and lastly, the promotion of a clean sportsmanship and sporting environment. The image of tennis itself, also called the 'white sport' in reference to its noble roots, a sport that is in essence based on fair-play, risked being compromised by bringing an athlete that had basically cheated back into the spotlight. Endorsing or tolerating dishonest behaviors could lead to mutations to the sport's core ethos and values and their depreciation with utilitarian and pragmatic attitudes.

Concluding ideas

In Maria Sharapova's case, many people, including the event organizers, coaches, supporters and the large public have long ignored an obvious truth, to the advantage of the sportswoman's image and notoriety. Post-hoc moral justifications or worse, the tolerance of dishonest behaviors contrast the heightened sensitivity to injustice seen in the other presented cases, jointly pointing to the arbitrariness with which moral values are becoming protected and endorsed within the field of tennis. In the Young-Harrison and Begu-Garcia incidents, the public ignored truth and justice while weighting the facts, which lead to the interpretation of the situations in the light of pre-existing beliefs regarding prejudice, and in the light of social stereotypes grounded in a long history. This kind of approach is affecting the correct perception of reality and also altering the potential moral reactions and actions in response to events. Such phenomena are most visible in sports, and in tennis in particular, a world that is supposed to thrive based on the respect of indisputable regulations, of its core values and the promotion of equality, and fair-play. In fact, sports and tennis in particular should have a determining social role. Sports competitions represent more than entertainment for the masses, or simple athletic competitions, and idealized, cosmetic representations of the top athletes, to be consumed by the large audiences.

Sports in general, and tennis in particular, should focus not only on harnessing physical mastery, but also on modelling values and strengthening moral character (through traits such as dedication, tenacity, fairness, respect for norms and rules). And the athlete should desirably function as a moral exemplar. Sports were never conceived as simple entertainment for its public. Especially when we consider a noble sport like tennis, who is surrounded in a historical aura of elitism, and promoted as a space for fair-play and etiquette, an environment conducive to personal development. In contrast to these aspirations, the perpetuation of prejudice and stereotypes only access, as we could see, some preexisting patterns of biased understanding. This is leading to moral confusions and ultimately condenses the specific consequences of the consumer society. These consequences are visible in a weakening of personal accountability, and a distortion of an authentic moral vision.

To summarize, we are talking about a temporary deferral of fundamental values when we interpret or understand phenomena such as those analyzed in the present paper (a deferral especially visible in the realm of social media). Our judgment is often obscured by certain schemas, customs or interpretation biases. This is, in the end, a problem of indiscriminate consumption of information, which directs the understanding of facts away from reality, towards the consideration of non-value criteria: ignorance, lack of authentic references, individualism, moral numbness, the avoidance of personal responsibility. And the effects are mainly observed in the moral realm, especially through the lack of direct and personal acknowledgement of one's own thoughts and actions. Values such as truth, fairness, humaneness, and solidarity are replaced by societally predefined thinking patterns, patterns that are specific to the postmodern man, that make this distorted perception of reality so readily 'available'.

Everything seems to boil down to a 'correct conceptual identification,' a simple categorization of different events and phenomena. We are not internalizing anymore, we are not willing to execute an accurate moral deliberation, in the sense proposed by Patrick Stokes, but we only align to a set of social habits of understanding and interpretation. We are contextualizing values based on predefined frames of reference provided by certain societal segments, especially through the press, through mass media. We ultimately get to see evidence of racism where there isn't any in reality; we promote counterfeit 'truths' especially due to the moral numbness and confusion, which are generated by the indiscriminate way in which we consume everything that we are offered. In essence, this attitude is an attribute of consumer society. This offers, similar to the malls, an abundance of 'culture', food, arts, entertainment and wellbeing. On the TV screens, we are offered relaxing advertisements, mixed with images of ecological disasters, shiny ads and plastic happiness.

Lastly, we also enjoy the anonymity and the immediate reaction that the virtual world of internet and social media provide us. This change of paradigm, that allows switching certain values with notoriety for instance, is generated by the existence, especially in social media of the hater or troll, who aims to attack and compromise any kind of value. Self-awareness and individual responsibility are anesthetized by understanding of wellbeing as a natural right, and also by the safety and comfort offered by our couch from where we are watching, critiquing, analyzing the world, in a distant and uninvolved way. The solution should nevertheless be a simple one: looking for authentic values and objectively following them when faced with morally infused judgments or actions, in tandem with the removal of interpretative biases and patterns interfering with these processes.

NOTES

[1] Here is what Bruno the ballboy had to say: "I'm right there, and I didn't hear it. No one heard a racial comment; no one on my side, no one on Young's side. No racial terminology whatsoever. It was pretty nasty, some of the things I was reading. For people to just jump on the bandwagon and start, like, really damaging someone's character without hearing any evidence or details of the conversation, it didn't sit right with me." (as quoted in Waldstein 2018):
[2] Fiind more information in (Bonesteel and Soong 2018):
[3] More information about the subject can be foud here: "Players Divided on Sharapova Wild Cards." (2018):
[4] More on the issue is to be found here: "Channel Seven Is a Global Laughing Stock." (2018):

REFERENCES

Baudrillard, Jean. 1998. *The Consumer Society*: *Myths & Structures*. London: SAGE Publications.

Bonesteel, Matt, and Kelyn Soong. 2018. "ATP Finds No Evidence That Ryan Harrison Directed Racist Remarks at Donald Young." *The Washington Post, WP Company*, February 16.www.washingtonpost.com/news/early-lead/wp/2018/02/13/donald-young-claims-ryan-harrison-made-racist-comment-during-new-york-open-match/?noredirect=on&utm_term=.b16987f42de9

Brady, William, and Molly Crockett. 2019. "How Effective is Online Outrage?" *Trends in Cognitive Sciences* 23 (2): 79-80.

Brady, William J., Julian A. Wills, John T. Jost, Joshua A. Tucker, and Jay J. Van Bavel. 2017. "Emotion Shapes the Diffusion of Moralized Content in Social Networks." *Proceedings of the National Academy of Sciences* 114 (28): 7313–7318. doi:10.1073/pnas.1618923114.

"Channel Seven Is a Global Laughing Stock." 2018. *NewsComAu*, January 12.

www.news.com.au/sport/tennis/channel-7-slammed-for-sharapova-suck-up/news-story/9155b97f7c44c2cffc222113d00df0fd

Crockett, M. J. 2017. "Moral Outrage in the Digital Age." *Nature Human Behaviour* 1 (11): 769–771. doi:10.1038/s41562-017-0213-3.

Doug, Smith. 2001. "Williams' Father Says Booing Racially Motivated." USATODAY, March 28.

http://usatoday30.usatoday.com/sports/tennis/stories/2001-03-26-williams.htm

Eccleshare, Charlie and Simon Briggs. 2018. "Serena Williams Unleashes Furious Rant at Umpire as She Loses US Open 2018 Final to Naomi Osaka." *The Telegraph, Telegraph Media Group*, September 8.

www.telegraph.co.uk/tennis/2018/09/08/serena-williams-vs-naomi-osaka-us-open-2018-final-live-score/

Jordan, J.J., and D.G. Rand. 2019. "Signaling When Nobody is Watching: A Reputation Heuristics Account of Outrage and Punishment in One-Shot Anonymous Interactions." *Forthcoming* at *Journal of Personality and Social Psychology* Available at SSRN: https://ssrn.com/abstract=2969063 or http://dx.doi.org/10.2139/ssrn.2969063.

Kleef, Gerben A. Van, and Agneta H. Fischer. 2015. "Emotional Collectives: How Groups Shape Emotions and Emotions Shape Groups." *Cognition and Emotion* 30 (1): 3–19.

doi:10.1080/02699931.2015.1081349.

Lerner, Jennifer S., and Larissa Z. Tiedens. 2006. "Portrait of the Angry Decision Maker: How Appraisal Tendencies Shape Angers Influence on Cognition." *Journal of Behavioral Decision Making* 19 (2): 115–137. doi:10.1002/bdm.515.

Lutz, Tom. 2018. "Donald Young Accuses Fellow US Player Ryan Harrison of Racism during Match." *The Guardian, Guardian News and Media*, February 13.

www.theguardian.com/sport/2018/feb/13/donald-young-ryan-harrison-racism-allegations-tennis-new-york-open

"Players Divided on Sharapova Wild Cards." 2017. *Sport*, March 10. www.sport24.co.za/Tennis/WTATour/tarpishchev-slams-sharapova-wild-card-critics-20170310

Spring, Victoria L., C Daryl Cameron and Mina Cikara. 2018. "The Upside of Outrage." *Trends in Cognitive Sciences* 22 (12): 1067–1069. doi:10.1016/j.tics.2018.09.006.

Stokes, Patrick. 2010. *Kierkegaard's Mirrors. Interest, Self and moral vision.* New York: Palgrave Macmillan.

Waldstein, David. 2018. "A Flash of Anger, a Charge of Racism and a Witness Who Says It Didn't Happen." *The New York Times,* February 16.

www.nytimes.com/2018/02/16/sports/tennis/donald-young-racism-ryan-harrison.html

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Critical Thinking in the Contemporary Education: A Historical Approach

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Abstract

The aim of the present text is to analyse the place held by critical thinking in the current education system, showing at the same time the changes of perspective that have taken place during the last decades in the manner this discipline is being approached. Critical thinking has been focusing for decades on logical and informal argumentation, but nowadays it also approaches subjects as psycho-cognitive biases, skills in evaluating numbers and having the sense of proportions, the use of language, understanding of the world, having interdisciplinary knowledge, moral, legal and aesthetic reasoning. We may notice lately that the authors who theoreticize critical thinking go back to antique logic and to the Greek philosophy of education, with the purpose to draw connections between them.

Keywords: critical thinking, education, applied logic, Plato, Aristotle, educational subjects

1. Introduction

When it comes to contemporary education, we can notice a salient emphasis on teaching the abilities of critical thinking at every educational stage. The wave started in the 70's, when in the USA, a general dissatisfaction started among students concerning the inutility of formal logic, especially in the political situation of that time. They found that logic did not help them to understand the premises of the Civil War (Fisher 2019, 17). Concerning this situation, the first author who introduces a course of "Contemporary Logic and Argumentation" for university level is Howard Kahane. The author backs away from formal logic's language and uses an

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informal approach to present the basic concepts, but very applied to reality. His book is famous today and it was edited until the 14th edition this year being one of the best introductory books in critical thinking.

In the following years, critical thinking and informal logic started to be introduced in schools' curriculum and the preoccupation how to teach critical thinking raised considerably among educators. Critical thinking has started to be seen as crucial even from the early stages of schools. Matthew Lipman has put the basis of the "Philosophy for Children" program where he included exercises and applications that stimulated children's thinking in primary school (Fisher 2019, 13).

However, the differences in classifying and teaching the abilities of critical thinking made Peter Facione run a research whose purpose was to find a consensus regarding what critical thinking is and how it must be taught in schools. Together with a team of 46 critical thinking experts, Facione published a common definition of what critical thinking is: "purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological, or contextual considerations upon which that judgment is based" (Facione 1990, 3). This definition is followed by a list of skills and sub-skills that are required to be taught in schools in order to form critical thinkers. These skills pass the border of just evaluating arguments. This research was a real fundament for critical thinking to be studied in schools and universities as a separate object. Nowadays, critical thinking cannot be missed in education curriculum, regardless the level of education.

2. The concept of critical thinking

The way we know critical thinking today as a research subject has its basis in the scientific work of the American philosopher John Dewey who, in his book *How we think* defines reflective thinking as the "active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends" (Dewey 1910, 6). In his studies concerning the human mind's mechanisms that determine the educational process, Dewey identifies many types of thinking: stream of consciousness, belief, imagination and, the most important, reflection. Even if reflection is the starting point for rational thinking, the philosopher does not deny the other types of thinking's importance and utility. For example, he considers that imagination is the key for finding innovative solutions during the reflective process (Rodgers 2002, 844). This will be the warrant for later debates between critical thinking and creative thinking.

John Dewey sees the process of reflective thinking as a scientific method: the experience, the interpretation of the experience, identifying the questions that rise from the experience, finding the possible explanations, transforming the explanations in hypotheses and, in the end, testing these hypotheses (Rodgers 2002, 856-861). We can see that there is a very systematic way of how critical thinking can be put into practice.

Edward Glaser developed 30 years later together with Goodwin Watson the very well-known critical thinking test, *Watson-Glaser Critical Thinking Appraisal.* His definition of critical thinking continues the vision of Dewey's focusing on three essential elements: knowing the methods of applying logic and reasoning, owning the skills required for applying these methods and an attitude of being open to analysing everything an individual encounters in his lifetime, in other words, to have a critical spirit (Fisher, 2019, 9). We can notice here the focus on the individual to be willing to question the information he encounters and to be aware about the limits of his knowledge. That is to say critical thinking does not seem to be for everyone.

Seeing how the interest in teaching critical thinking raised, Robert Ennis proposes a model which analyses the critical thinking's components, a contribution that is worth being mentioned in our research. Ennis makes the distinction between dispositions and skills. Concerning the disposition to critical thinking, Ennis continues the idea of Dewey and Glaser, but offers a specific list of modalities of how an individual can show this disposition. For example, always trying to be informed, using credible sources and mentioning these sources, considering everything around in a thoughtful way, taking into consideration the other's opinion in a serious manner and also being careful concerning the other's feelings, level of knowledge and level of intelligence. The other component of critical thinking is represented by a very laborious list of skills and sub-skills Ennis proposes which focuses on good reasoning abilities as elementary clarification, basic support, inference, advanced clarification, strategies and tactics (Ennis 1989, 4). We can see that the main part of critical thinking puts emphasis on the informal logic and argumentation abilities, but also starts to approach other psychosomatic components and does not ignore the feelings.

Nowadays, critical thinking is approached in a broader manner and does not limit itself to logic reasoning skills. The most recent books in critical thinking present subjects as: argumentation, logic reasoning, fallacies, psycho-cognitive biases, skills in evaluating numbers and having the sense of proportions, the use of language, understanding of the world, having interdisciplinary knowledge, moral, legal an aesthetic reasoning. We can see that the meaning of critical thinking extended comparing to the first definitions of the concept, but at the same time, its content does not seem to be very new. Even if numerous components of critical thinking are more specific today, we can find in a more refined way the principals of education that we encounter in Plato and Aristotle's work.

3. Critical thinking and the return to the ancient philosophy

Concerning the main object of analysis in critical thinking – formal and informal logic – we have to pay tribute to Plato and Aristotle. Plato showed a preoccupation for logic and dialectics in his famous *Dialogues*. He also recommended that dialectics had to be thought after 20 years, in *Akademia*. Plato approached some basic principles of logic as the law of non-contradiction or law of excluded middle, but the one that defined logic in a more systematic way was Aristotle. Aristotle developed the *syllogism*, a method to formalize a reasoning, to state new arguments and to appraise arguments based on formal validity (Thayer-Bacon 2000, 23). However, critical thinking was born from the need of better appliance of logic to reality. Formal logic is essential, but there are contexts when the form is not enough to appraise an argument and in this way informal logic and critical thinking started to be shaped.

Both Plato and Aristotle put emphasis on moral education and they considered the virtue the main guide in life. In modern critical thinking's courses, the moral aspect is present in both indirect and direct ways. Concerning the indirect ways, we can state that there is a moral cloud upon all critical thinking lifestyle. The main moral principle is to not mislead the others. The most popular way of approaching this is through analysis of the fallacies in reasoning. The differences between approaching fallacies in nowadays critical thinking and ancient logic is that logic focuses on formal fallacies as affirming the consequent and denying the antecedent, whereas critical thinking approaches a various list of fallacies which is still updating today. Beside classic non-formal errors in reasoning as *ad personam* or *ignoratio elenchi*, critical thinking analyses the most common fallacies of daily life. For example, there are fallacies for almost every emotion such as hope appeal or envy appeal (Dobre 2014, 137-138). In most of the critical thinking books, the fallacies are not presented just to be careful of the other's reasoning and to sanction the faulty reasoning. They are presented in order to make aware and, most importantly, not to employ them against the others. Knowing the fallacies does not give us the right to use them in order to manipulate the ones standing in front of us or in any other immoral way.

In strong connection with the idea of not misleading, there is another frequently approached aspect in the theory of critical thinking: the use of credible sources. The sources we get our information from must be reliable, credible, and objective. Most of critical thinking literature approaches the next problems concerning the sources: the knowledge, the expertise of the source, the objectivity, the accuracy and the impartiality of the source (Moore and Parker 2012, 118). Concerning the expertise of the source, things are not new since the argument of authority is precisely analysed in rhetoric. The new aspect in critical thinking could be found in the substantial approach of analysing media and internet sources since the fake news phenomena has a great impact today. In this sense, it was raised the problem of a digital literacy that became necessary for every individual.

Concerning the direct presence of moral dimension, the latest critical thinking books have separate chapters on moral reasoning. As a short outline, critical thinking literature focuses on value judgments and the difference between moral and amoral value judgments. They usually put emphasis on the two principles of moral reasoning: “If separate cases aren’t different in any relevant way, then they should be treated the same way, and if separate cases are treated the same way, they should not be different in any relevant way”, which can be called the consistency principle; “If someone appears to be violating the consistency principle, then the burden of proof is on that person to show that he or she is in fact not violating the principle” (Moore and Parker 2012, 441-442). There are also presented the major perspectives in moral reasoning starting, of course, from the virtue ethics of Aristotle.

Regarding the content taught in his school, Aristotle promoted many subjects as philosophy, grammar, gymnastics, nature sciences or music. He believed in interdisciplinary approach and valued especially the natural sciences. What can we extract from here concerning critical thinking? Critical thinking is not limited to a single domain. Critical thinking involves a highly complex theory that can also be seen as useful a tool to other areas. That is why today critical thinking has gained independence and became a core subject of teaching. For example, in the medical field, critical thinking became a necessary tool for an efficient diagnosing activity (Papp et al. 2014, 715).

Critical thinking requires both easy and complex skills whose appliance is made difficult if the individual does not posses some general knowledge or some specific knowledge in the area. The interdisciplinary aspect is seen as necessary for a critical thinker since he must notice if something is inaccurate in the information he encounters. One must pay attention both to the formal part of reasoning, but also to the content of the arguments.

In today's critical thinking we can find the concept of creative thinking as being in close kinship with Aristotle's thought. Even though at the first sight creative thinking might seem opposed to critical thinking, the latest researches proved their inseparability. Creativity implies the process of conception, while critical thinking assesses and evaluates what was already produced. Even if we talk about the most pragmatic outputs belonging to a most rational mind or the imaginative thoughts of an artist, the creative and the critical part work together. Anthony Weston states that throughout creativity the world of possibility is opened before our eyes and starting from this world of possibility, new ideas arise, new solutions to problems appear. With critical thinking these solutions are assessed and reasoned (Weston 2006, 10-11). Aristotle recommends activities like music studies or any kind of art in order to enhance our creativity. The same activities prove to augment the critical thinking abilities.

Plato and Aristotle were thinking that it is enough for an individual to rely on his mind capabilities, especially on logical reasoning, to discover knowledge and did not believe in the necessity of teamwork (Thayer-Bacon 2000, 26). Today's critical thinker is not exactly a loner. Of course, most of the processes imply working on your own. Authors like Richard Paul whose research in critical thinking relies especially on the assessing of our own thoughts have developed the theory of metacognition (Fisher 2019, 13). But then again some authors like J. Thayer-Bacon propose a new model of critical thinking where empathy has a crucial role in understanding ideas. They believe that critical thinking can be developed by working together and finding solutions by constant intellectual interactions (Thayer-Bacon 1993, 323).

In modern critical thinking, the authors continuously repeated that a critical thinker needs a natural disposition of analysing in a thoughtful way what he encounters on a daily basis, that the individual must be aware of its mind and aware of the things outside the mind. John Dewey uses in its comparisons rather a strong word for non-critical thinkers calling them "savages". Currently, the contributions to critical thinking assume the belief that it can answer to a general necessity of our time. The problem of the growing irrationality of our societies (some authors as Daniel Kahneman and Dan Ariely believe that we are more irrational than rational) received an in-depth exploration in critical thinking, particularly from the psycho-cognitive point of view concerning biases.

With the recent necessity of introducing critical thinking in curriculum we observed that the basic elements of critical thinking have abundant affinities with the principles of education evoked by Plato and Aristotle. The ancient learning of grammar (in a higher levels) developed the proto-skills of today's critical approach to language; the studying of dialectics is echoed in today's logic and informal argumentation within critical thinking; the principles of teaching that merged the transmission of moral values with the intellectual skills are still assumed in the practice of critical thinking. The interdisciplinary is also promoted by critical thinking in many forms. Of course there are some differences concerning how a critical thinking is seen today and if critical thinking is for everyone. There are also some subjects that cannot be directly tied to critical thinking, for example gymnastics. Of course, in an indirect way, a critical thinker should be aware of his mind, as well of his body. In the multidisciplinary application of critical thinking there are also plenty of studies that prove its importance to other sports. Plenty of the educational principles formulated by the ancient Greek philosophers still have a significant application in today's world and, despite the fact that there were times when they seemed to be forgotten, nowadays there is a significant tendency to recover them in the studies and practice of critical thinking.

REFERENCES

Dewey, John. 1910. *How we think*. Boston: D.C. Heat & Co.

Dobre, Marius. 2014. „Teoria sofismelor în abordări contemporane de gândire critică". *Revista de Filosofie* LXI (2): 133-144.

Ennis, Robert H. 1989. "Critical Thinking and Subject Specificity: Clarification and Needed Research". *Educational Researcher* 18 (3): 45-48.

Facione, Peter. 1990. *Critical Thinking: A statement of expert consensus for purposes of educational assessment and instruction*. Newark DE: American Philosophical Association.

Fisher, Alec. 2019. "What Critical Thinking Is". In *Studies in Critical Thinking*, ed. by J. Anthony Blair, Vol. 8 of *Windsor Studies in Argumentation*, 7-32.

Moore, Brooke Noel and Richard Parker. 2012. *Critical Thinking*. Tenth Edition. New York: Mc Graw Hill.

Papp, Klara K, et al. 2014. "Milestones of Critical Thinking. A Developmental Model for Medicine and Nursing". *Academic Medicine, Journal of the Association of American Medical Colleges* 89 (5): 715-720.

Rodgers, Carol. 2002. "Defining Reflection: Anther Look at John Dewey and Reflective Thinking". *Teachers College Record* 104 (4): 842-866.

Thayer-Bacon, Barbara J. 1993. "Caring and its Relationship to Critical Thinking". *Educational Theory* 43 (3): 323-340.

Thayer-Bacon, Barbara J. 2000. *Transforming Critical Thinking: Thinking Constructively*. New York: Teacher's College Press.

Weston, Anthony. 2006. *Creativity for Critical Thinkers*. New York: Oxford University Press.

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